Univerzitet u Beogradu – Građevinski fakultet
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Studijski program: **GRAĐEVINARSTVO**

Modul: **PŽA, HVE, MTI**

Godina/Semestar: **III godina / V semestar**

Naziv predmeta (šifra): **Betonske konstrukcije 1**

(B2S3BK, B2H3BK, B2M3BK, B1S3BK)

Nastavnik: **Jelena Dragaš**

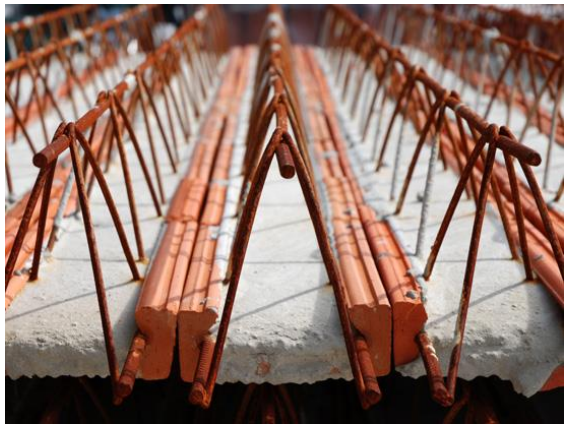
Naslov predavanja: **Polumontažne ploče, ploča stepeništa.**
Priprema za ispit.

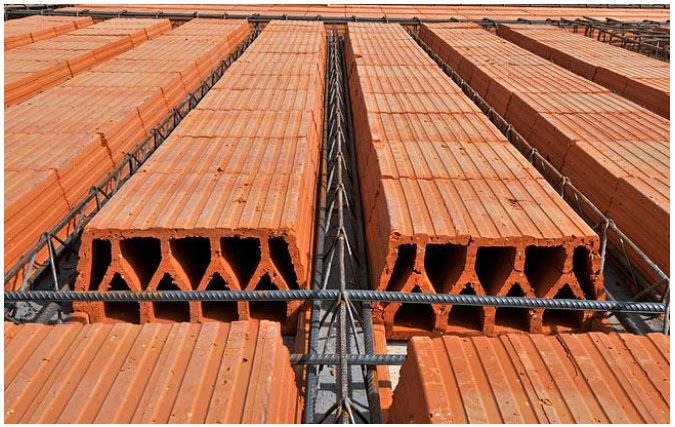
Datum : 12.01.2023.

Beograd, 2021.

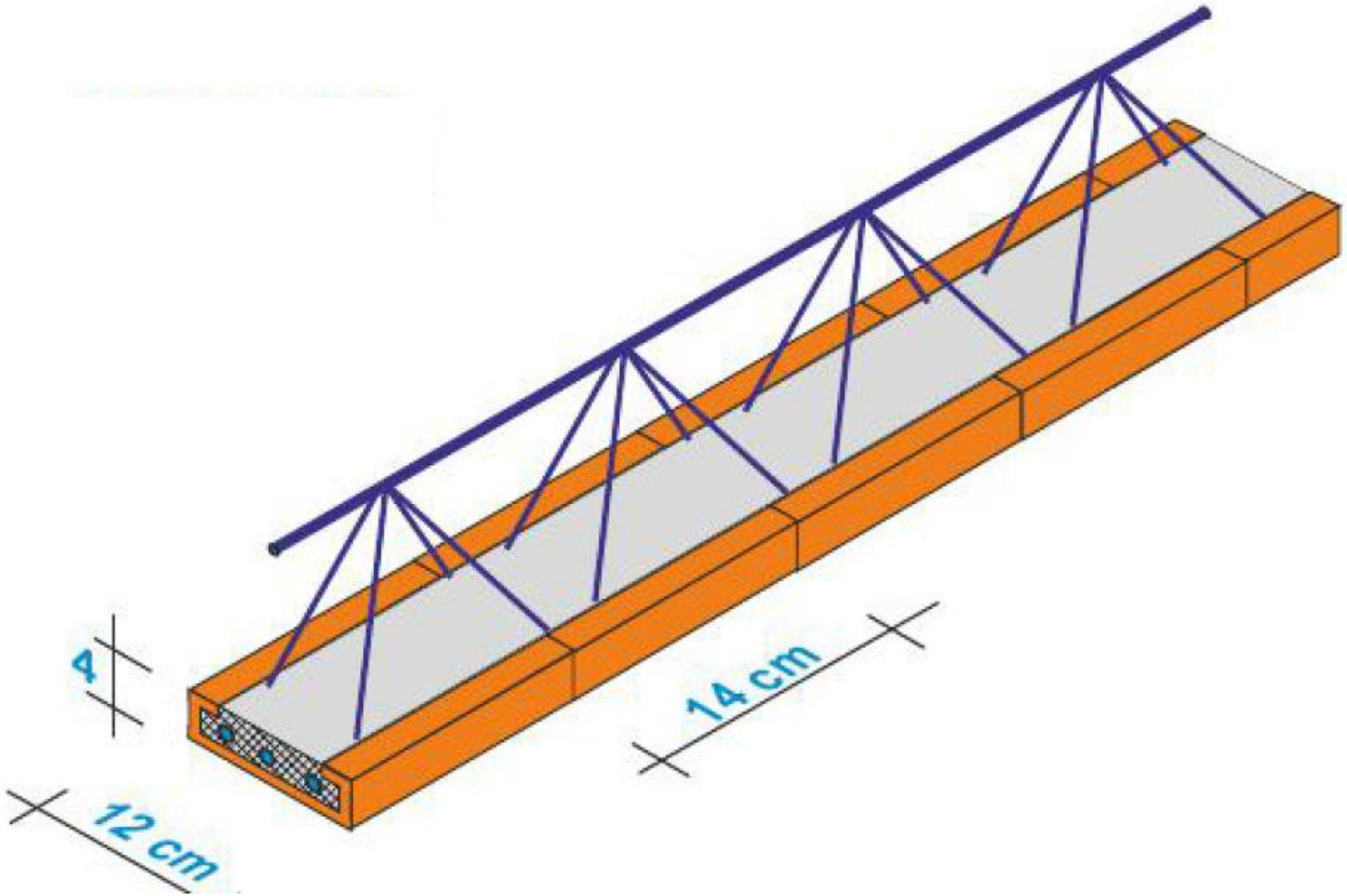
Polumontažne tavanice - LMT tavanice tipa FERT

- *Fert gredice se koriste za izradu lako-montažnih tavanica (LMT) - međuspratnih konstrukcija.*
- *Fert gredice su noseći elementi tako da moraju zadovoljiti određene statičke uslove. Pored osnovne armature (**rešetkasti nosač BINOR sa 2Ø7 ili 2Ø8** u donjoj zoni), kod većih raspona ugrađuje se dodatna armatura prema statičkom proračunu.*
- *Međuspratne konstrukcije izrađene od fert gredica se primenjuju u stanogradnji gde je isključivo prisutno statičko opterećenje i samo za sisteme prostih greda.*





Fert gredica



Opekarski element ispune

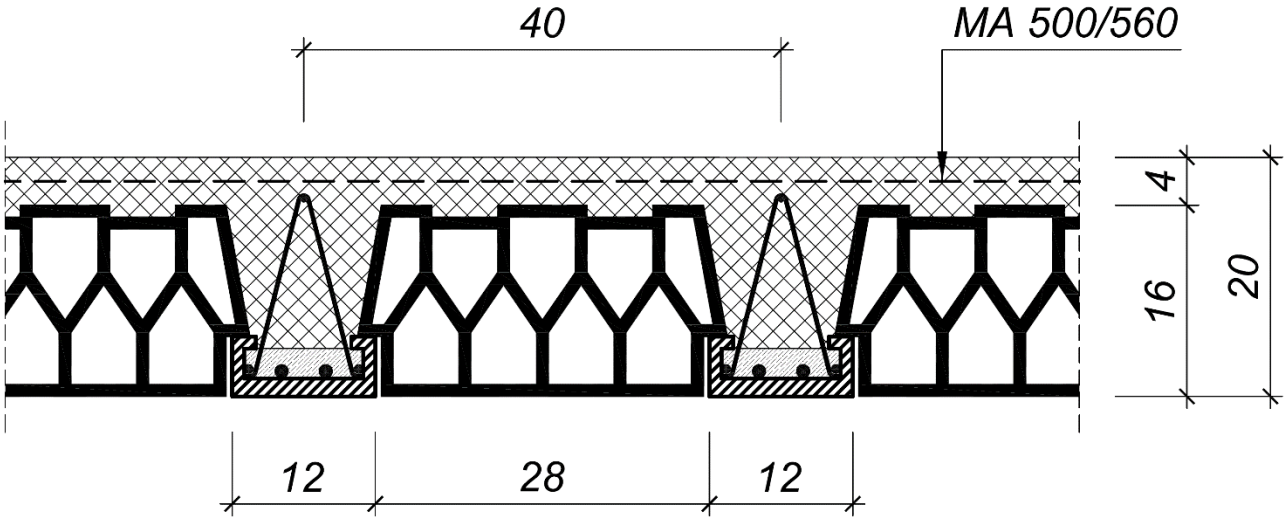
Dimenzije (cm): ***25×28×16(20)***

Masa: ***≈ 7.5-9.0 kg***

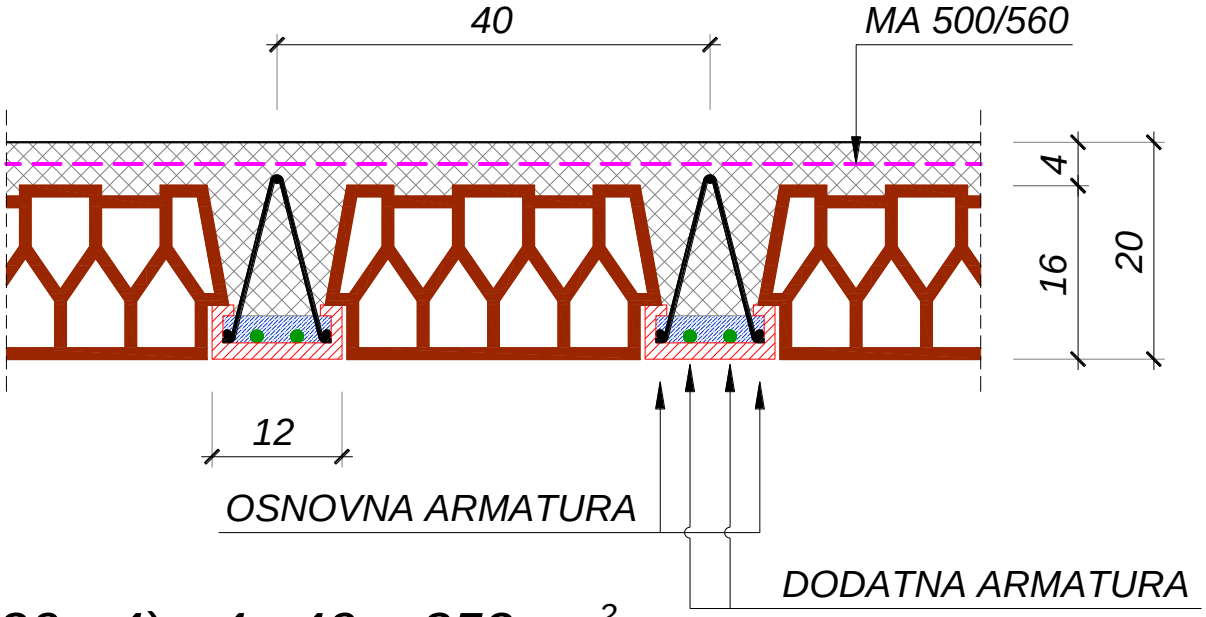
Potrošnja opeke:

$$n = \frac{1.0\,m^2}{0.40 \times 0.25} = 10\,\frac{kom.}{m^2}$$

Osovinski razmak FERT nosača je 40 cm.



Rešetkasti armaturni nosač BINOR



$$A_b = 12 \times (20 - 4) + 4 \times 40 = 352\,cm^2$$

$$\frac{A_b}{e} = \frac{352}{0.4} = 880\,\frac{cm^2}{m} \Rightarrow g_{SW}^{AB} = 880 \times 10^{-4} \times 25 = 2.2\,\frac{kN}{m^2}$$

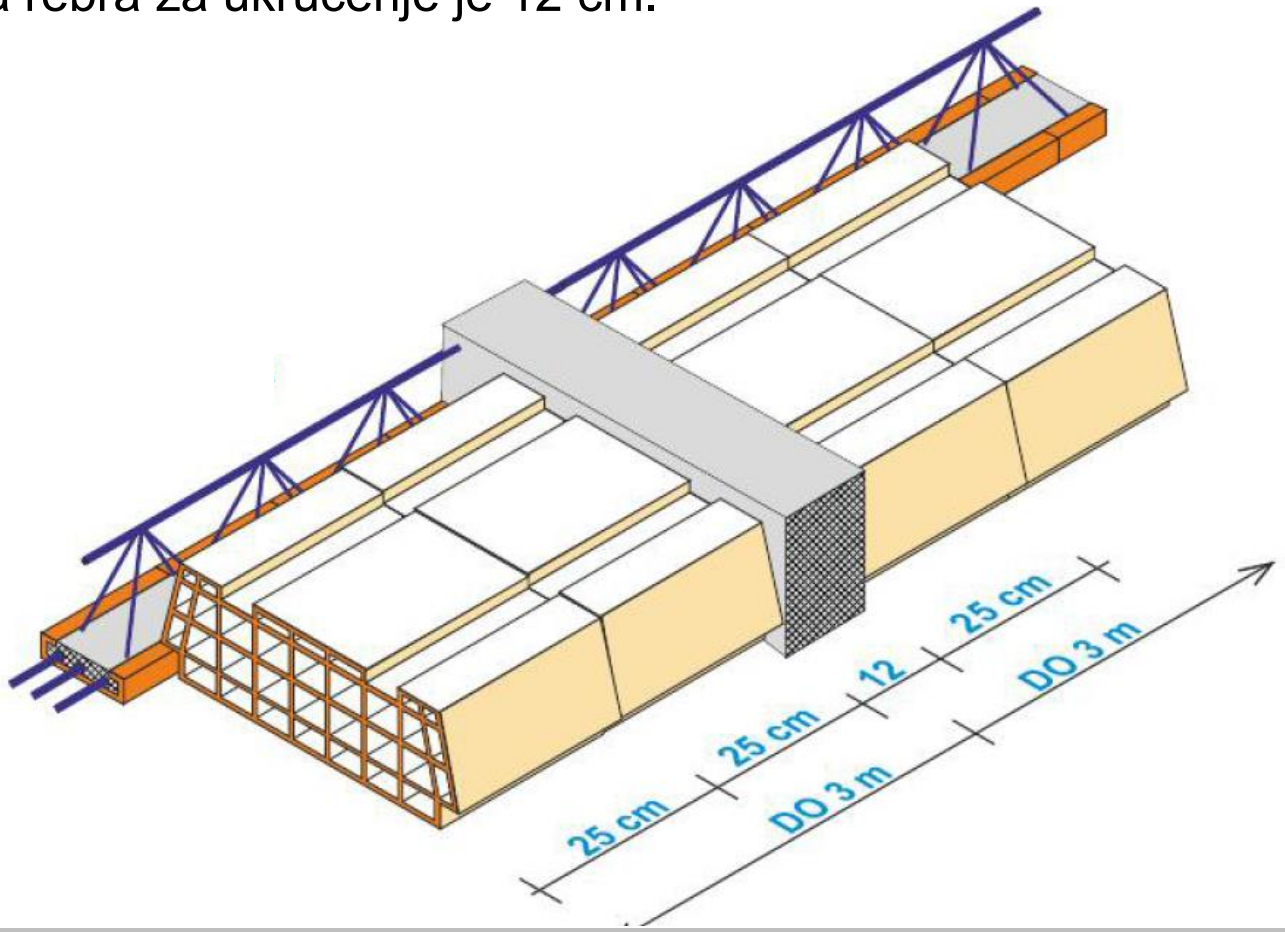
$$g_{SW}^{op.} = 10\,\frac{kom.}{m^2} \times 7.5\,kg = 0.75\,\frac{kN}{m^2}$$

$$g_{SW} = 2.2 + 0.75 = 2.95\,\frac{kN}{m^2}$$

$$g_{SW} \approx 3.0\,\frac{kN}{m^2}$$

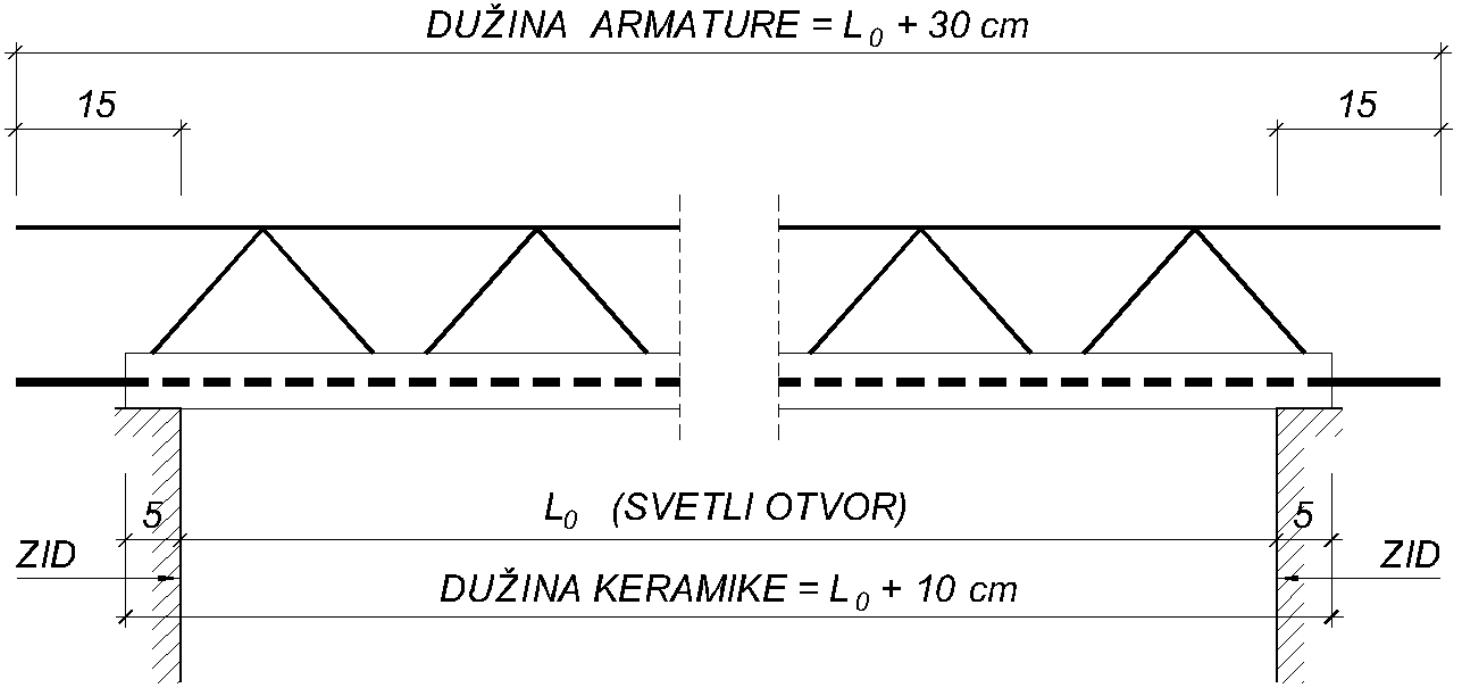
Redosled operacija kod montaže

- Kod raspona preko 4.0 m potrebno je formirati rebro za ukrućenje u sredini raspona. Za raspone do 6.7 m rebra za ukrućenje formirati u trećinama raspona, u visini međuspratne konstrukcije. Minimalna dimenzija rebra za ukrućenje je 12 cm.



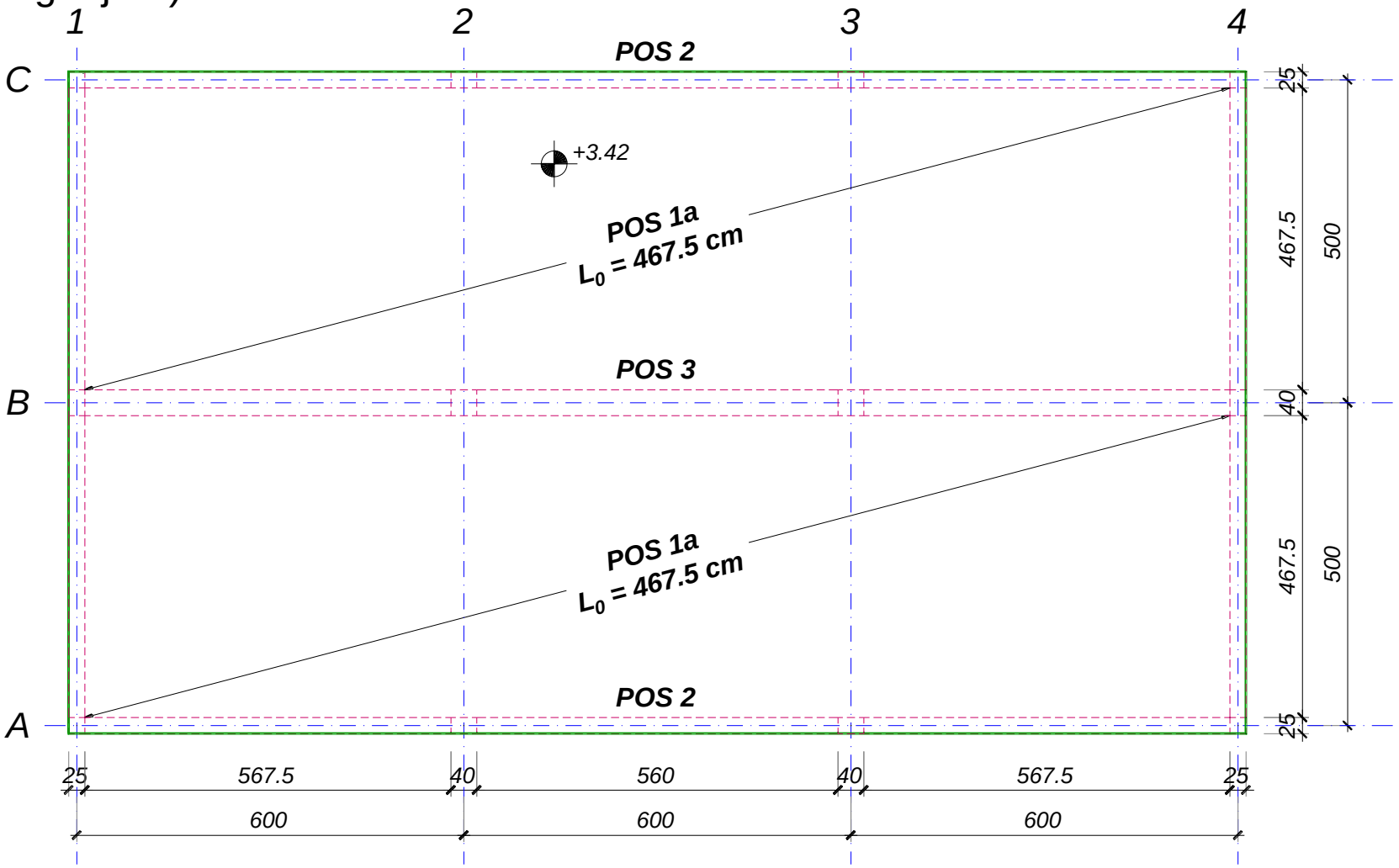
Redosled operacija kod montaže

- Keramički deo FERT gredice mora naleći na konstruktivne zidove minimalno 50 mm. Armatura gredice mora biti upletena u armaturu serklaža nosećih zidova
- Betoniranje rebara i ploče debljine 4 (5) cm obaviti istovremeno. Ukupna debljina MK treba da bude 20 (25) cm.



Zadatak – LMT fert

Dimenzionisati polumontažnu ploču prikazanu na skici ispod. Težina podova i pregradnih zidova je 2.5 kN/m^2. Korisno opterećenje za stambene objekte (kategorija A).



Zadatak – LMT fert

1. Analiza opterećenja

- Stalno opterećenje

$g = g_{st} + \Delta g = 3.0 + 2.5 = 5.5\ kN/m^2$

- Korisno opterećenje

$q = 2.0\ kN/m^2$ (kategorija A)

- Granično opterećenje

$p_{Ed} = 1.35 \cdot 5.5 + 1.5 \cdot 2.0 = 10.425\ kN/m^2$

2. Statički uticaji

- Statički sistem: prosta greda raspona 5.0 m
- Maksimalni moment:

$M_{Ed,max} = 10.43 \cdot 5.0 \cdot 5.0/8 = 32.58\ kNm$

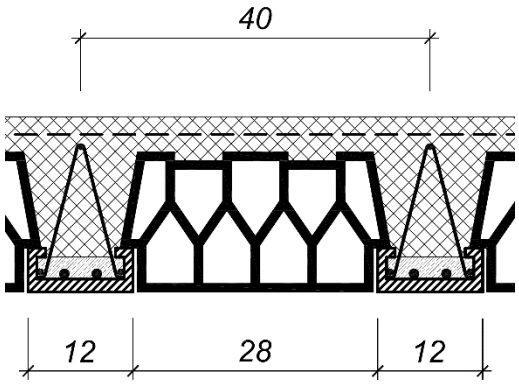
Zadatak – LMT fert

3. Dimenzionisanje

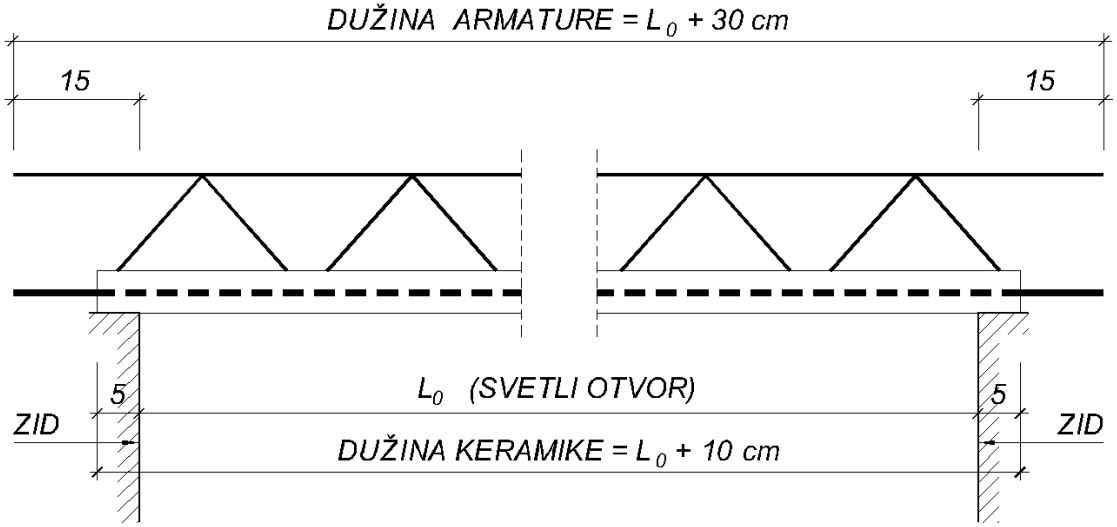
- d_1=3 cm d= 20 – 3 = 17 cm
- C25/30, f_{cd} = 0.85 · 25 / 1.5= 14.2 MPa = 1.42 kN/cm^2
- B500 B, f_{yd} = 500 / 1.15 = 435 MPa = 43.5 kN/cm^2

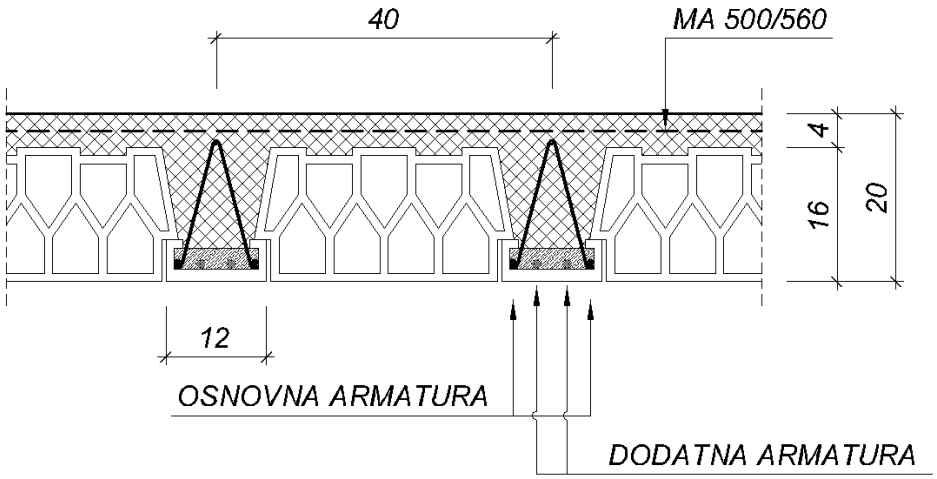
$$k = \frac{d}{\sqrt{\frac{M_{Ed}}{b_{eff} \cdot f_{cd}}}} = \frac{17}{\sqrt{\frac{3258}{100 \cdot 1.42}}} = 3.55$$

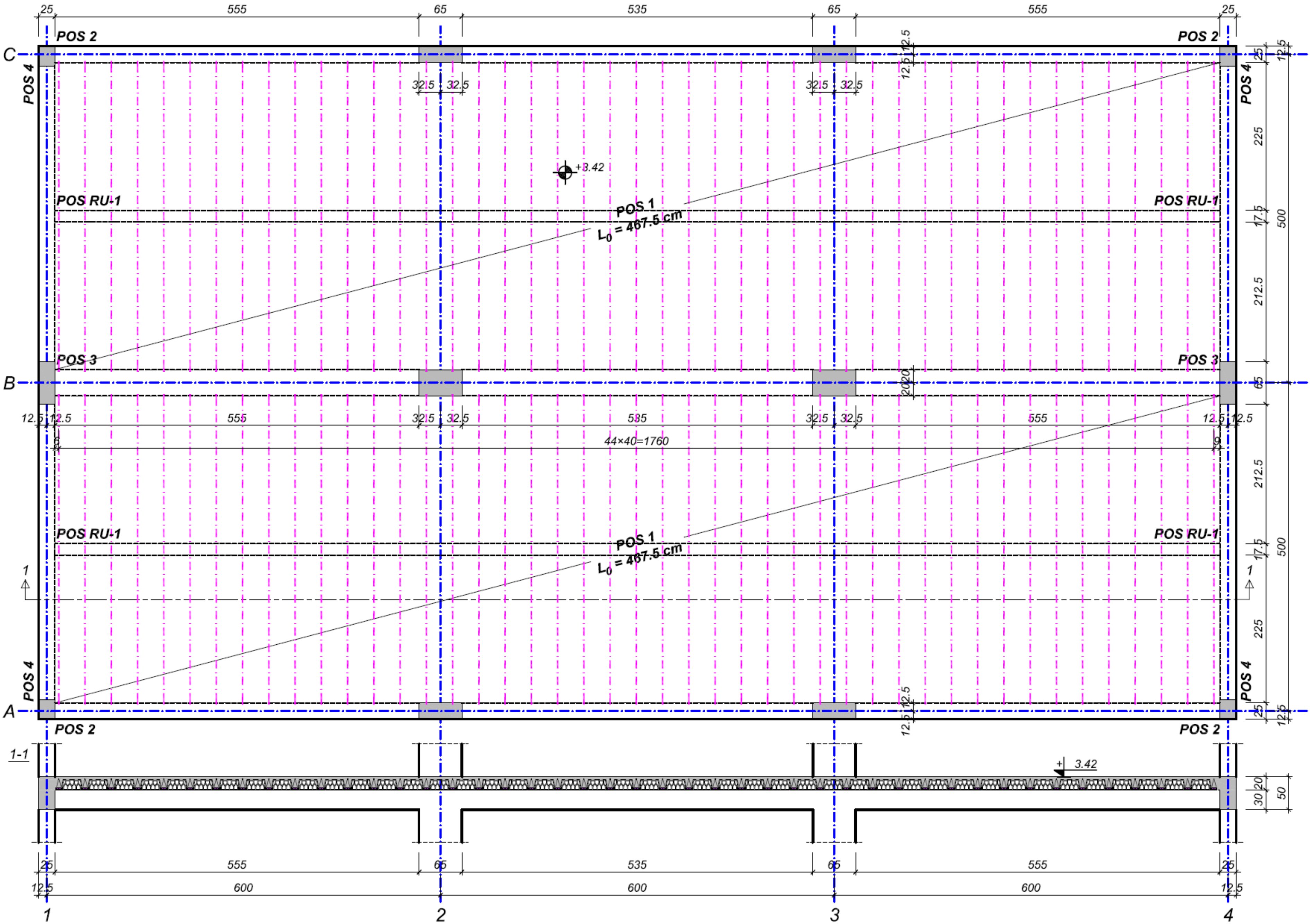
$$A_{s1} = 8.458 \times 17 \times \frac{1.42}{43.5} = 4.70\, cm^2 / m$$



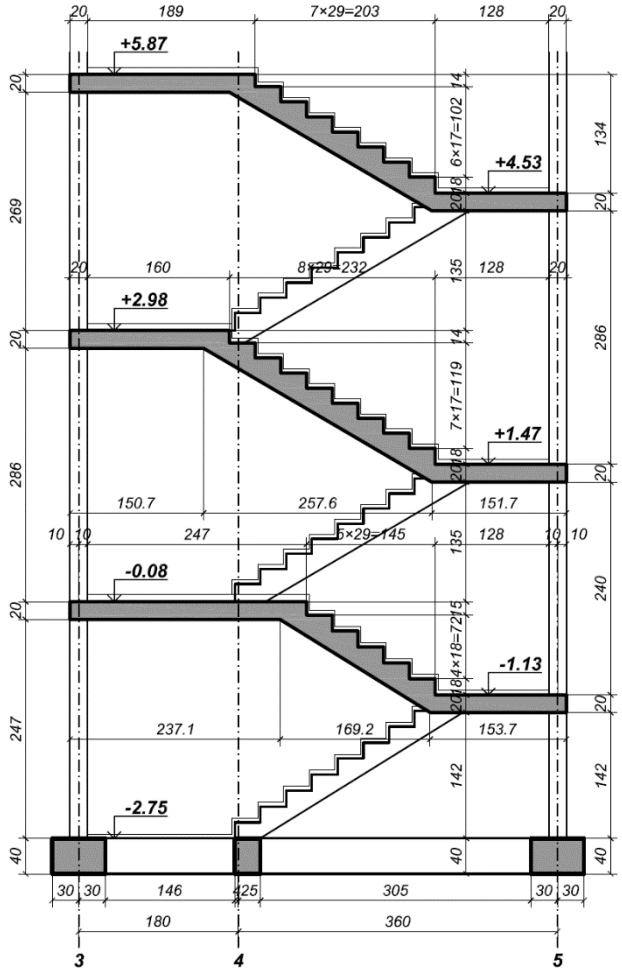
- *Za jedno rebro: A_{s1} = 0.40×4.7 = 1.88 cm^2*
- *Ugrađena armatura rešetke: **2Ø8** = 1.00 cm^2*
- *Potrebna dodatna armatura: ΔA_s =1.88 – 1.0 = 0.88 cm^2*
- *Usvojeno **2Ø8** (1.0 cm^2)*

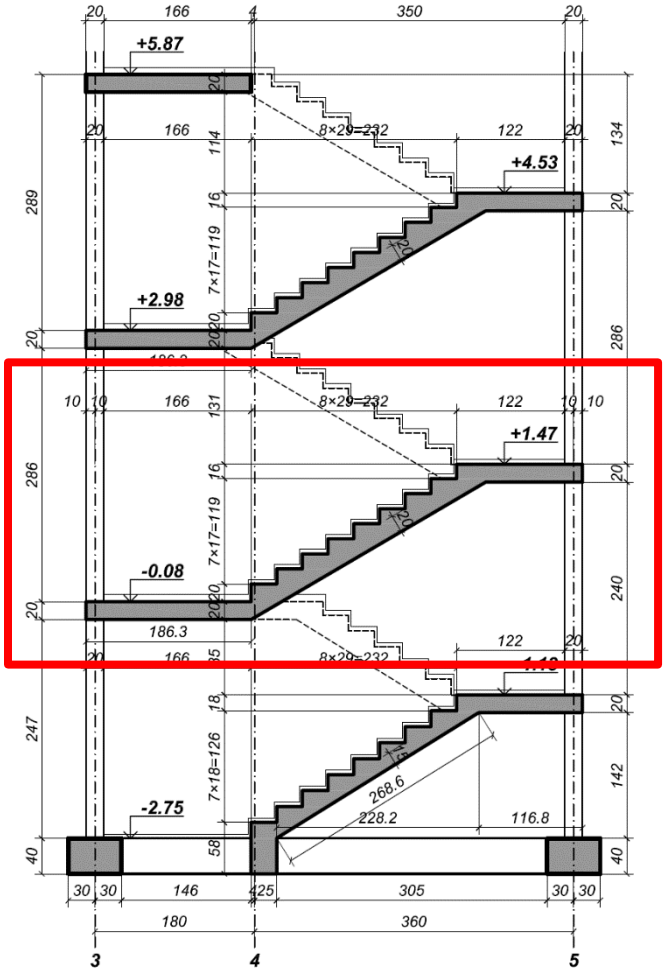

DUŽINA ARMATURE = L_0 + 30 cm
15
15
5
L_0 (SVETLI OTVOR)
5
ZID
DUŽINA KERAMIKE = L_0 + 10 cm
ZID

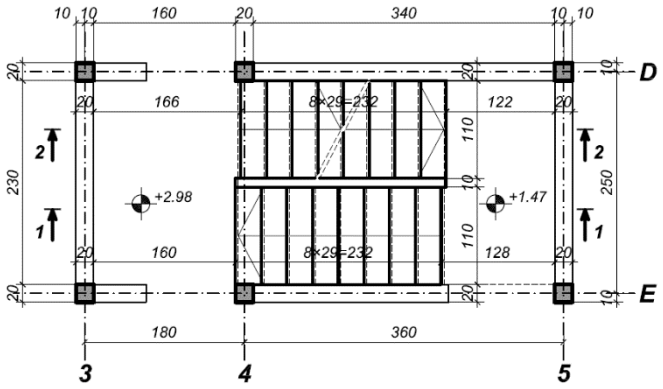

40
MA 500/560
4
16
20
12
OSNOVNA ARMATURA
DODATNA ARMATURA

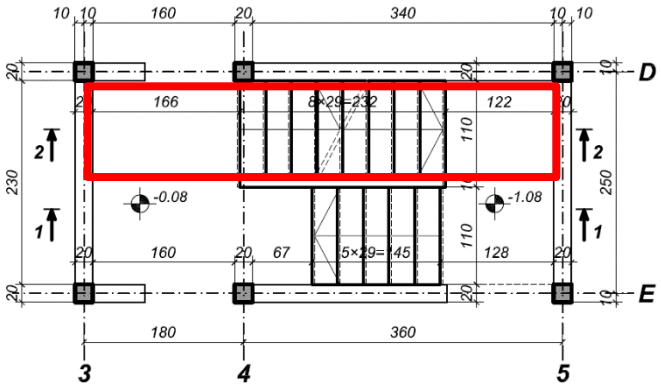


POS 2
POS 4
POS RU-1
POS 1
L_0 = 467.5 cm
POS 3
+3.42
44×40=1760
1-1
3.42
A
B
C
1
2
3
4
25
555
65
535
600
12.5
32.5
225
212.5
17.5
500
20
30
50


+5.87
+4.53
+2.98
+1.47
-0.08
-1.13
-2.75
6×17=102
8×29=232
7×17=119
4×18=72
5×29=145
150.7
257.6
151.7
237.1
169.2
153.7
3
4
5


+5.87
+4.53
+2.98
+1.47
-0.08
-1.13
-2.75
8×29=232
7×17=119
7×18=126
186.3
268.6
228.2
116.8
3
4
5

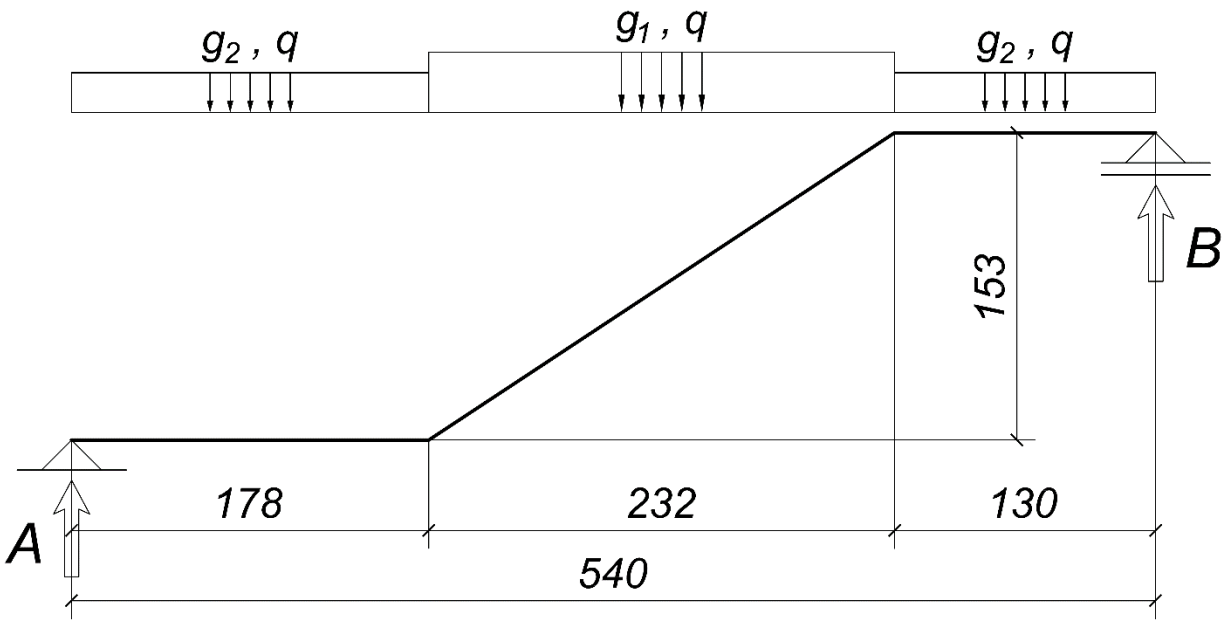

D
E
1
2
+2.98
+1.47
8×29=232
3
4
5

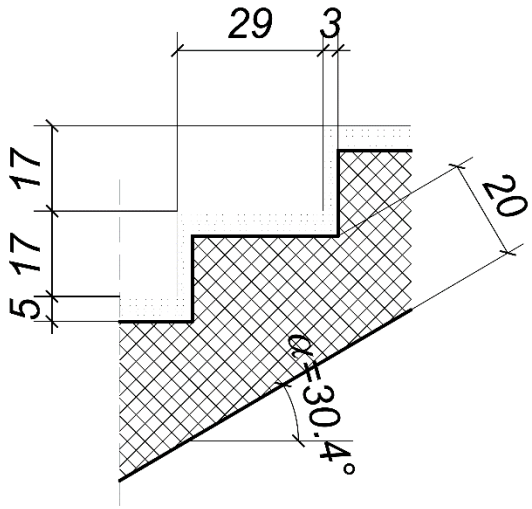

D
E
1
2
-0.08
-1.08
8×29=232
5×29=145
3
4
5

Ploča stepeništa

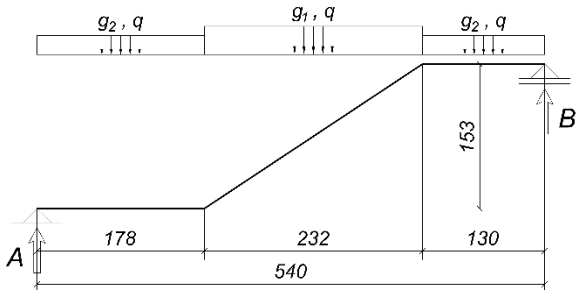
Dimenzionisati ploču dvokrakog stepeništa prikazanog na skici ispod.

- *Stepenište je statičkog sistema kolenaste ploče, debljine h_{pl} = 20 cm. Stepenici su dimenzija 17/29 cm. Debljina vertikalne obloge stepenika je 3 cm, a horizontalne 5 cm.*
- *Dodatno stalno opterećenje usvojiti na osnovu debljina obloga (γ=25 kN/m^3)*
- *Težinu poda i plafona na horizontalnom delu usvojiti kao 1.5 kN/m^2*
- *Korisno opterećenje usvojiti prema standardu EN1992-1-1*





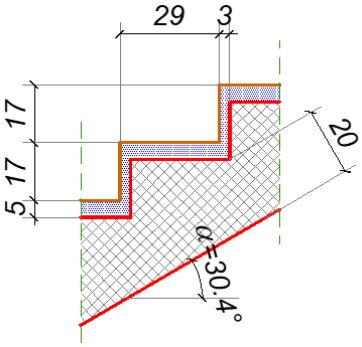
Analiza opterećenja



Kos deo:

- *stalno opterećenje*

težina ploče	$0.20 \times 25.0/\cos 30.4^\circ$	$= 5.80$ kN/m²
težina stepenika	$0.5 \times 0.17 \times 24.0$	$= 2.04$ kN/m²
horizontalna obloga	0.05×25	$= 1.25$ kN/m²
vertikalna obloga	$17/29 \times 0.03 \times 25$	$= 0.44$ kN/m²
ukupno:		**$g_1 = 9.53$ kN/m²**



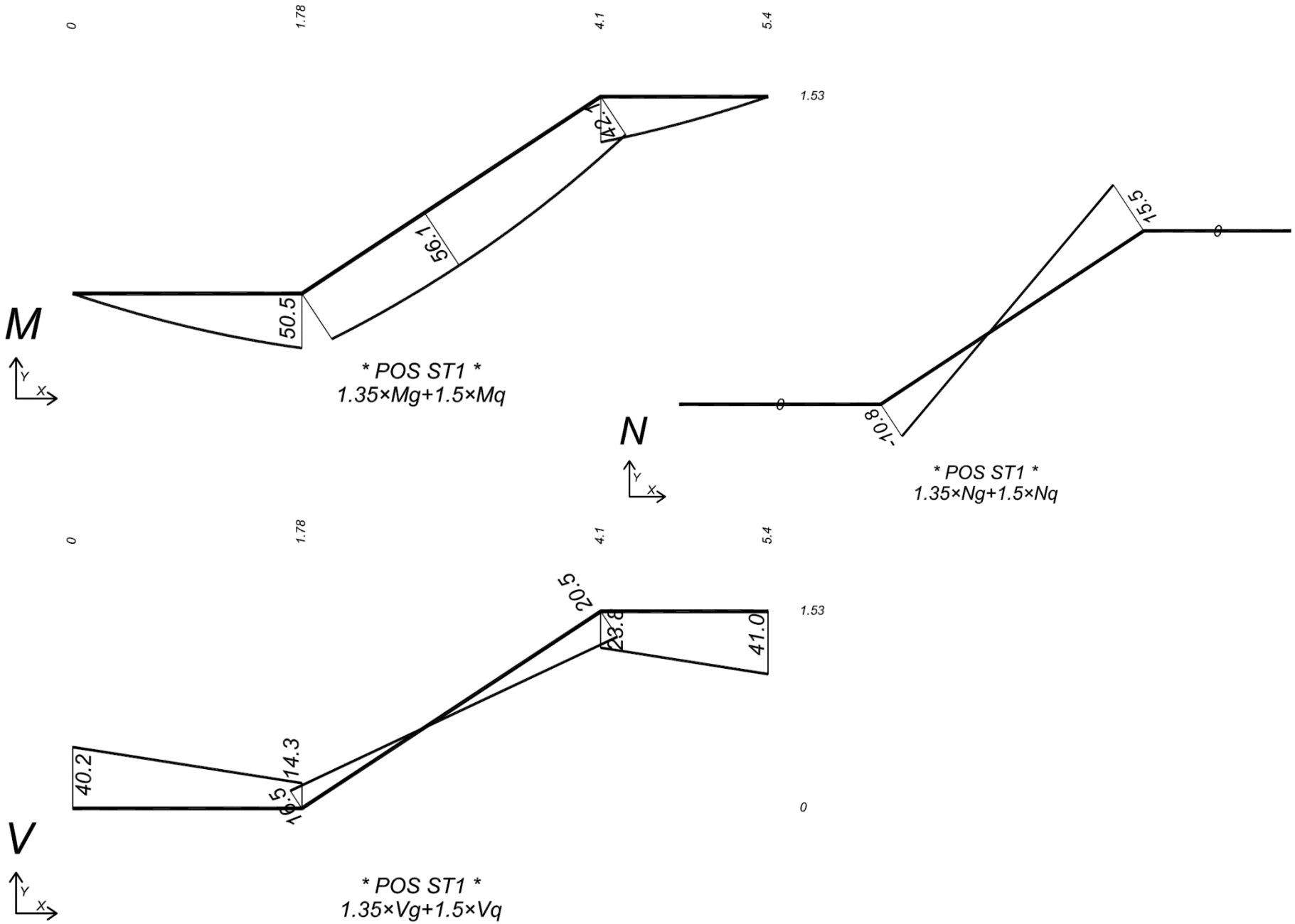
- *povremeno opterećenje* **$q = 3.00$ kN/m²**

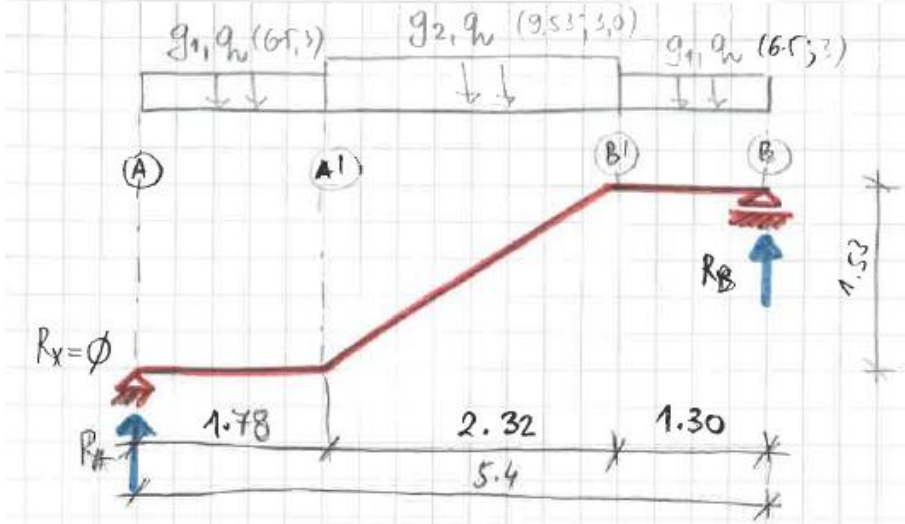
Horizontalni delovi:

- *stalno opterećenje*

težina ploče	0.20×25.0	$= 5.00$ kN/m²
pod, plafon		$= 1.50$ kN/m²
ukupno:		**$g_2 = 6.50$ kN/m²**

- *povremeno opterećenje* **$q = 3.00$ kN/m²**


0
1.78
4.1
5.4
1.53
M
50.5
66.1
42.2
Y
X
* POS ST1 *
1.35×Mg+1.5×Mq
N
0
10.8
15.5
0
* POS ST1 *
1.35×Ng+1.5×Nq
V
40.2
16.5
14.3
20.5
23.8
41.0
0
* POS ST1 *
1.35×Vg+1.5×Vq



$g_1 = 9.53\ kN/m^2$

$g_2 = 6.5\ kN/m^2$

$q_h = 3.0\ kN/m^2$

$\alpha = 30.4°$

$\cos\alpha = 0.8625 \Rightarrow V$

$\sin\alpha = 0.506 \Rightarrow N$

① STALNO OPTEREĆENJE

$\Sigma M_A^{\curvearrowright +} = 0: \quad 6.5 \cdot 1.78^2/2 + 9.53 \cdot 2.32 \cdot (2.32/2 + 1.78) + 6.5 \cdot 1.3 \cdot (5.4 - 1.3/2) = R_B^g \cdot 5.4$

$$R_B^g = 21.38\ kN/m'$$

$\Sigma Y = 0: \quad R_A^g + 21.38 = 6.5 \cdot (1.78 + 1.3) + 9.53 \cdot 2.32$

$$R_A^g = 20.75\ kN/m'$$

A1 $M_{A1}^g = 20.75 \cdot 1.78 - 6.5 \cdot 1.78^2/2 = 26.64\ kNm/m'$

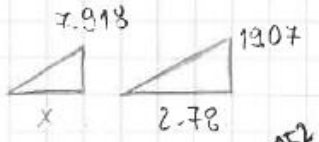
$V_{A1}^g = 20.75 - 6.5 \cdot 1.78 = 9.18\ kN/m'$

B $M_B^g = 21.38 \cdot 1.3 - 6.5 \cdot 1.3^2/2 = 22.30\ kNm/m'$

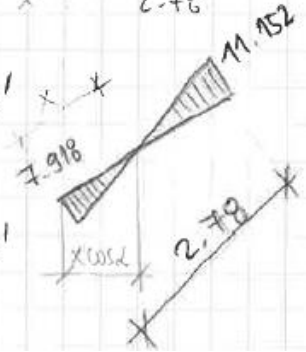
$V_B^g = 21.38 - 6.5 \cdot 1.3 = 12.93\ kN/m$

$x \cdot \cos\alpha = 0.9455\ m$

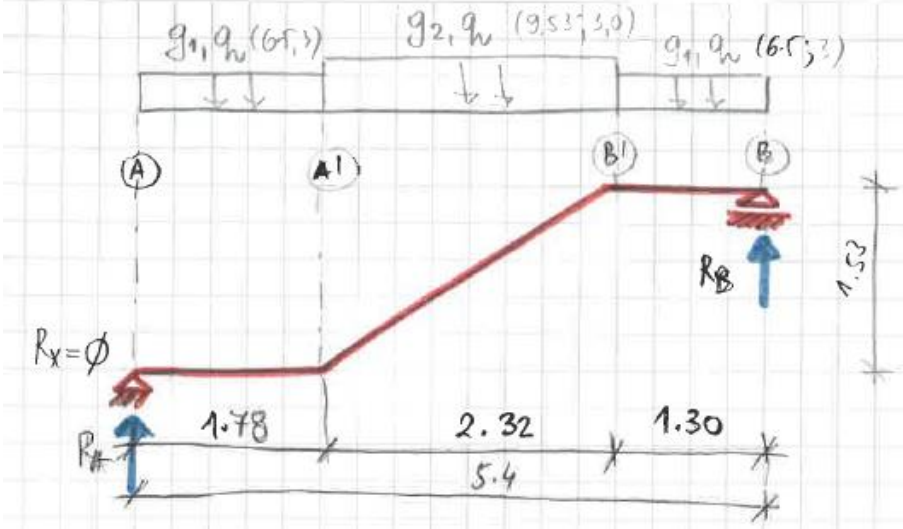
$x = 1.154\ m$



$V_{A1}^g \cdot \cos\alpha = 7.918\ kN/m'$



$V_B^g \cdot \cos\alpha = 11.152\ kN/m'$



$g_1 = 9.53\ kN/m^2$

$g_2 = 6.5\ kN/m^2$

$q_k = 3.0\ kN/m^2$

$\alpha = 30.4°$

$\cos\alpha = 0.8625 \Rightarrow V$

$\sin\alpha = 0.506 \Rightarrow N$

② KORISNO OPTEREĆENJE

$\Sigma M_A^{\curvearrowright +} = 0 \quad 3 \cdot 5.4^2/2 = R_B^{q_k} \cdot 5.4$

$$R_B^{q_k} = 8.1\ kN/m'$$

$\Sigma Y = 0;$

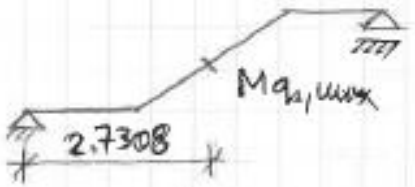
$$R_A^{q_k} = 8.1\ kN/m'$$

(A') $M_{A'}^{q_k} = 8.1 \cdot 1.78 - 3 \cdot 1.78^2/2 = 9.665\ kNm/m'$

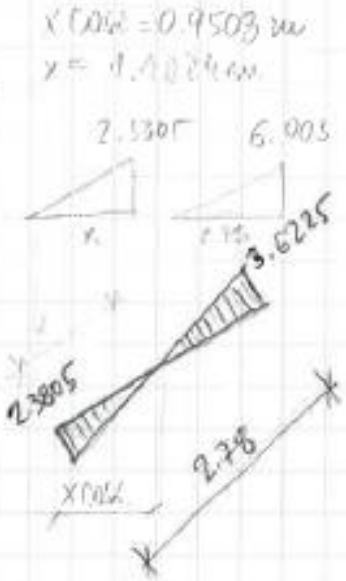
$V_{A'}^{q_k} = 8.1 - 1.78 \cdot 3 = 2.76\ kN/m'$

$M_{B'}^{q_k} = 8.1 \cdot 1.3 - 3 \cdot 1.3^2/2 = 7.995\ kNm/m'$

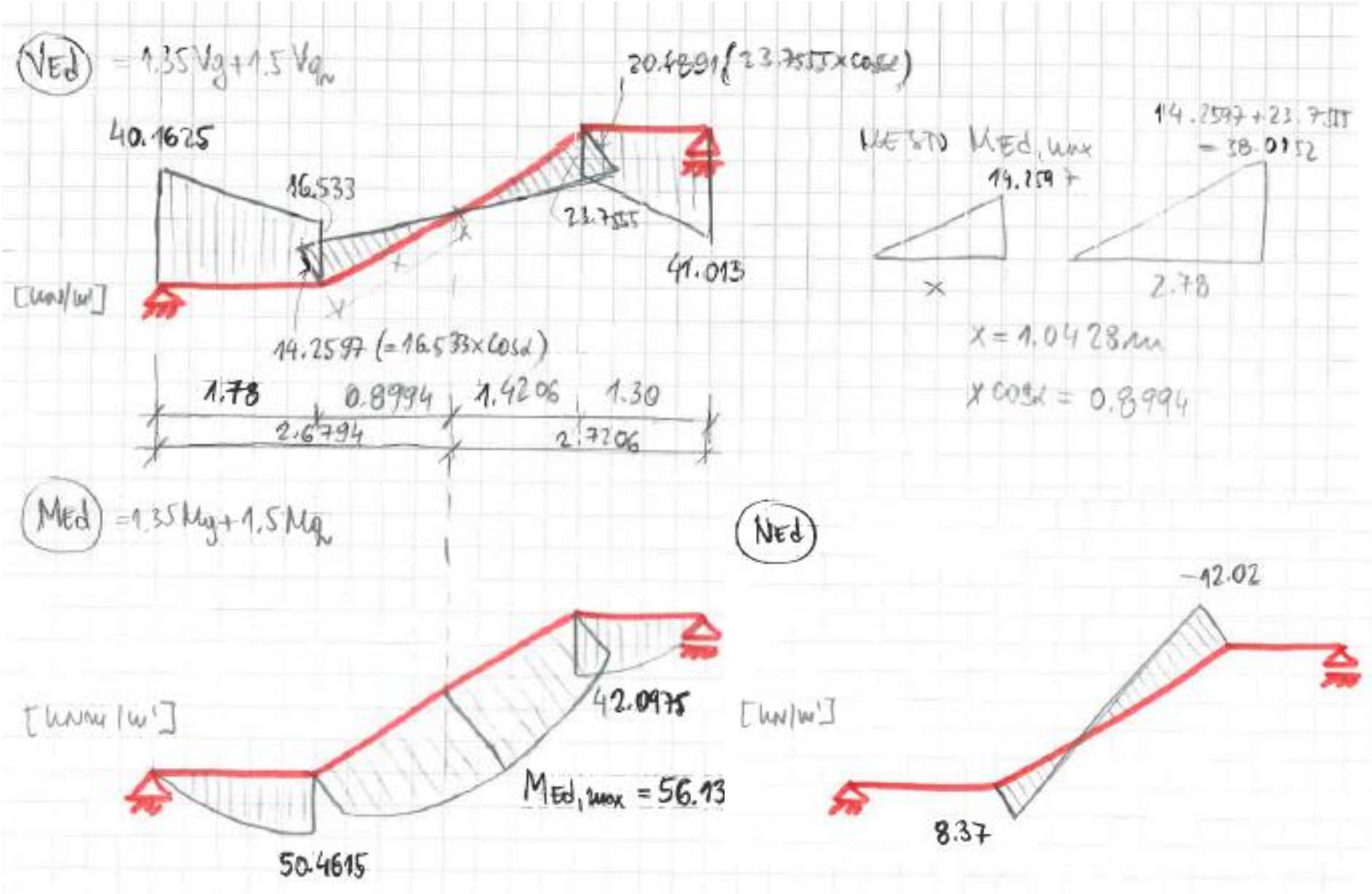
$V_{B'}^{q_k} = 8.1 - 1.3 \cdot 3 = 4.2\ kN/m'$



$V_{A'}^{q_k} \cdot \cos\alpha = 2.3805\ kN/m'$



$V_{B'}^{q_k} \cos\alpha = 3.6225\ kN/m'$


VEd = 1.35 Vg + 1.5 Vq
40.1625
16.533
20.4891 (23.7555 x cosα)
23.7555
41.013
[kN/m']
14.2597 (= 16.533 x cosα)
1.78
0.8994
1.4206
1.30
2.6794
2.7206
MEd = 1.35 Mg + 1.5 Mq
[kNm/m']
42.0975
MEd,max = 56.13
50.4615
NEd
[kN/m']
-12.02
8.37
MESTO MEd,max
14.259 +
14.2597 + 23.7555 = 38.0152
x
2.78
x = 1.0428 m
x cosα = 0.8994

Ploča stepeništa

C25/30 ➡ $f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 MPa = 1.42\ kN/cm^2$

B500 B ➡ $f_{yd} = 500 / 1.15 = 435\ MPa = 43.5\ kN/cm^2$

1. M_{Ed}=56.13 kNm
2. pretp. d_1 = 3.0 cm (C_{nom}=3.0 cm)

   $d = h - d_1 = 20 - 3.0 = 17.0$ cm

3. Računa se:

$$k = \frac{d}{\sqrt{\frac{M_{Ed}}{b \cdot f_{cd}}}} = \frac{17.0}{\sqrt{\frac{56.13 \cdot 10^2}{100 \cdot 1.42}}} = 2.700$$

ε_{s1} = 15.91‰ > 2.5‰, ω_1=14.594%

Ploča stepeništa

$$A_{s1} = 14.594 \cdot \frac{100 \cdot 17.0}{100} \cdot \frac{1.42}{43.5} = 8.1 cm^2 \ / \ m$$

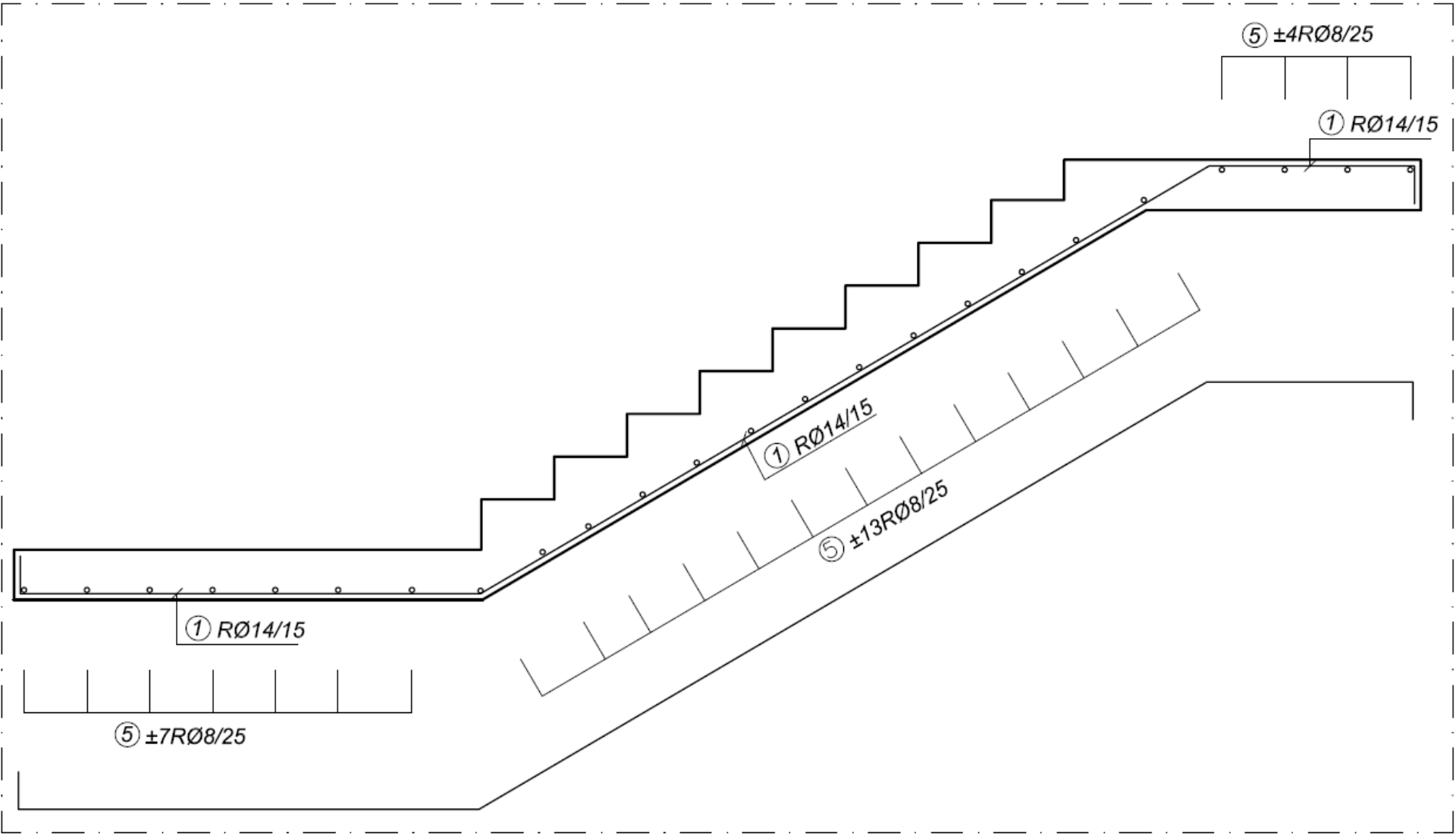
usvojeno: ***Ø 14/15*** *(10.26 cm²/m)*

Podeona armatura

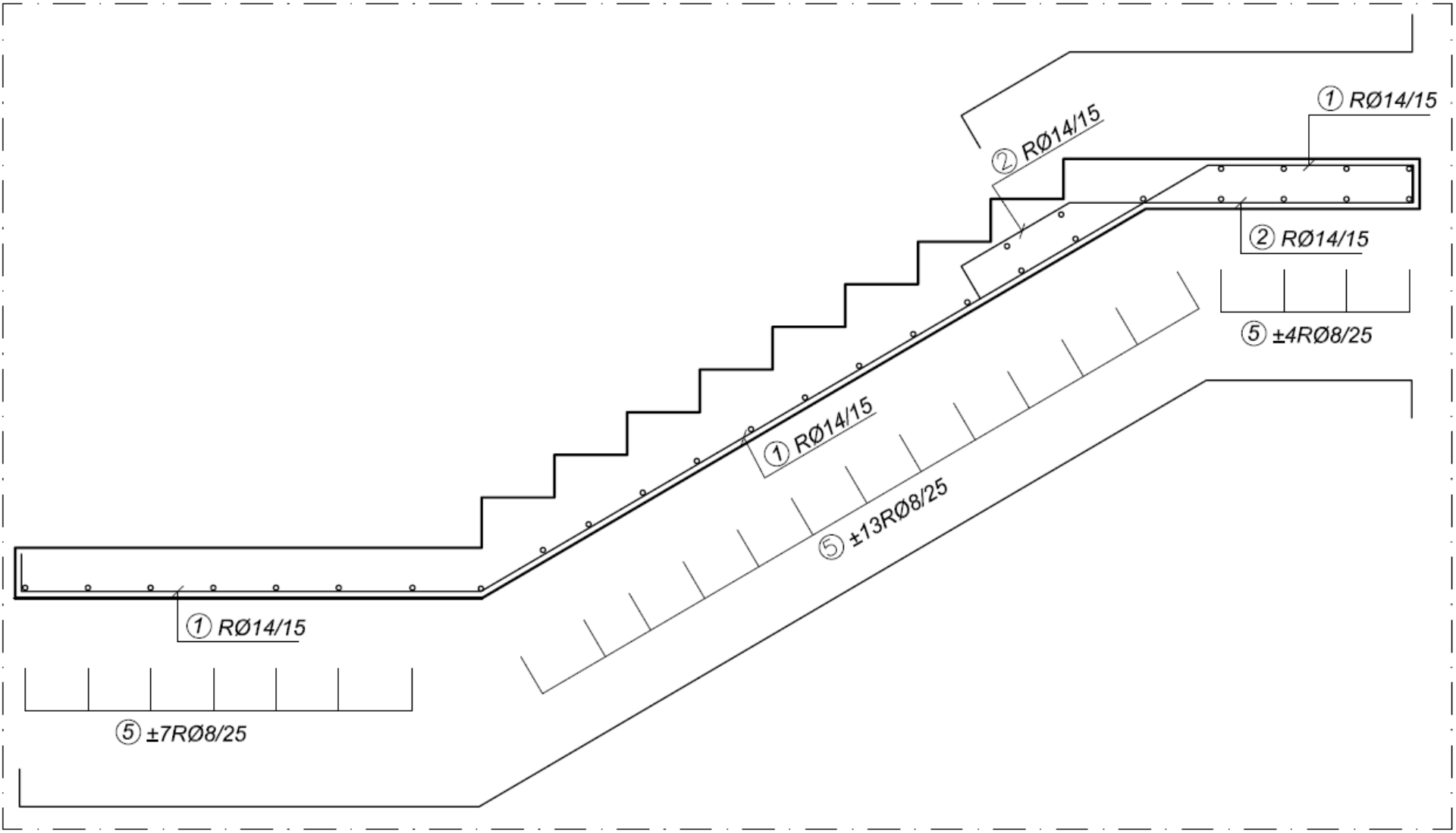
$A_{ap} = 0.2 \times 8.1 = 1.62$ *cm²/m*

usvojeno: ***Ø 8/25*** *(2.01 cm²/m)*

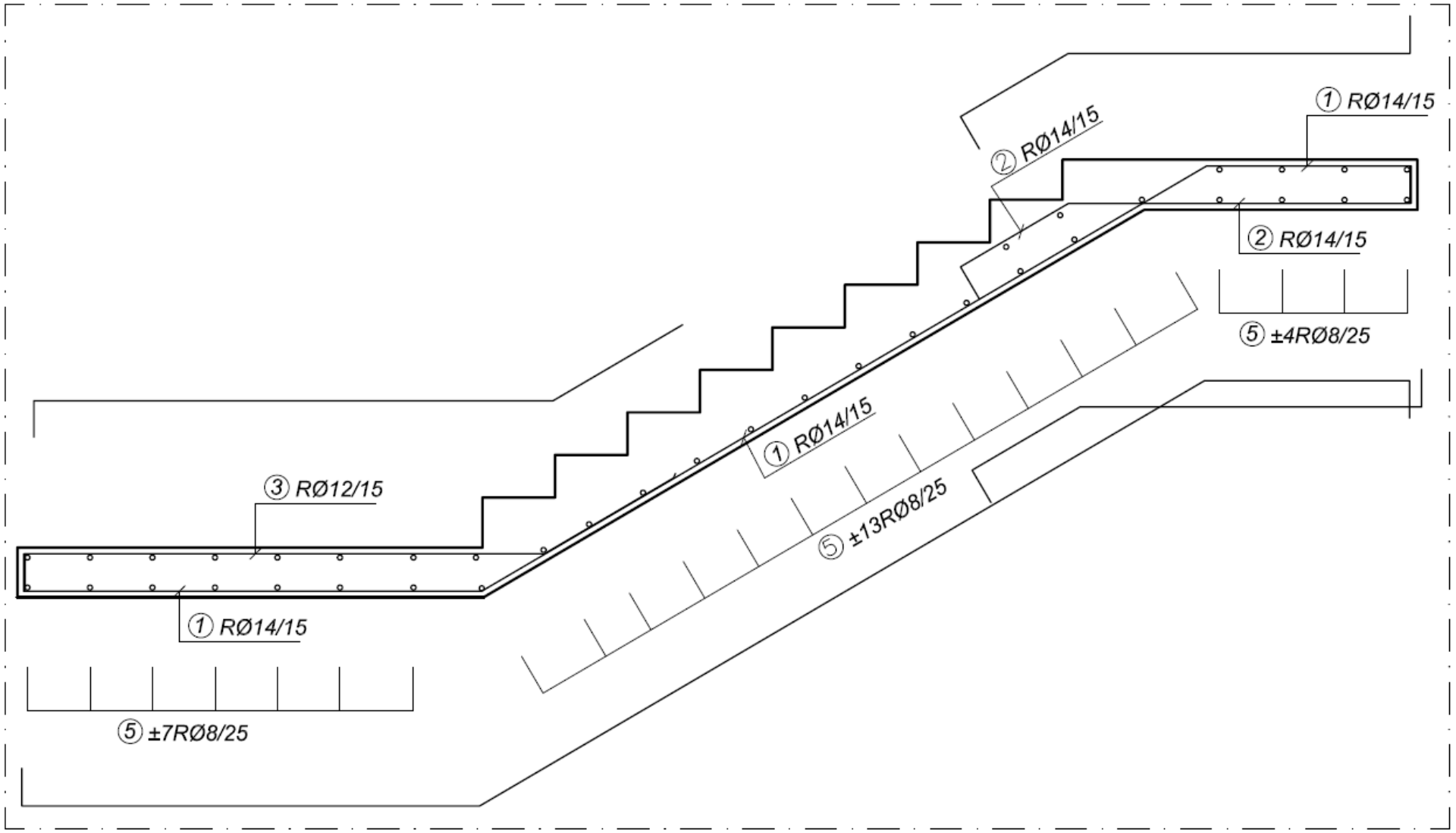
Ploča stepeništa



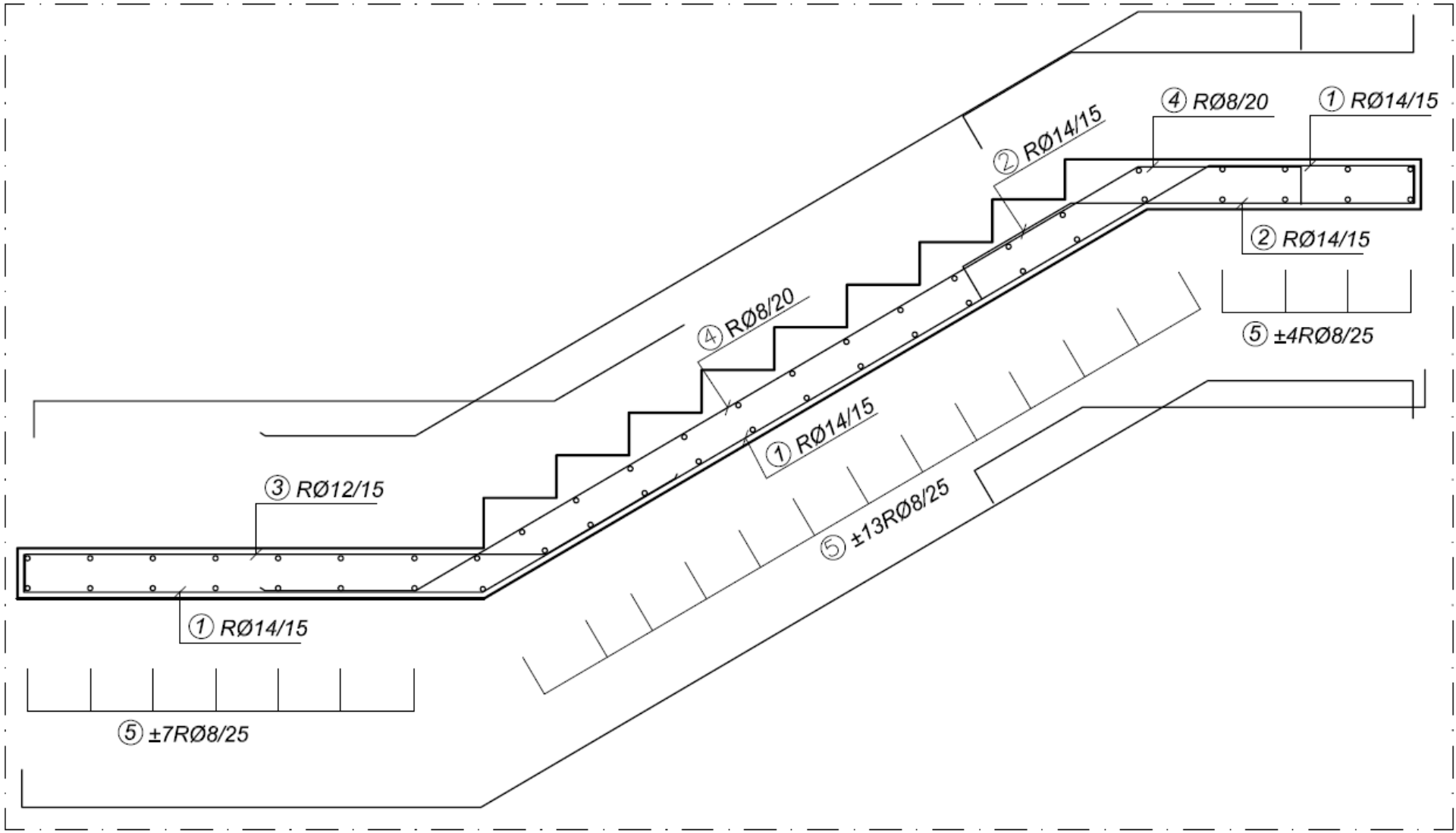
Ploča stepeništa



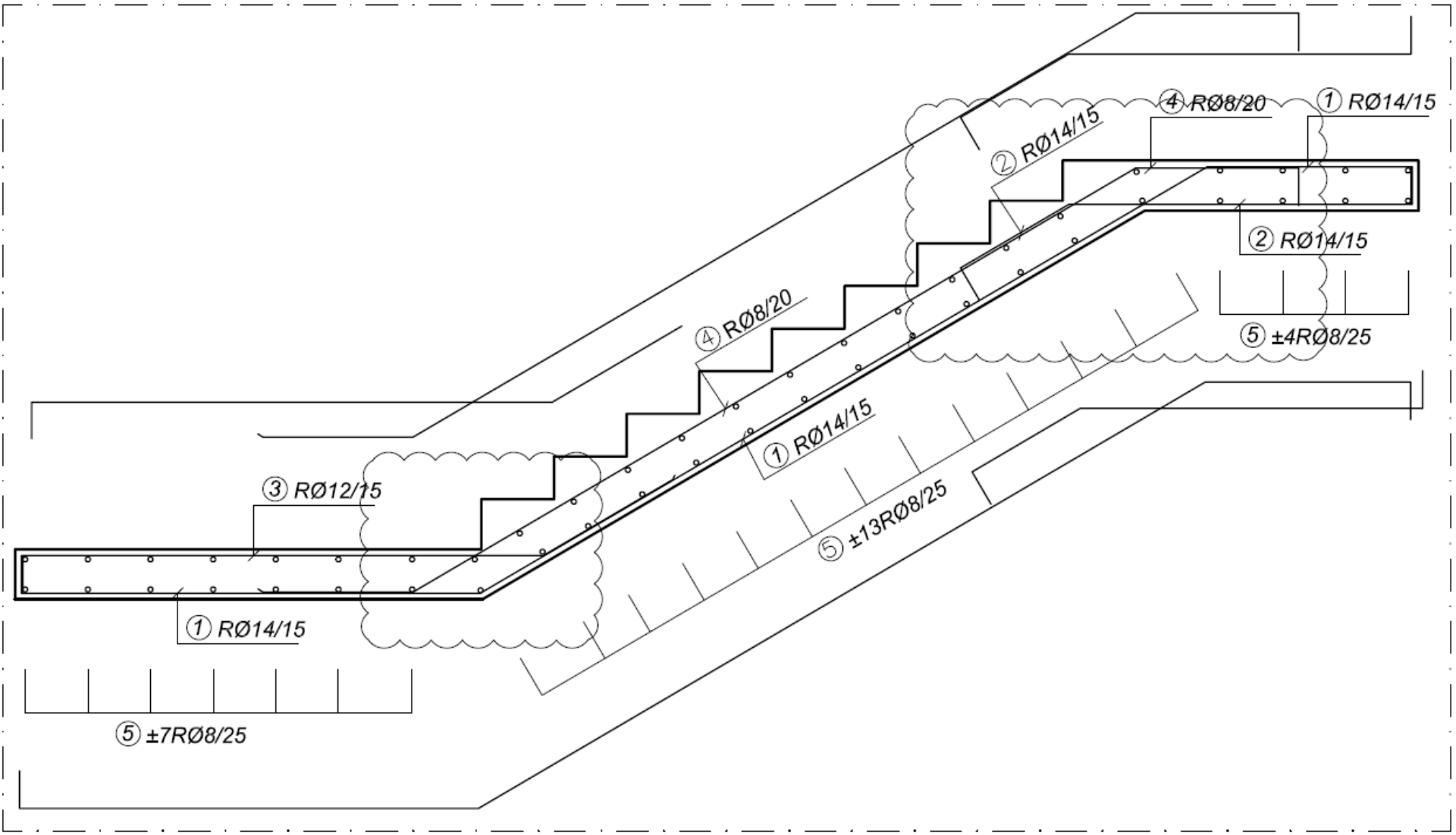
Ploča stepeništa



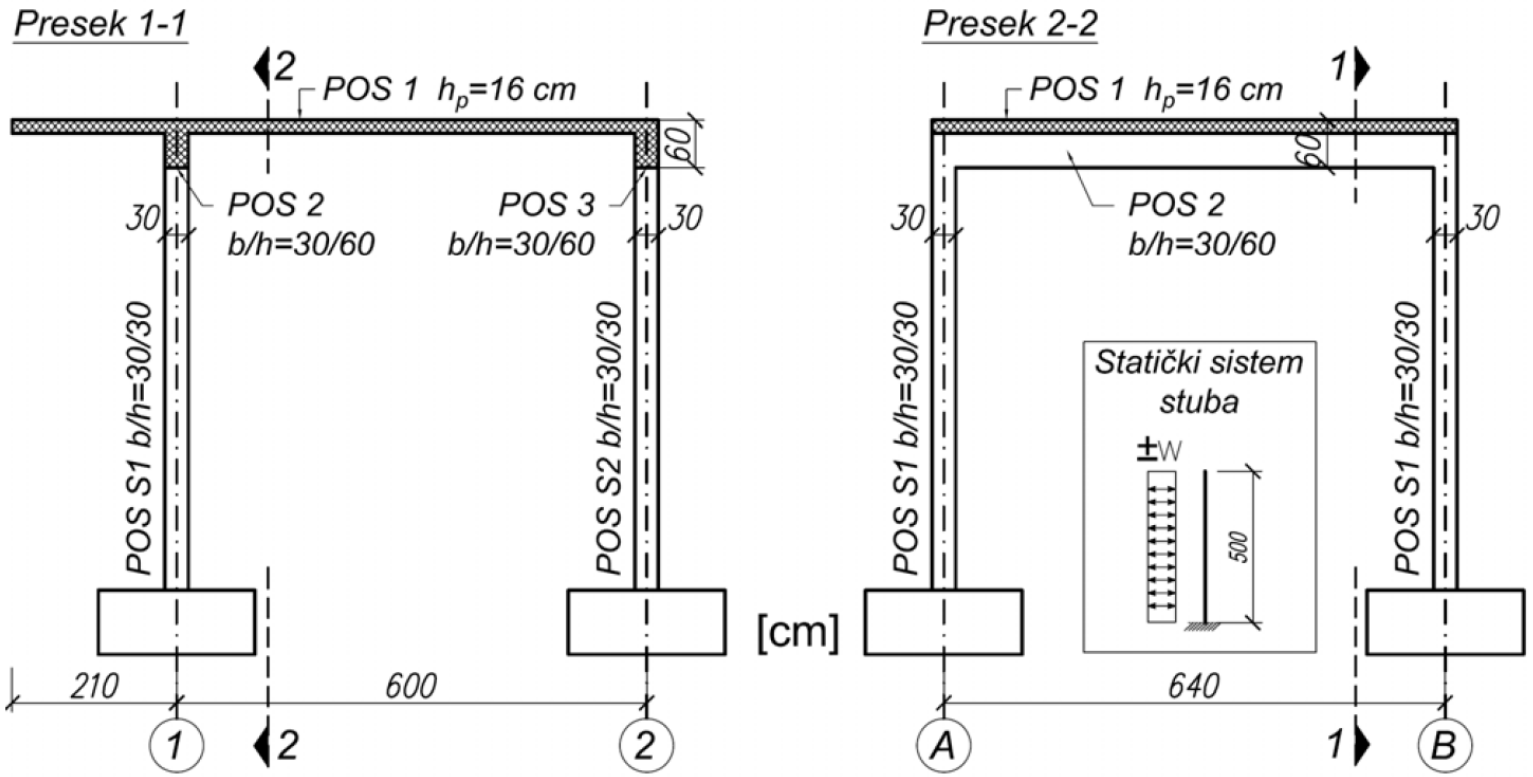
Ploča stepeništa



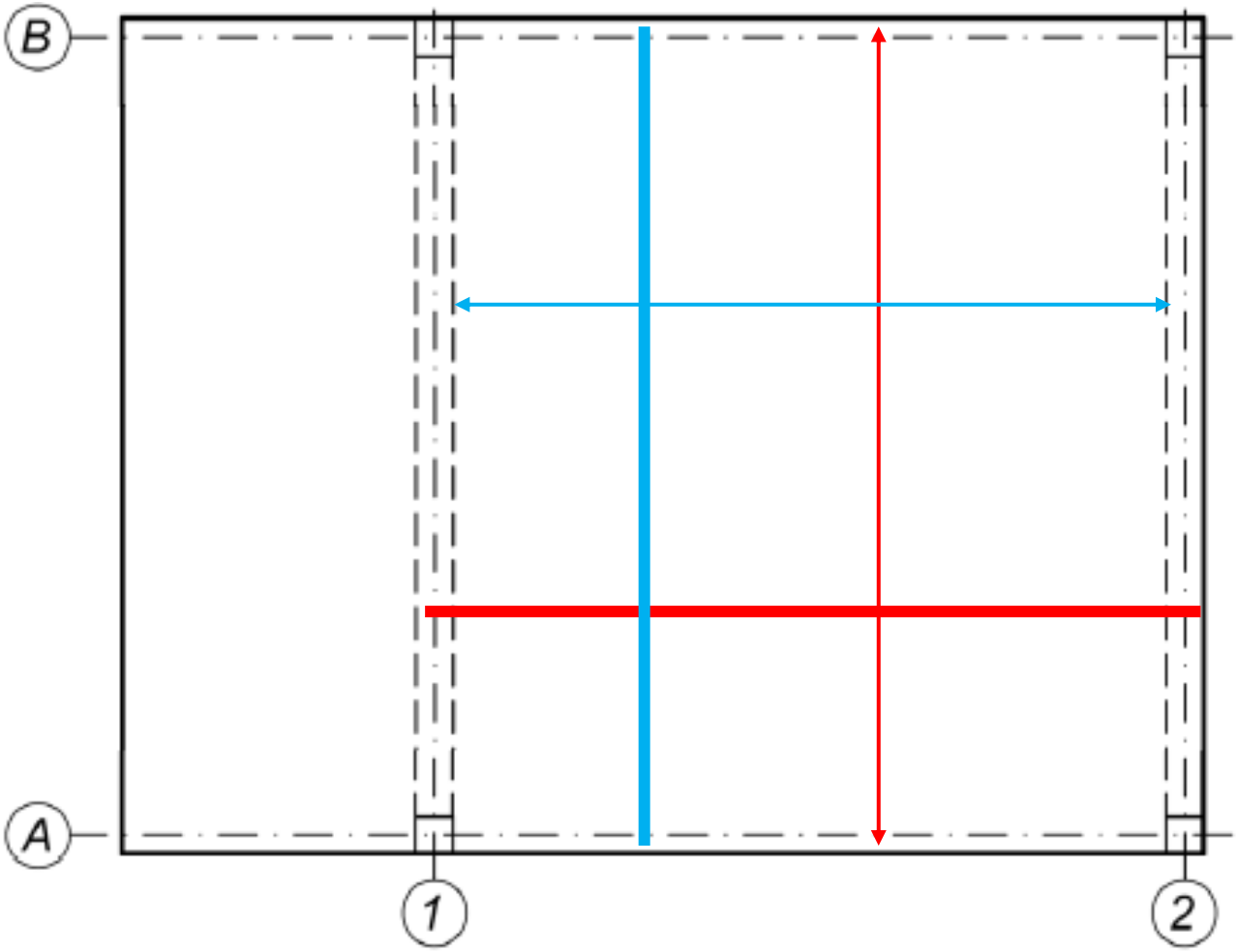
Ploča stepeništa



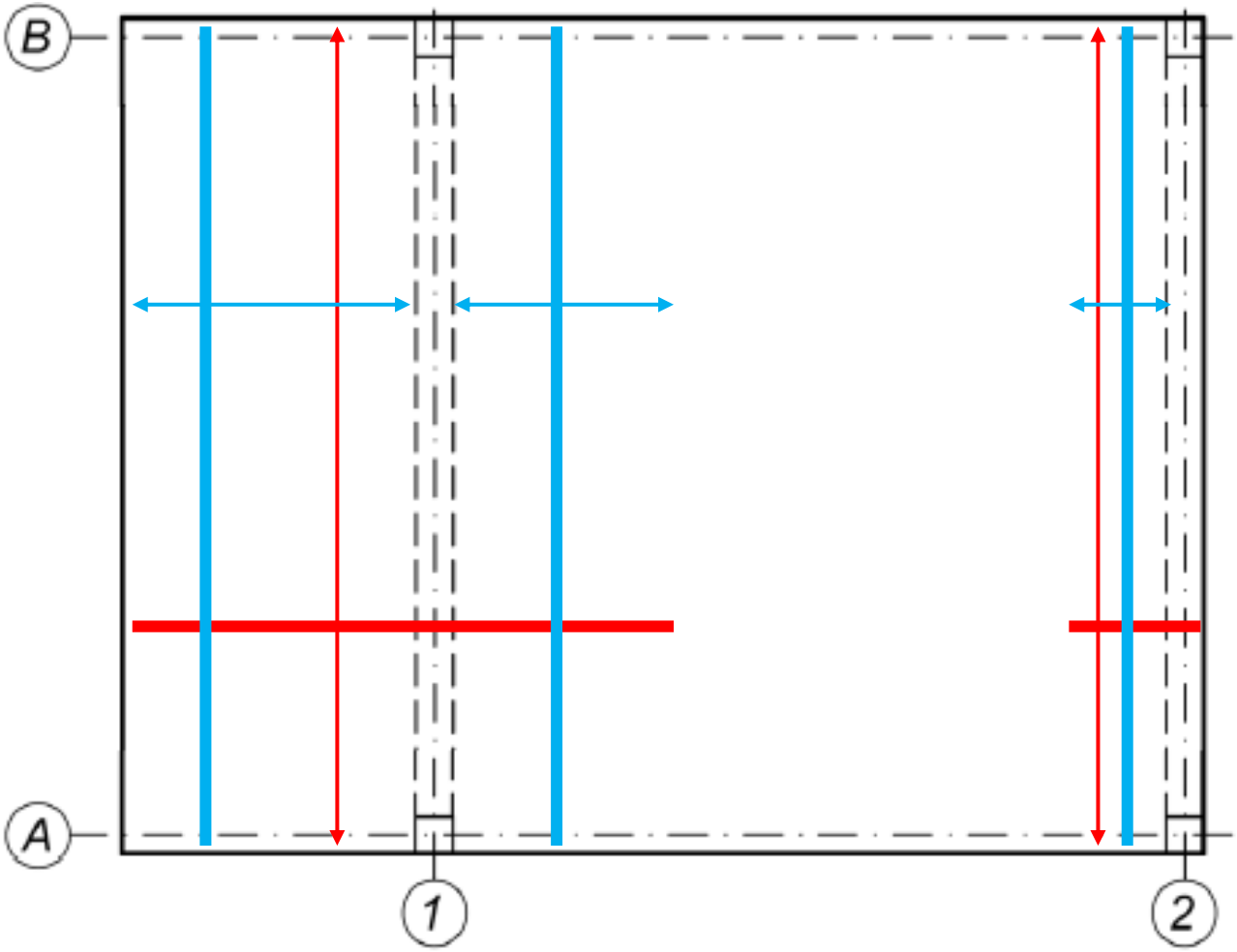
1. *Odrediti granične vrednosti momenata savijanja (M_{Ed}) za sve karakteristične preseke u ploči **POS1** (**h_p = 16 cm**) potrebne za dimenzionisanje donje i gornje zone ploče. Pored sopstvene težine, ploča je opterećena dodatnim stalnim opterećenjem **Δg** i promenljivim opterećenjem **q** koja deluju po čitavoj površini ploče. (10 poena)*
2. *Dimenzionisati ploču **POS 1** (**h_p = 16 cm**) u karakterističnim presecima. (30 poena)*
3. *Usvojeni raspored i dužinu šipki armature u gornjoj i donjoj zoni prikazati u osnovi (poleđina lista, nisu potrebne specifikacija i rekapitulacija armature, ni broj šipki). (25 poena)*
4. *Dimenzionisati stub **POS S1** (**b_s/h_s=30/30 cm**) za dejstvo stalnog, promenljivog opterećenja i vetra koji deluje na stub statičkog sistema prikazanog na skici ispod. Povremeno opterećenje i vetar ne moraju delovati istovremeno. Sopstvenu težinu stuba i efekte izvijanja stuba zanemariti. Nacrtati usvojeni raspored armature u poprečnom preseku stuba. Dimenzije svih greda su **b_g/h_g = 30/60 cm.** (35 poena)*



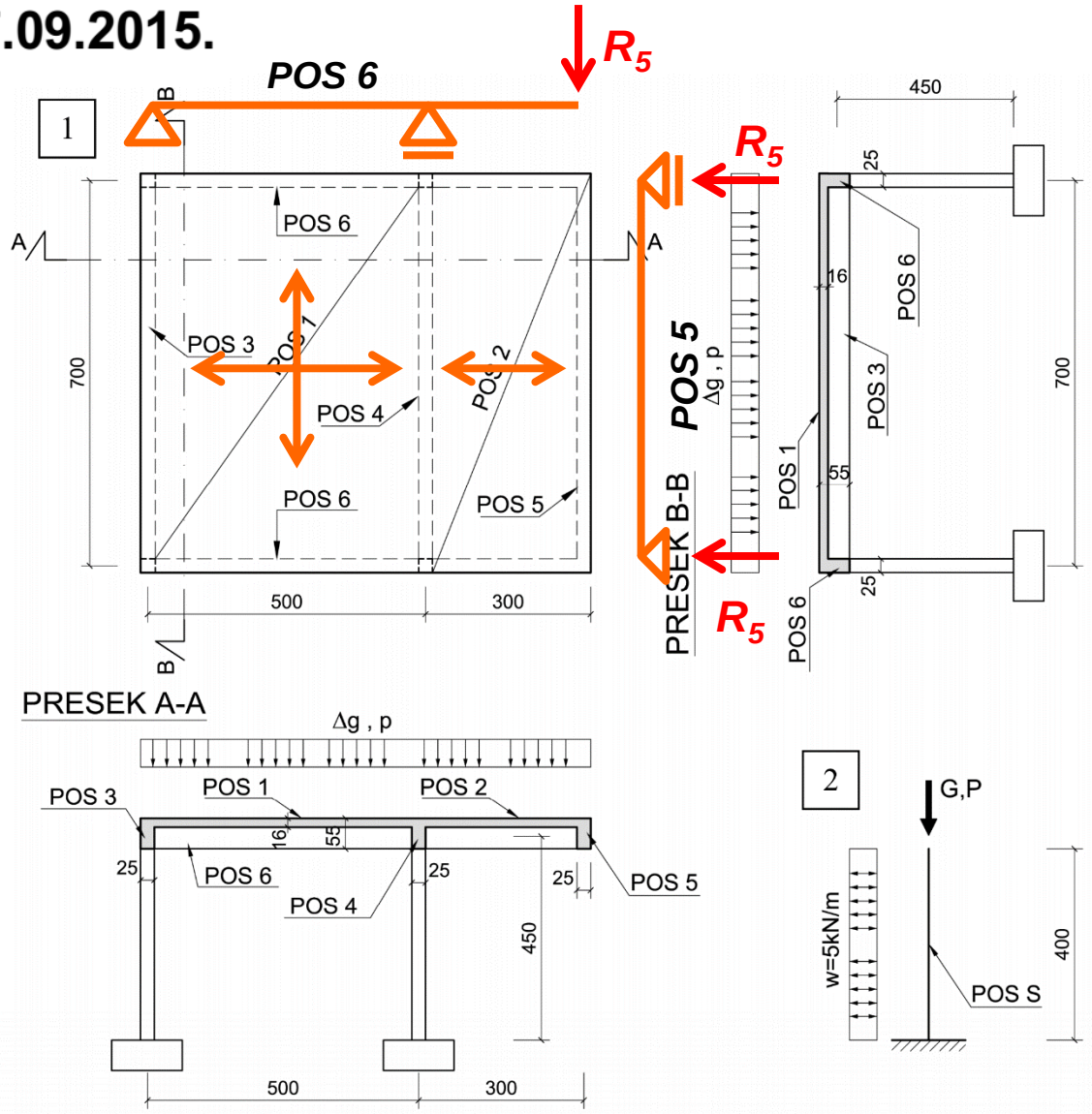
Donja zona ploce



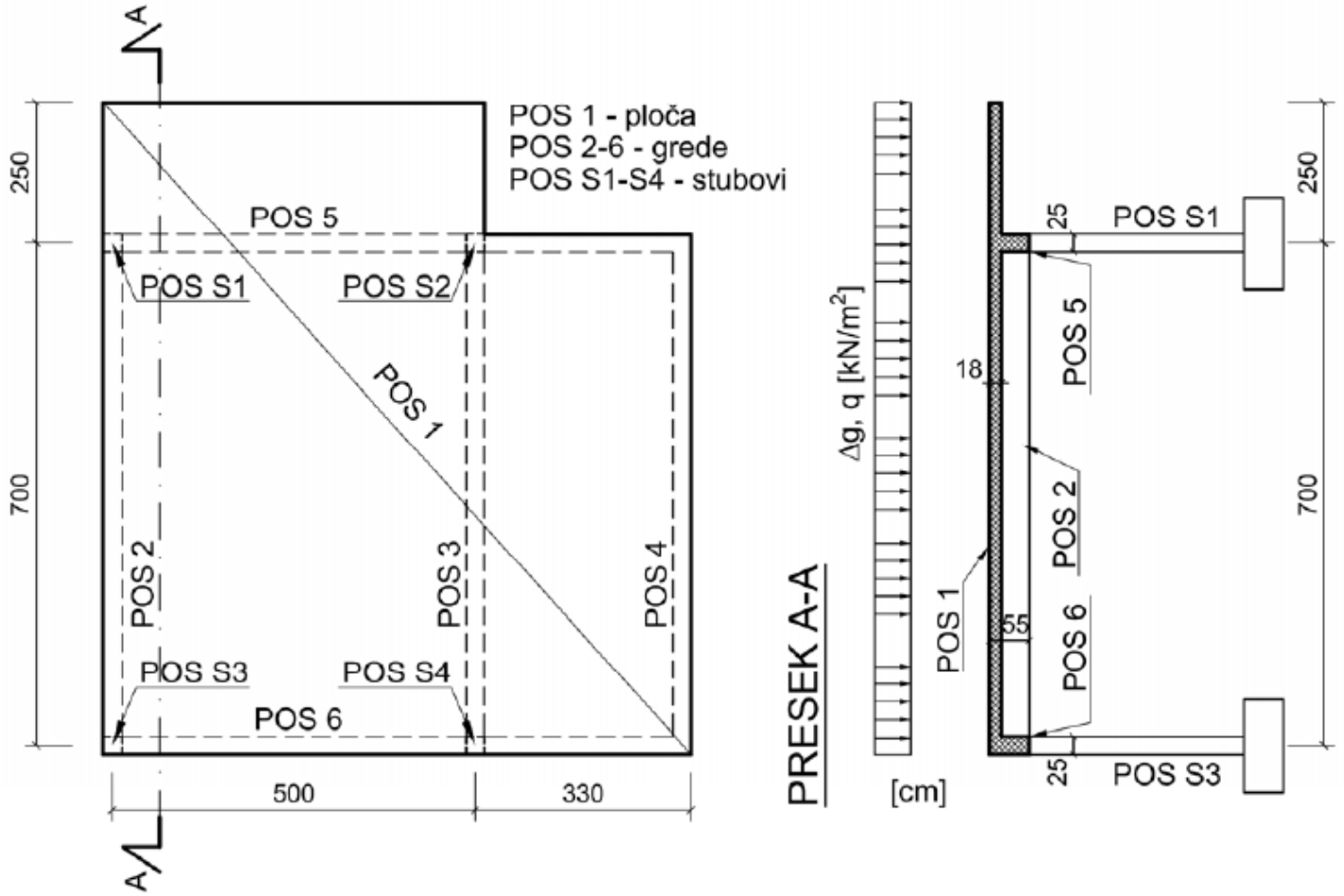
Gornja zona ploce



27.09.2015.


POS 6
R5
1
B
A
POS 6
POS 3
POS 7
POS 4
POS 2
POS 6
POS 5
700
500
300
B
R5
POS 5
Δg , p
PRESEK B-B
R5
450
25
POS 6
16
POS 3
POS 1
55
700
25
POS 6
PRESEK A-A
Δg , p
POS 3
POS 1
POS 2
16
55
25
POS 6
POS 4
25
25
POS 5
450
500
300
2
G,P
w=5kN/m
POS S
400

1. *Za armiranobetonsku konstrukciju koja je prikazana na skici ispod, potrebno je:*



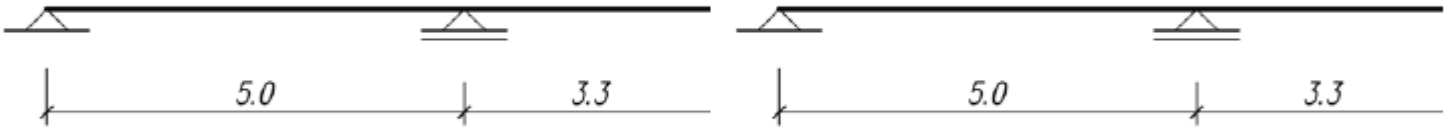
*1.1. Odrediti granične vrednosti momenata savijanja (**M_{Ed}**) za sve karakteristične preseke u ploči **POS1** (**h_p=18 cm**) potrebne za dimenzionisanje **gornje zone** ploče. Pored sopstvene težine, ploča je opterećena dodatnim stalnim opterećenjem (**Δg=2.5 kN/m²**) i promenljivim opterećenjem (**q=3.5 kN/m²**) koja deluju po čitavoj površini ploče.* *(15 poena)*

*1.2. Dimenzionisati ploču **POS 1** (**h_p = 18 cm**) u karakterističnim presecima samo u **gornjoj zoni**.* *(20 poena)*

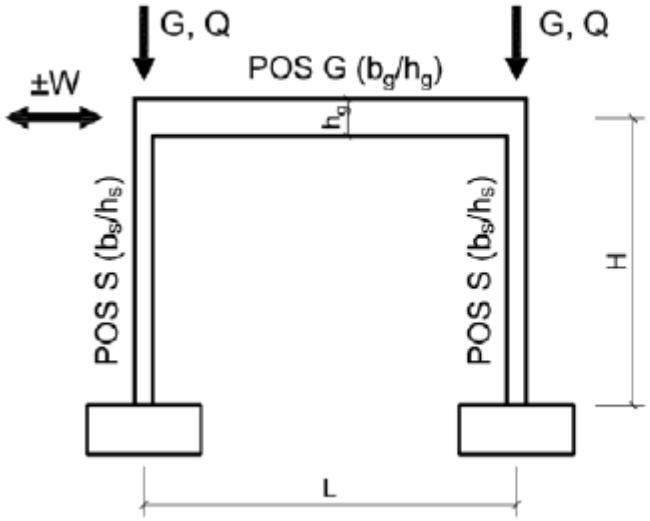
*1.3. Usvojeni raspored šipki armature u **gornjoj zoni** prikazati u osnovi (**poleđina lista**, nisu potrebne specifikacija i rekapitulacija armature, dužina ni broj šipki).* *(15 poena)*

1.4. *Izvršiti analizu opterećenja za gredu* ***POS 5,*** *posebno za stalno i promenljivo opterećenje, i prikazati ih na skicama ispod. Sve grede su dimenzija* ***b/h=25/55 cm****.* *(15 poena)*

POS 5 – stalno opterećenje ***POS 5 – promenljivo opterećenje***



2. *Smičući armiranobetonski ram prikazan na skici na poleđini lista opterećen je horizontalnom koncentrisanom silom usled vetra (**±W**) kao i vertikalnim koncentrisanim silama usled stalnog (**G**) i promenljivog (**Q**) opterećenja. Potrebno je dimenzionisati stub rama prema **M** i **N**. Usvojeni raspored armature skicirati u poprečnom preseku.* *(35 poena)*



L=6.2 m

H=4.5 m

Greda: ***POS G b_g/h_g=30/60 cm***

Stub: ***POS S b_s/h_s=30/30 cm***

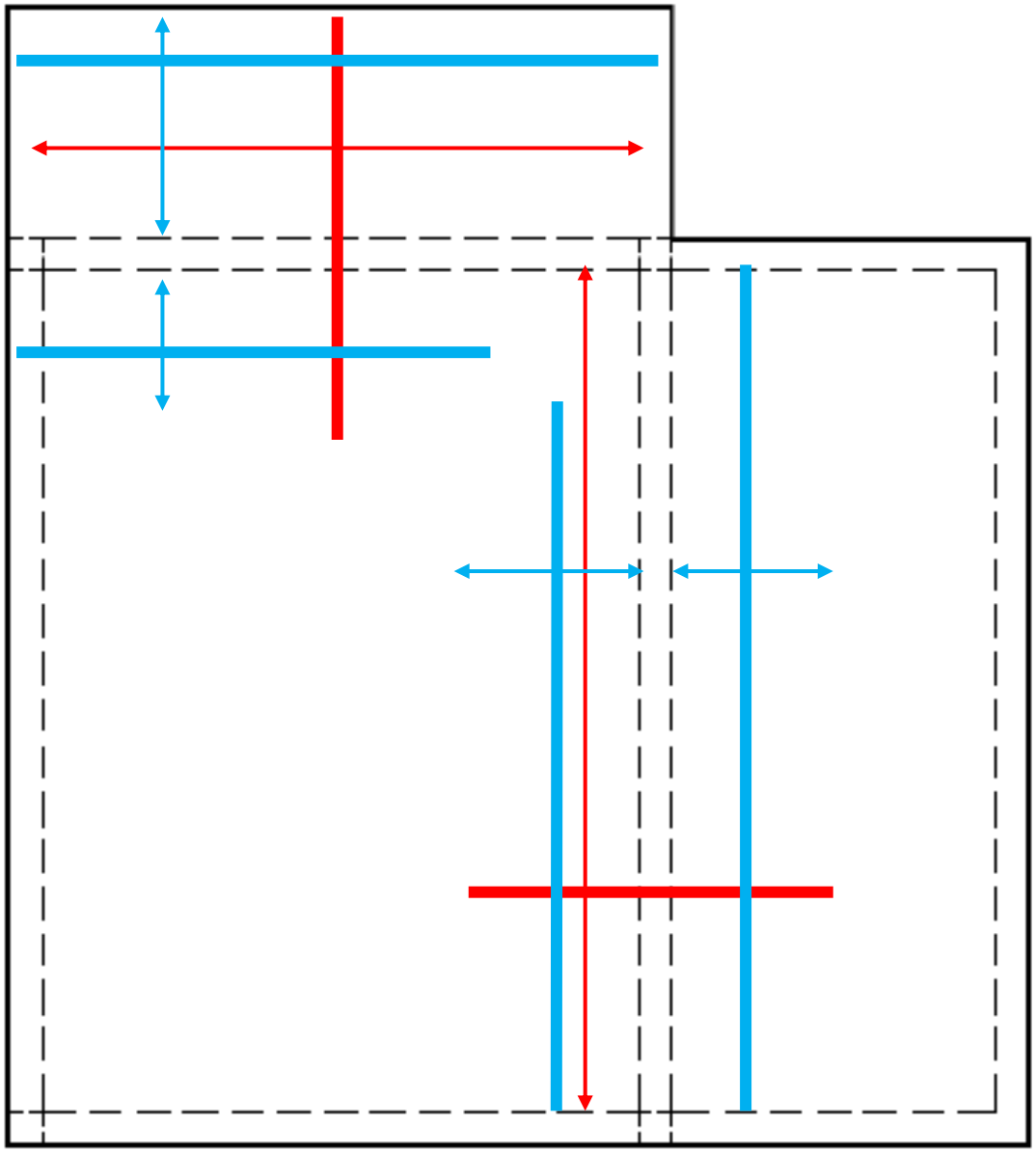
Stalno opterećenje: ***G=320 kN***

Promenljivo opterećenje: ***Q=580 kN***

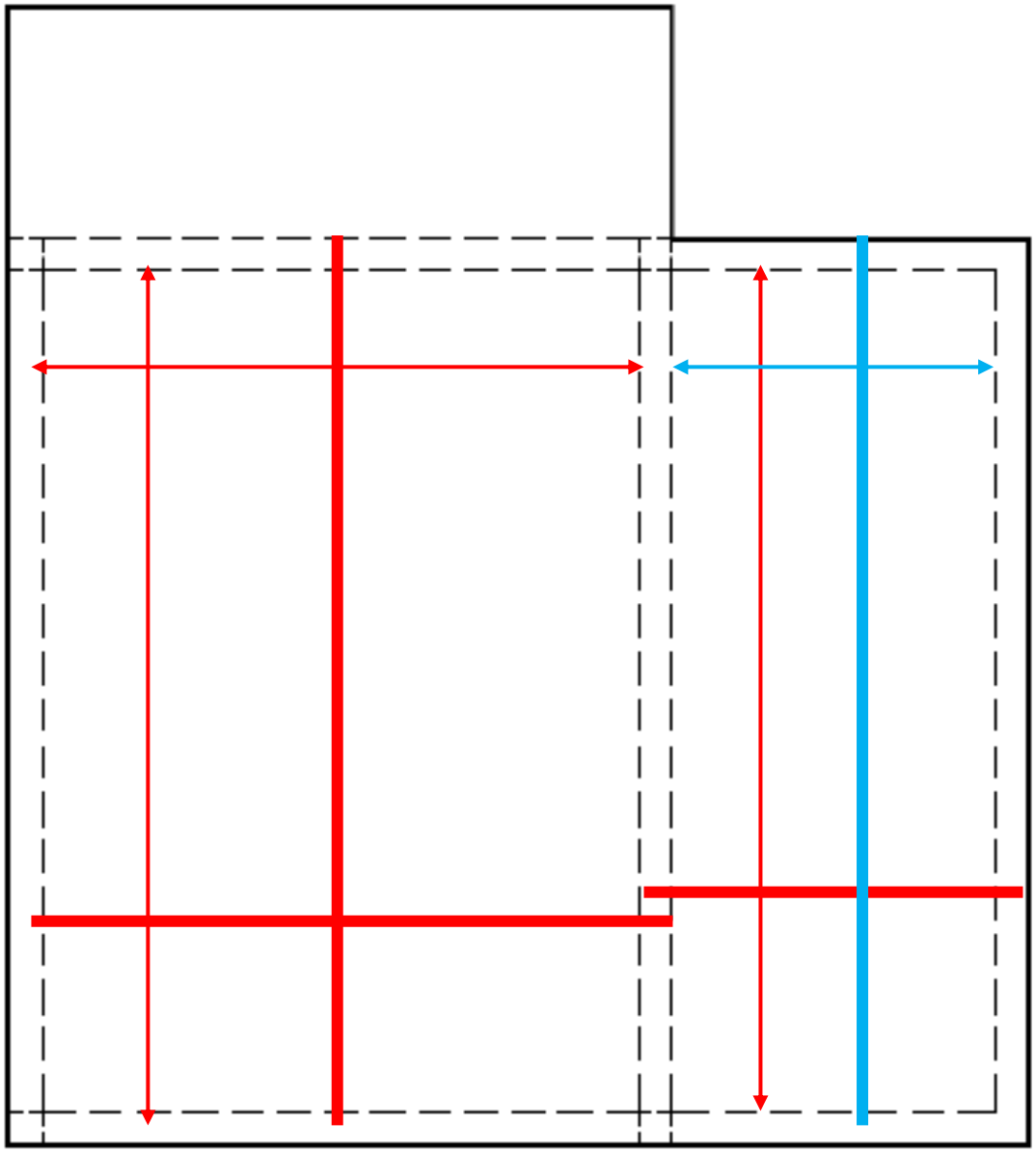
Vetar: ***W=±45 kN***

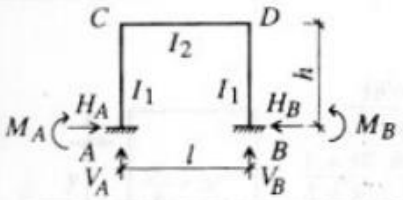
Napomena: Sopstvenu težinu grede POS G i stubova POS S zanemariti.

Gornja zona ploce

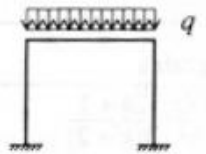


Donja zona ploce





$$k = \frac{I_2}{I_1}\frac{h}{l}$$

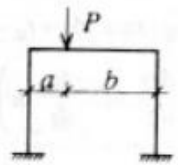


$$H = H_A = H_B = \frac{ql^2}{4h(k+2)}$$

$$V_A = V_B = \frac{ql}{2}$$

$$M_A = M_B = \frac{ql^2}{12(k+2)} = H\frac{h}{3}$$

$$M_C = M_D = \frac{ql^2}{6(k+2)} = -2H\frac{h}{3}$$



$$H = H_A = H_B = \frac{3Pab}{2hl(k+2)}$$

$$V_A = \frac{Pb}{l}\left[1 + \frac{a(b-a)}{l^2(6k+1)}\right]$$

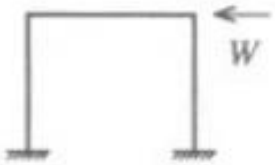
$$V_B = P - V_a$$

$$M_A = \frac{Pab}{2l^2}\,\frac{5kl - l + 2a\,(k+2)}{(k+2)(6k+1)}$$

$$M_B = \frac{Pab}{2l^2}\,\frac{7kl + 3l - 2a\,(k+2)}{(k+2)(6k+1)}$$

$$M_C = M_A - Hh$$

$$M_D = M_E - Hh$$

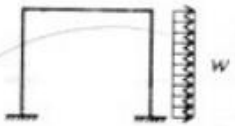


$$H_A = -H_B = \frac{W}{2}$$

$$V_A = -V_B = \frac{3Whk}{l(6k+1)}$$

$$M_A = -M_B = \frac{Wh}{2}\,\frac{3k+1}{6k+1}$$

$$M_C = -M_D = \frac{Wh}{2}\,\frac{3k}{6k+1}$$



$$H_A = \frac{wh}{8}\,\frac{2k+3}{k+2}$$

$$H_B = H_A - wh$$

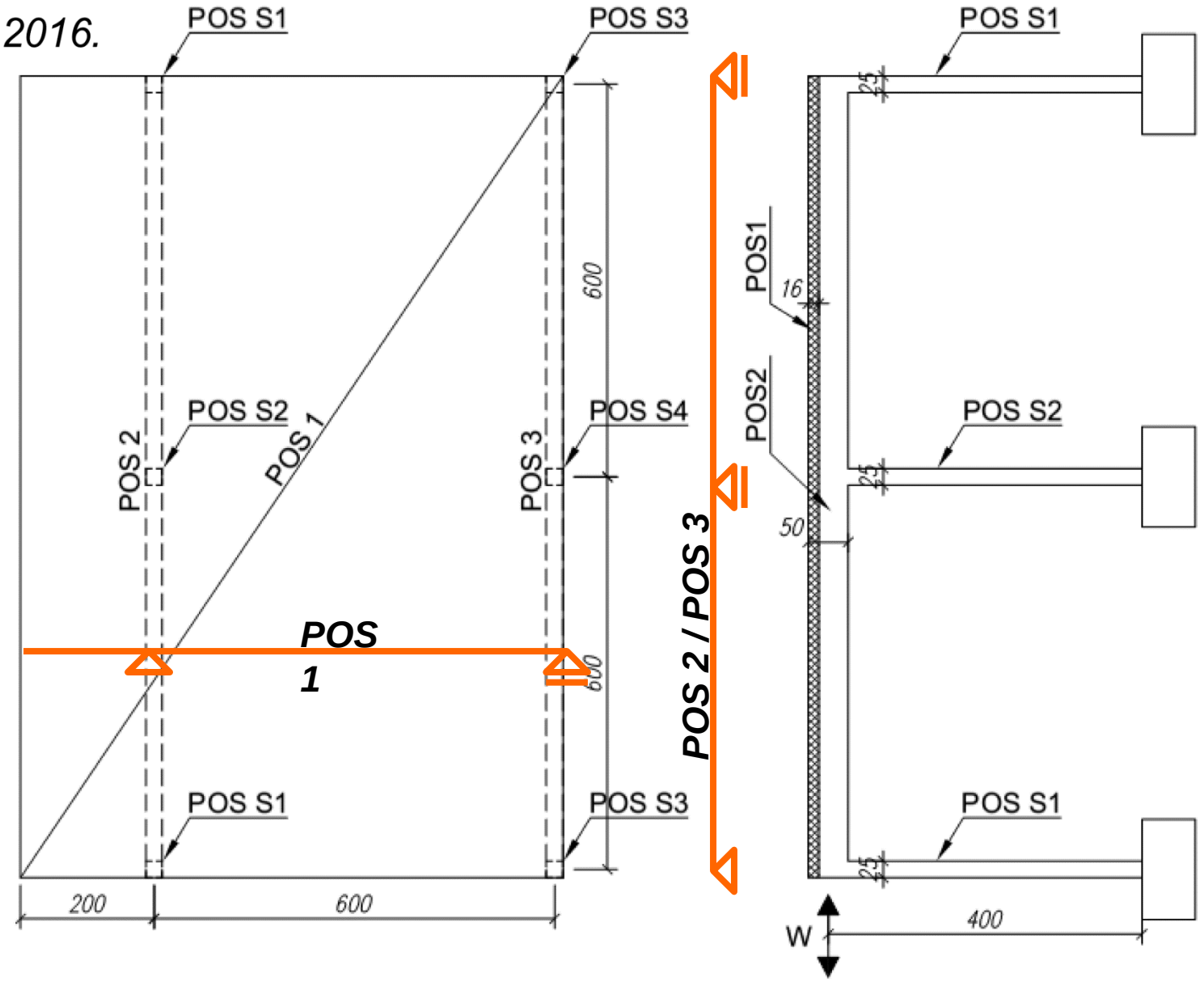
$$V_A = -V_B = \frac{wh^2k}{l(6k+1)}$$

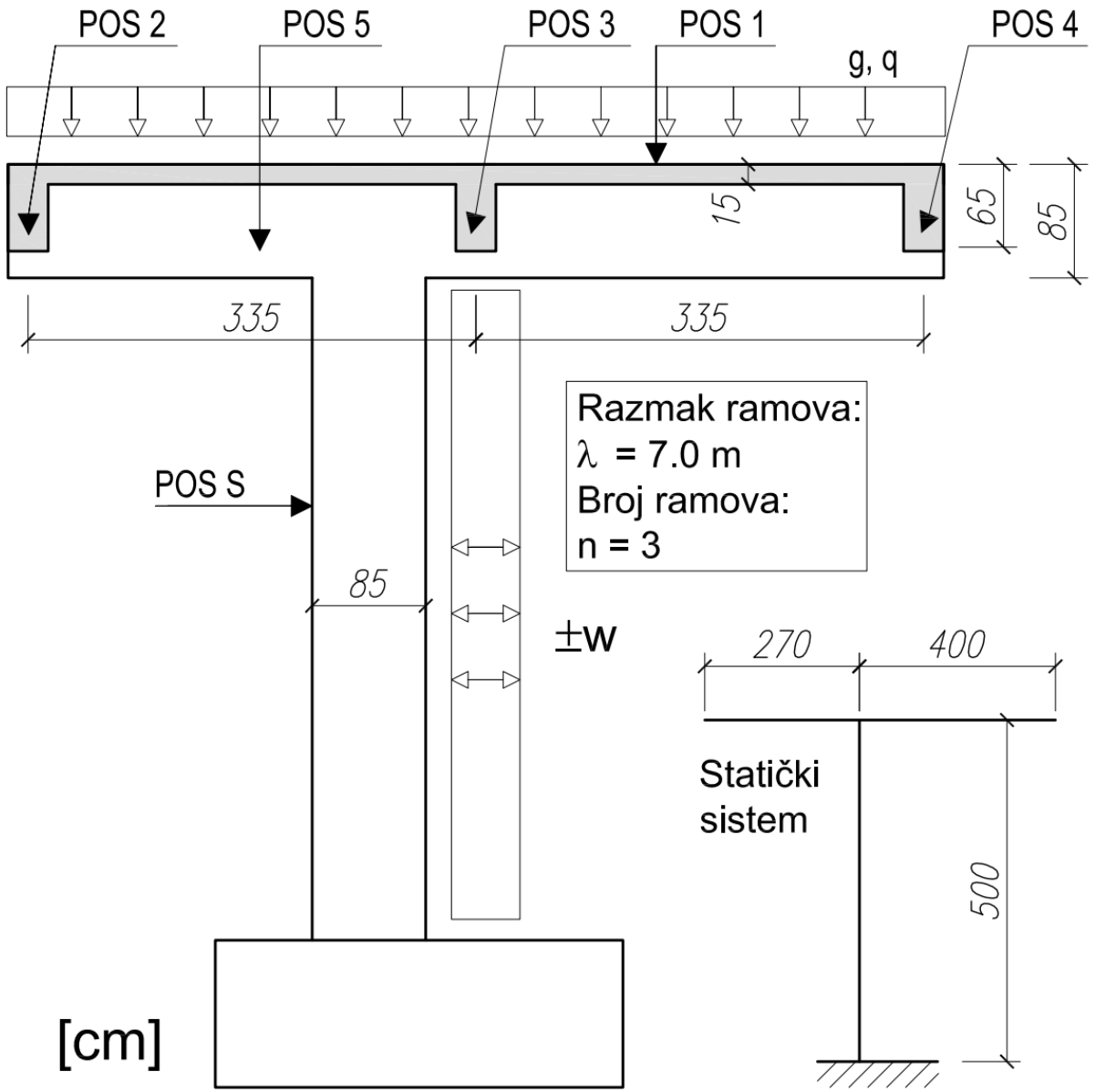
$$M_A = \frac{wh^2}{24}\left(\frac{5k+9}{k+2} - \frac{12k}{6k+1}\right)$$

$$M_B = -\frac{wh^2}{24}\left(12 - \frac{5k+9}{k+2} - \frac{12k}{6k+1}\right)$$

$$M_C = M_A - H_A h$$

$$M_D = M_B - Hh + \frac{wh^2}{2}$$

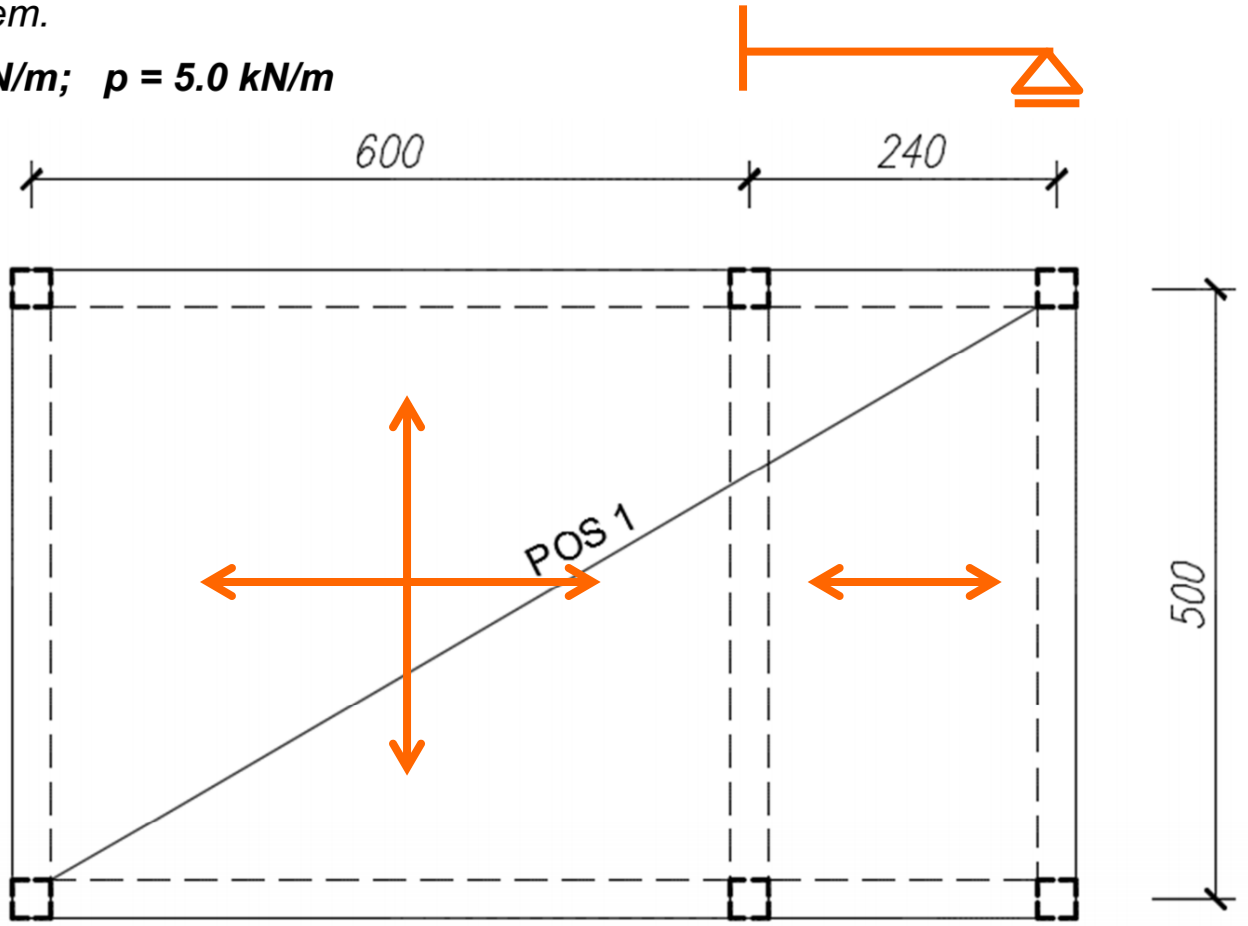

POS S1
POS S3
POS S2
POS S4
POS 1
POS 2
POS 3
POS S1
POS S3
600
600
200
600
POS 1
POS 2 / POS 3
POS S1
POS1
16
POS2
POS S2
50
POS S1
25
25
25
W
400


POS 2
POS 5
POS 3
POS 1
POS 4
g, q
15
65
85
335
335
Razmak ramova:
λ = 7.0 m
Broj ramova:
n = 3
POS S
85
±w
270
400
Statički
sistem
500
[cm]

15.06.2016.

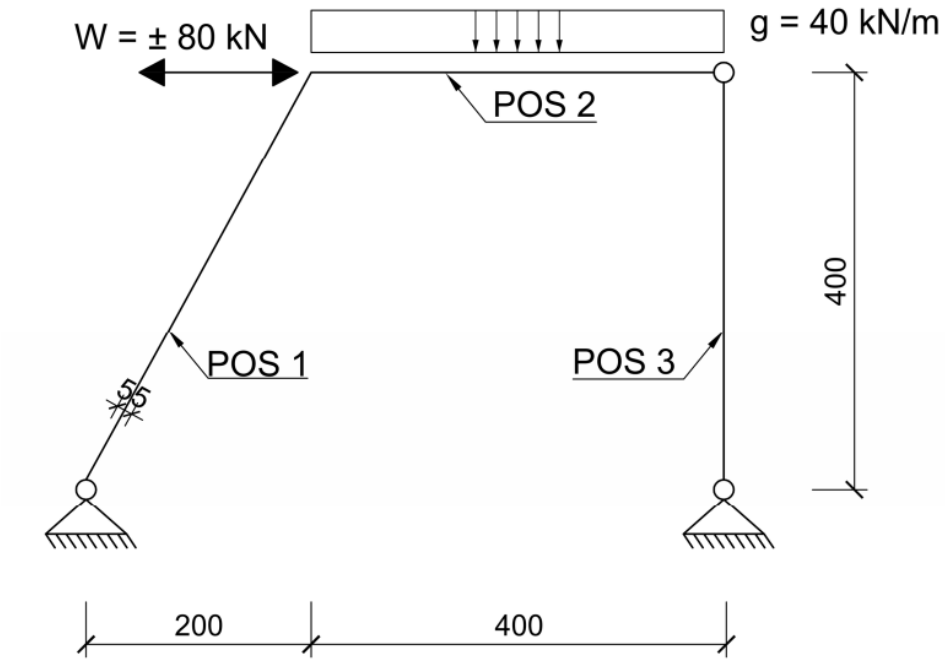
4. *Za konstrukciju prikazanu na skici potrebno je dimenzionisati ploču* ***POS 1*** *(**d_{pl} = 18cm**) prema* ***M*** *u karakterističnim presecima i nacrtati u odgovarajućoj razmeri plan armature gornje i donje zone. Pored sopstvene težine ploča je opterećena dodatnim stalnim i povremenim opterećenjem.*

Δg = 2.0 kN/m; p = 5.0 kN/m ***(40 poena)***



06.07.2016.

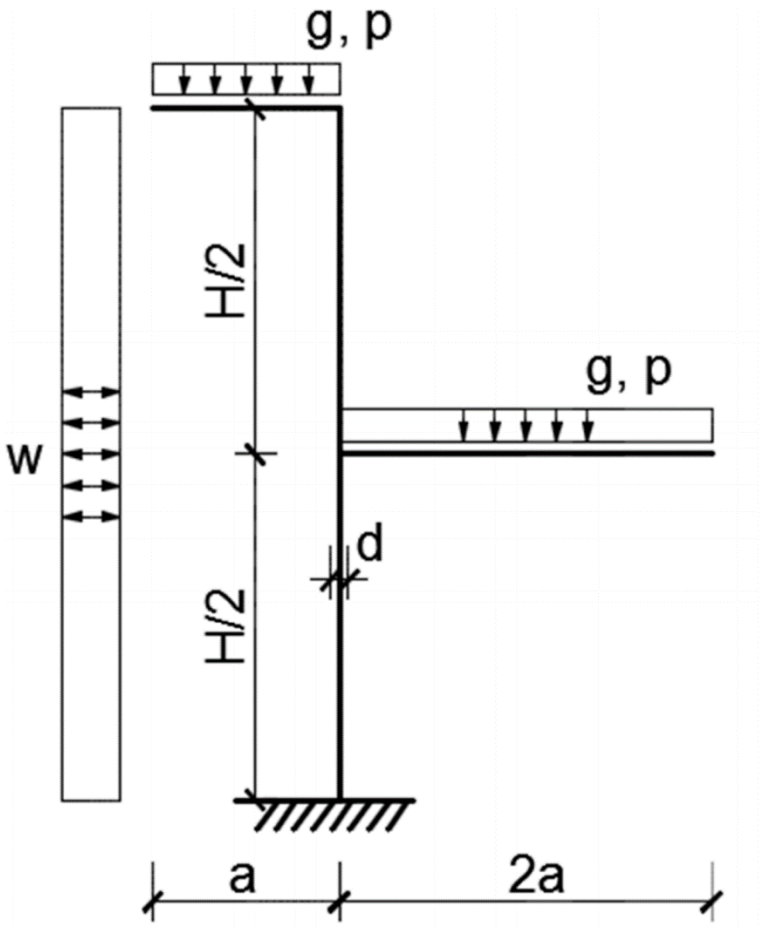
2. *Za konstrukciju prikazanu na skici potrebno je dimenzionisati stub **POS 1** (**b/d=30/55cm**). Sopstvenu težinu elemenata rama zanemariti.*

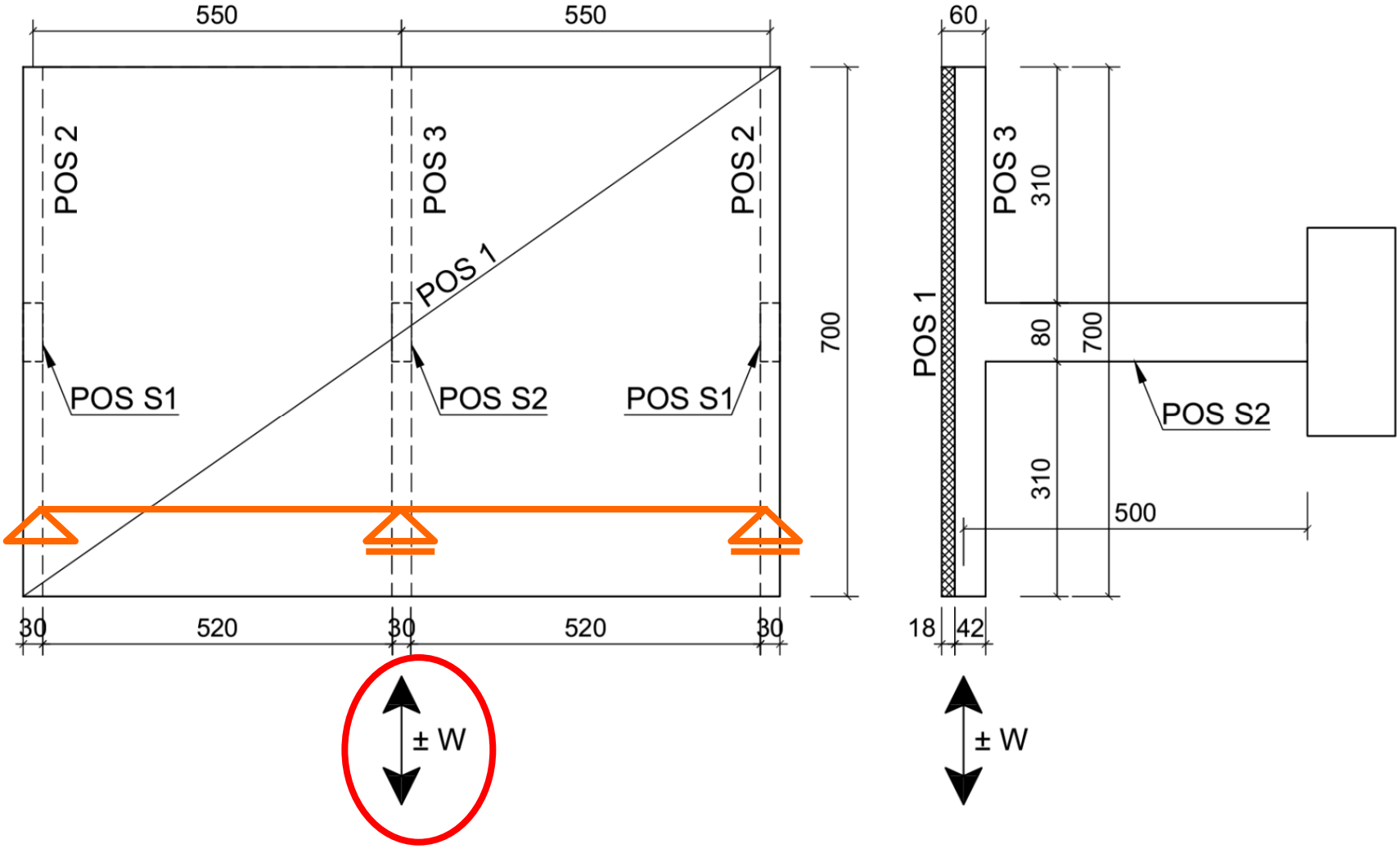


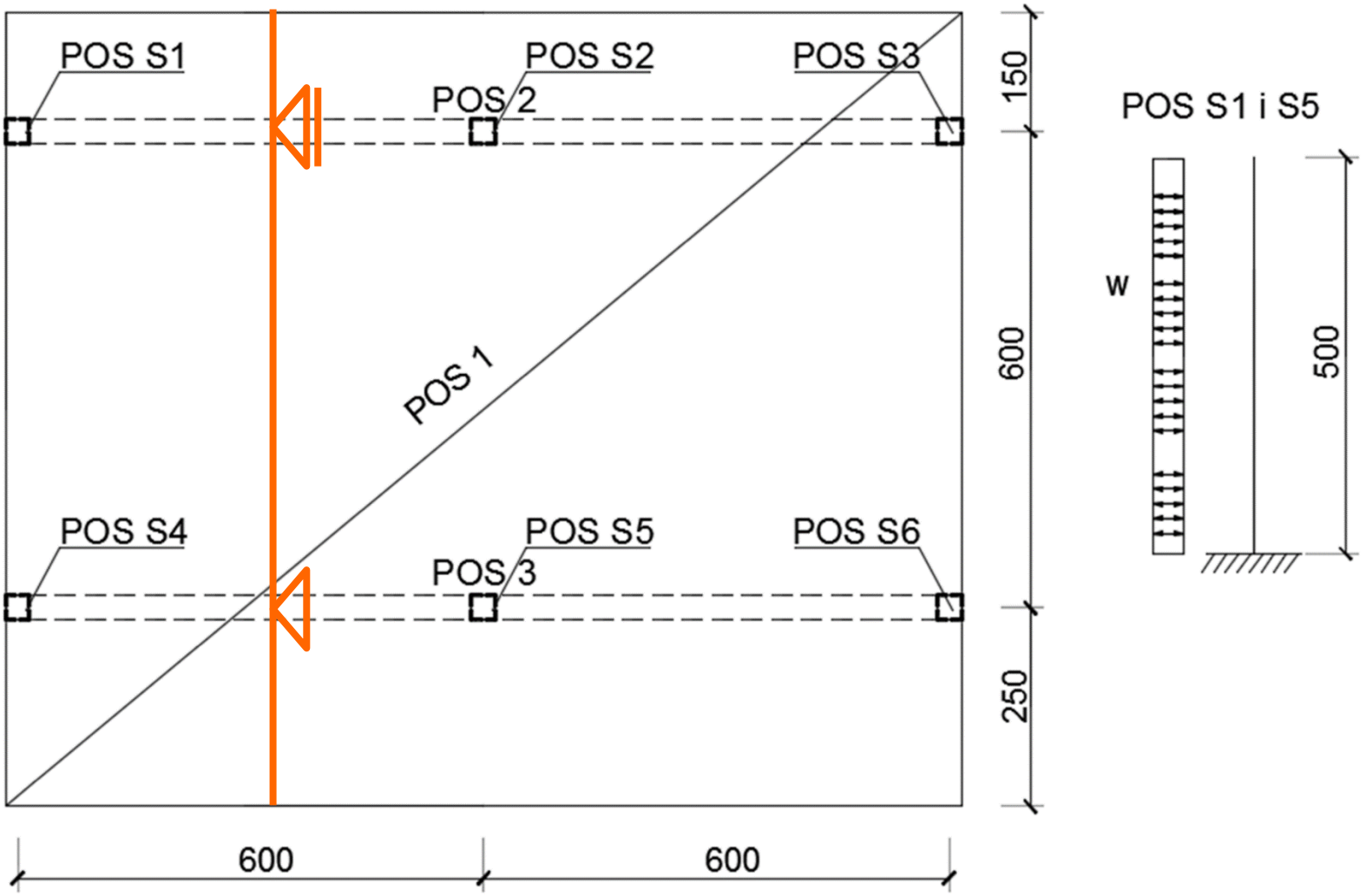
1. *Za konstrukciju prikazanu na skici desno potrebno je dimenzionisati prema* ***M*** *presek* ***u uklještenju*** *(**b/d = 30/80 cm**).*

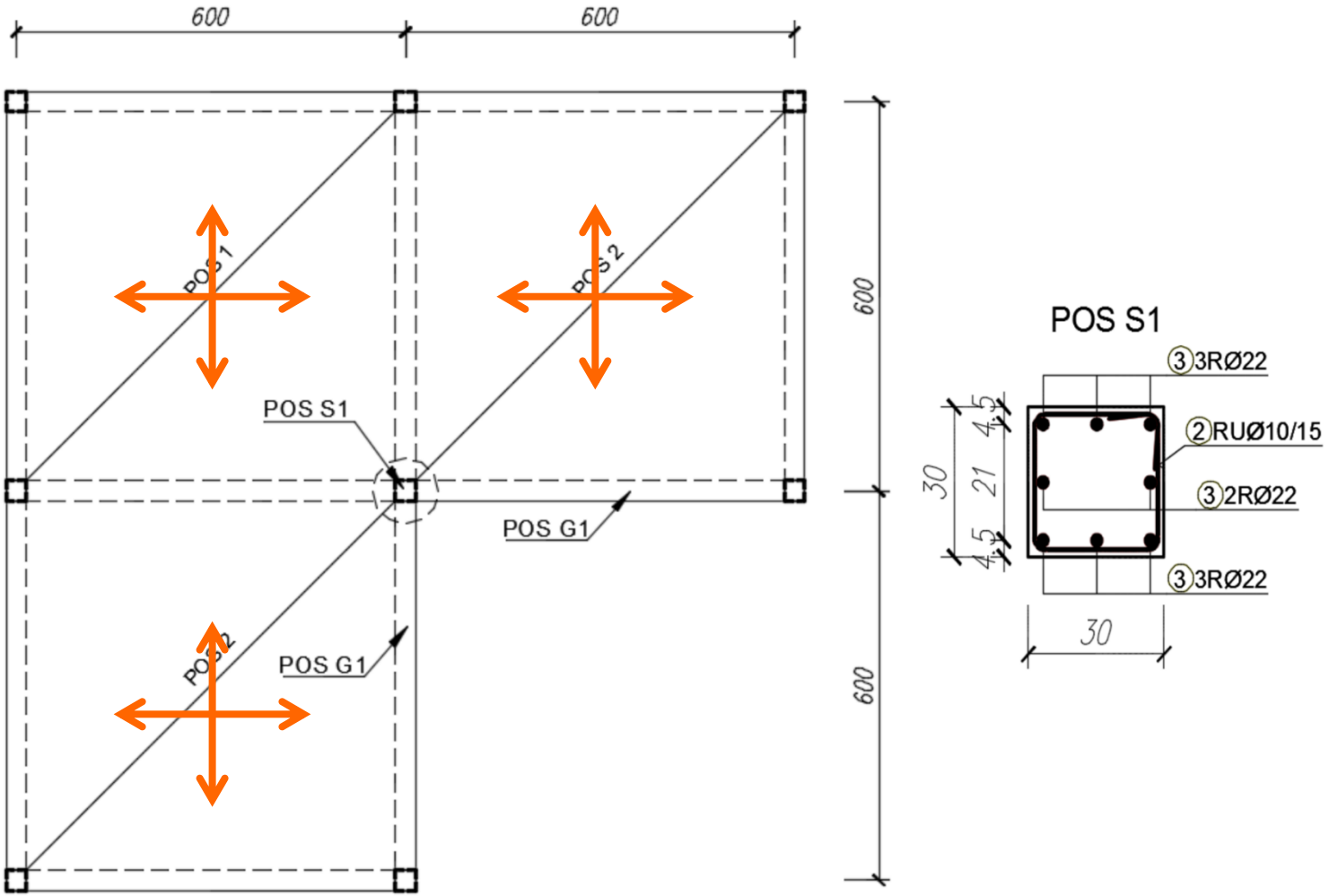
g = 30 kN/m; p = 50 kN/m; w = ±15 kN/m

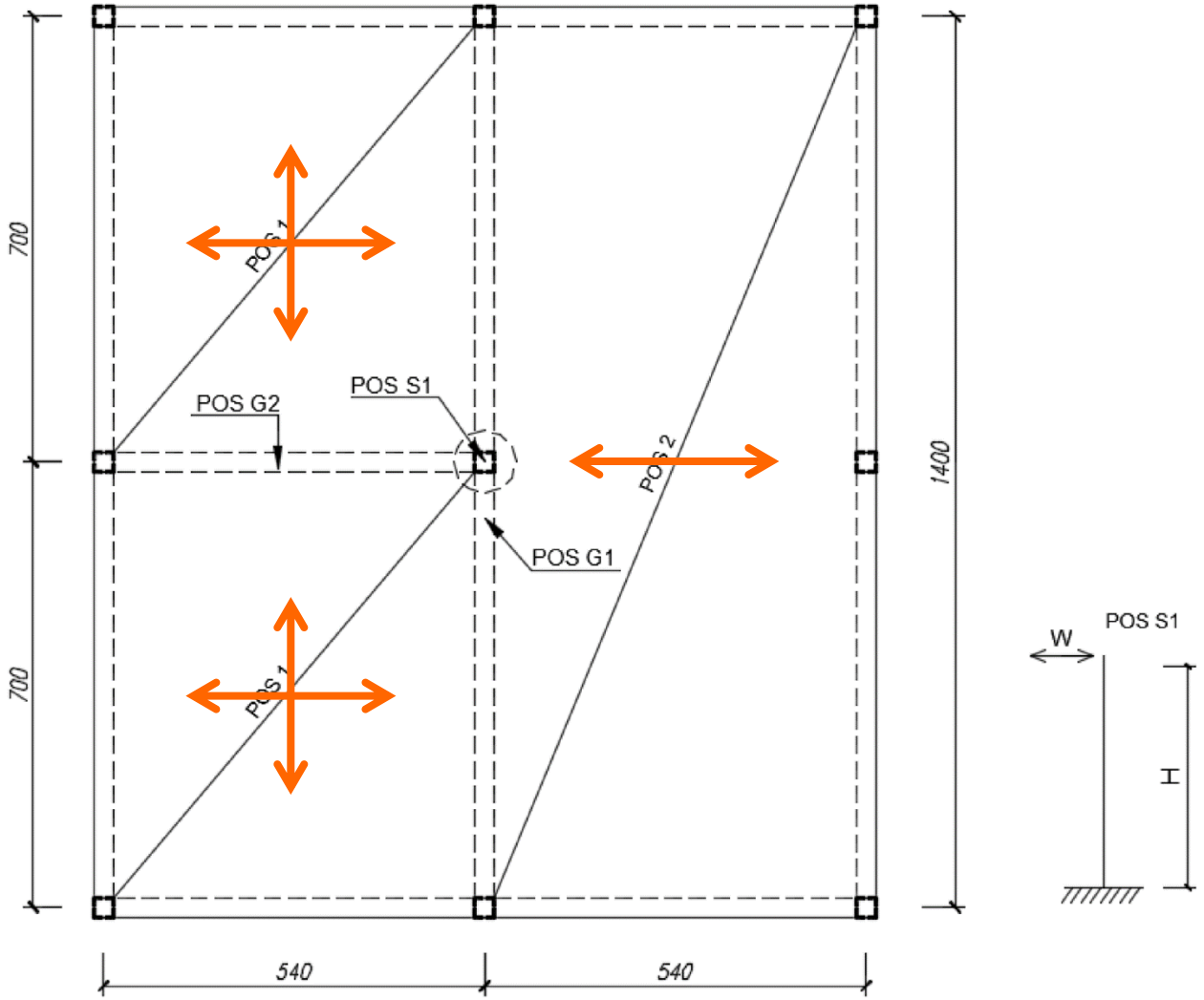
H = 6 m; a = 1.5 m




550
550
POS 2
POS 3
POS 2
POS 1
POS S1
POS S2
POS S1
700
30
520
30
520
30
± W
60
POS 3
310
POS 1
80
700
POS S2
310
500
18
42
± W

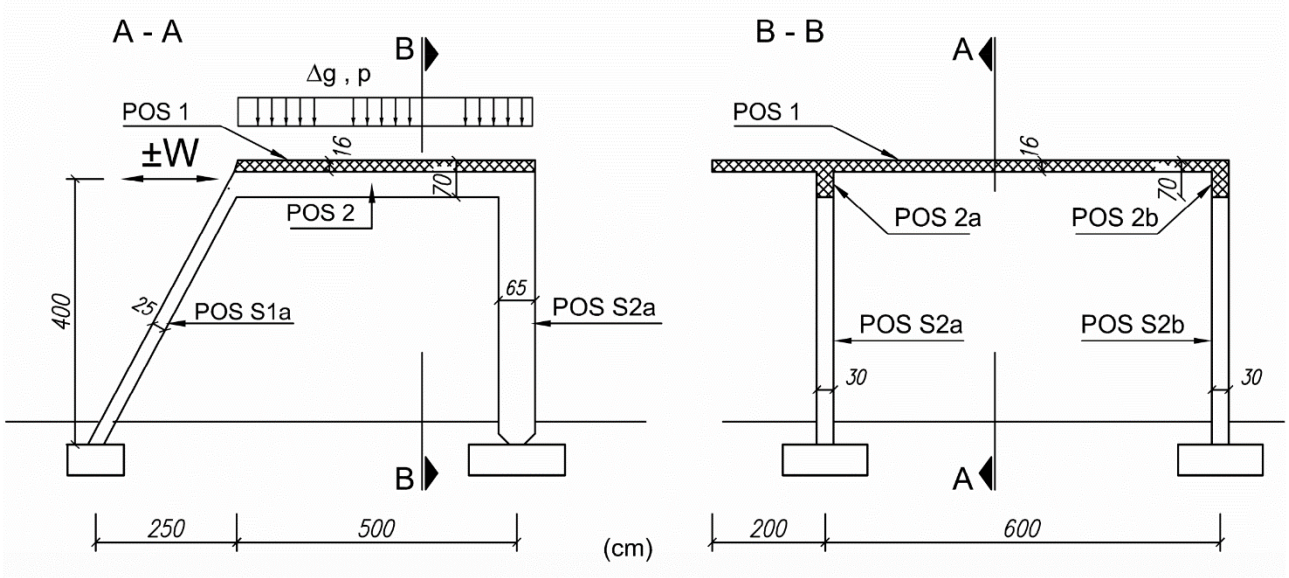

POS S1
POS S2
POS S3
POS 2
POS 1
POS S4
POS S5
POS S6
POS 3
150
600
250
600
600
POS S1 i S5
w
500


600
600
600
600
POS 1
POS 2
POS S1
POS G1
POS 2
POS G1
POS S1
③3RØ22
②RUØ10/15
③2RØ22
③3RØ22
30
4,5
21
4,5
30


700
700
1400
POS 1
POS S1
POS G2
POS 2
POS G1
POS 1
540
540

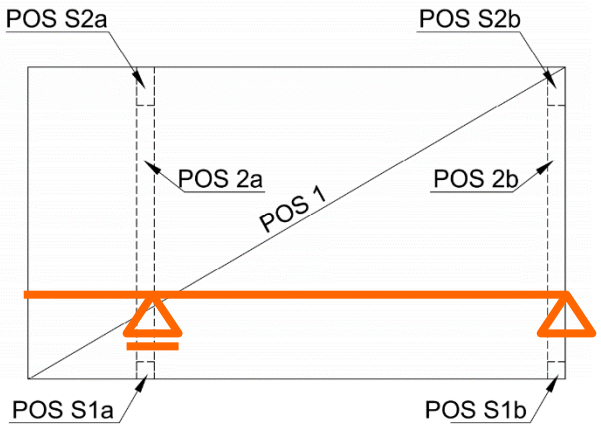
POS S1
W
H

11.02.2014.



Podaci za proračun:

MB 30
RA 400/500
Δg = 3.0 kN/m^2
p = 4.0 kN/m^2
W = ±80 kN
POS 2a b/d=30/70 cm
POS S2a b/d=30/65 cm



12.06.2013

4. *Za konstrukciju na skici potrebno je izvršiti analizu opterećenja i dimenzionisati gredu* ***POS4*** *prema* ***M*** *i* ***T****.*

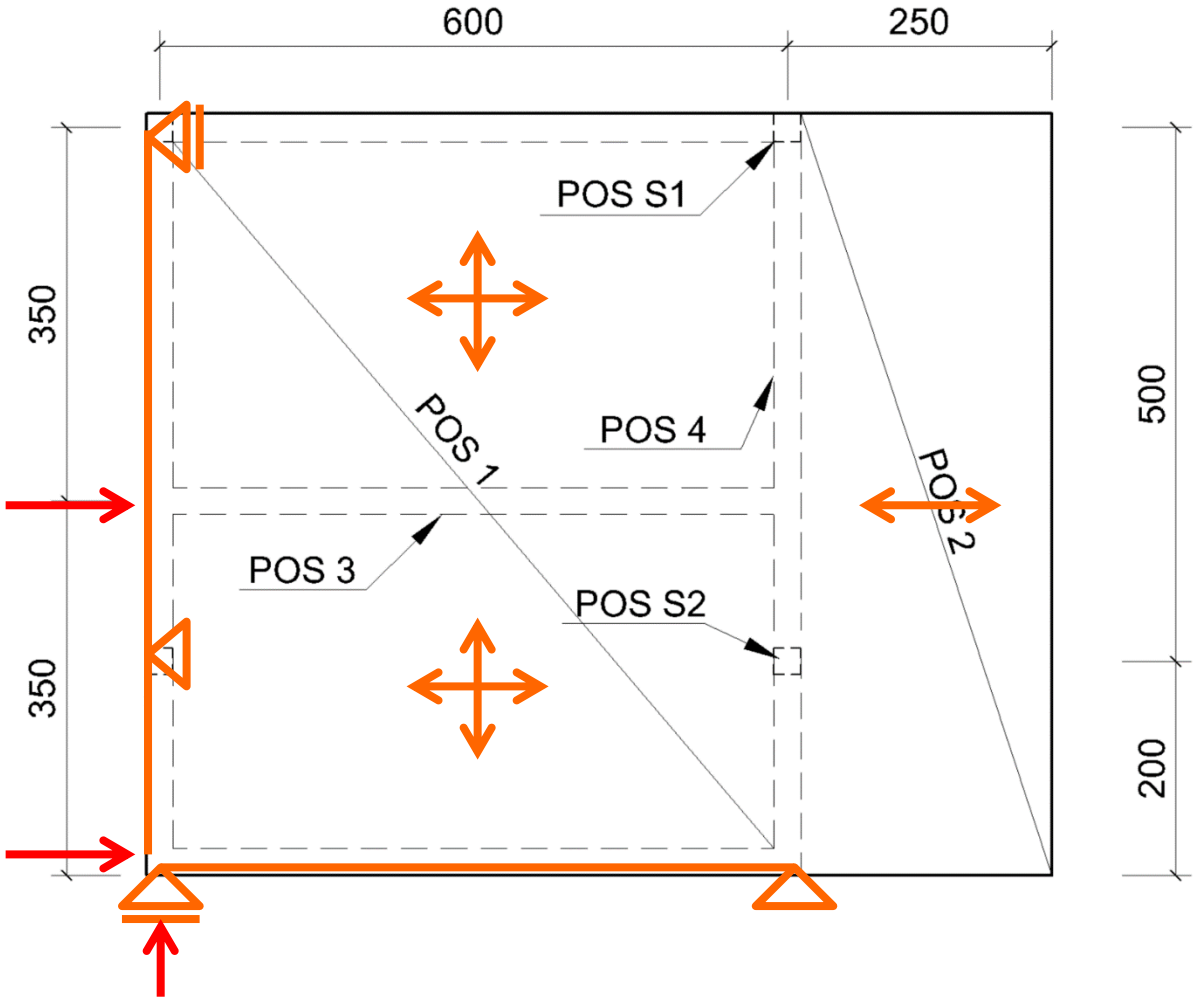
Podaci za proračun:

POS 1,POS 2 (d_p = 15 cm):

- *Δg = 1.5 kN/m^2*
- *p = 2.5 kN/m^2*

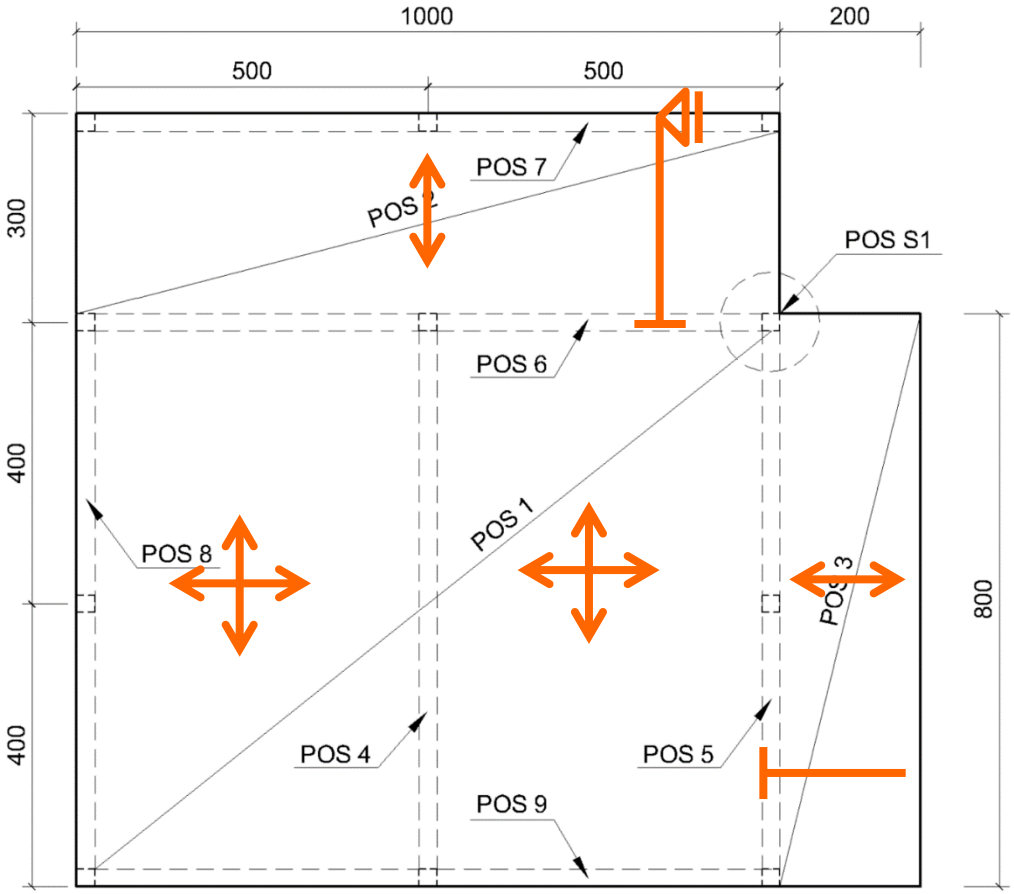
sve grede: b/d = 25/55 cm

svi stubovi: b/d = 25/25 cm



Nastaviti ovde!

06.07.2013



Podaci za proračun:

POS1, POS 2, POS 3 (d_p=16cm)

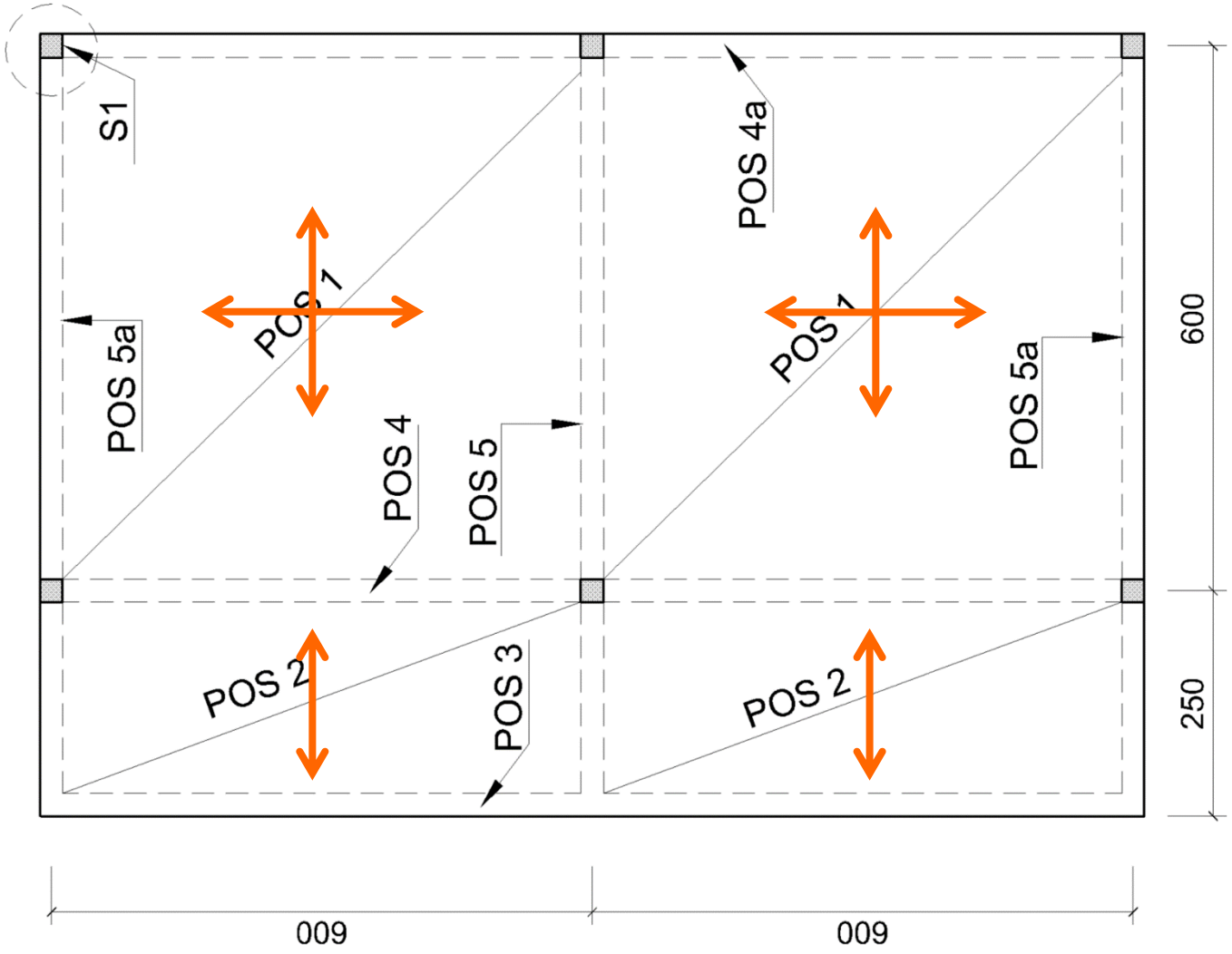
- *Δg = 2.0 kN/m^2*
- *p = 3.0 kN/m^2*

POS 4 b/d = 25/65 cm

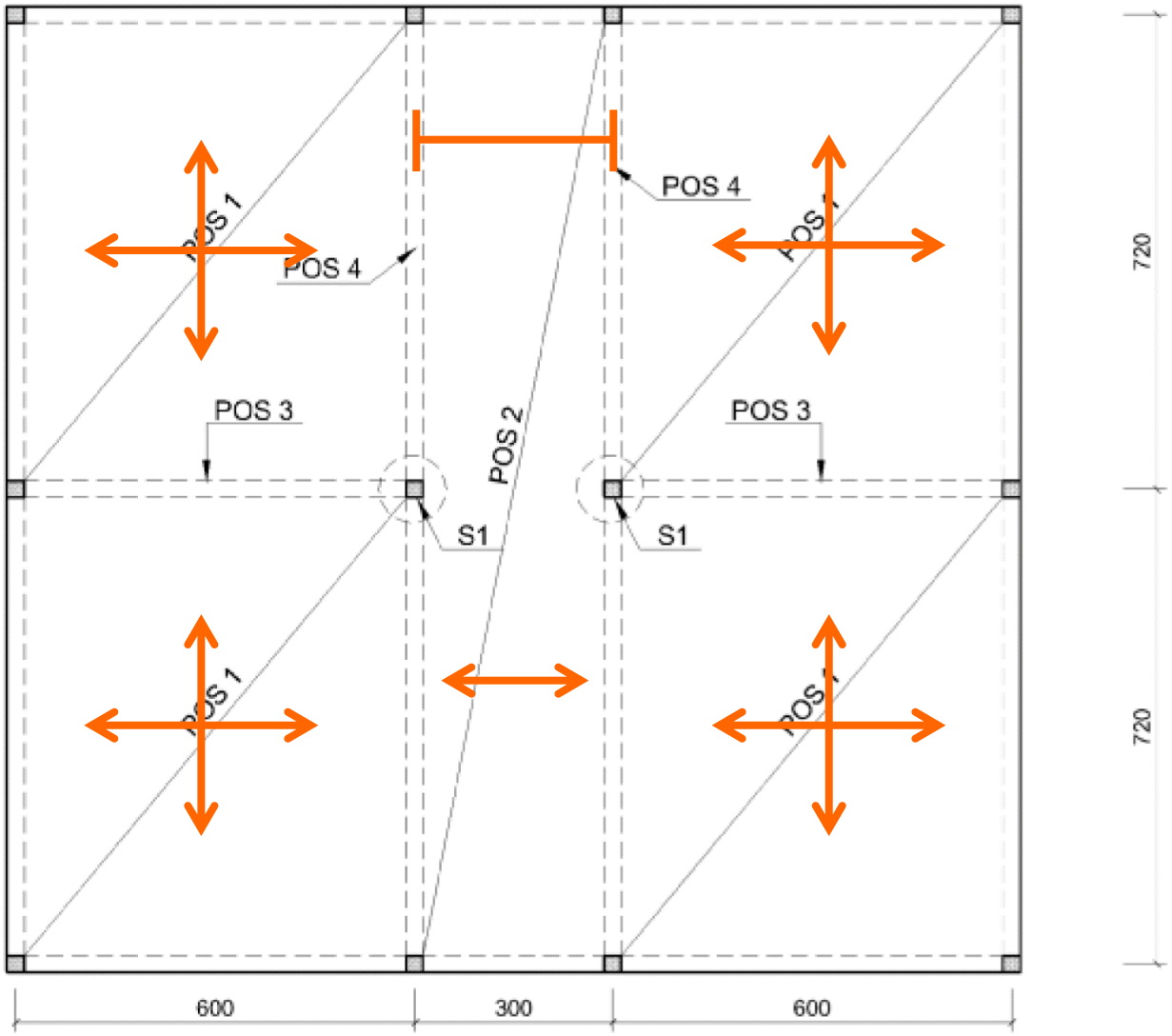
sve ostale grede: b/d = 25/55 cm

(36 poena)

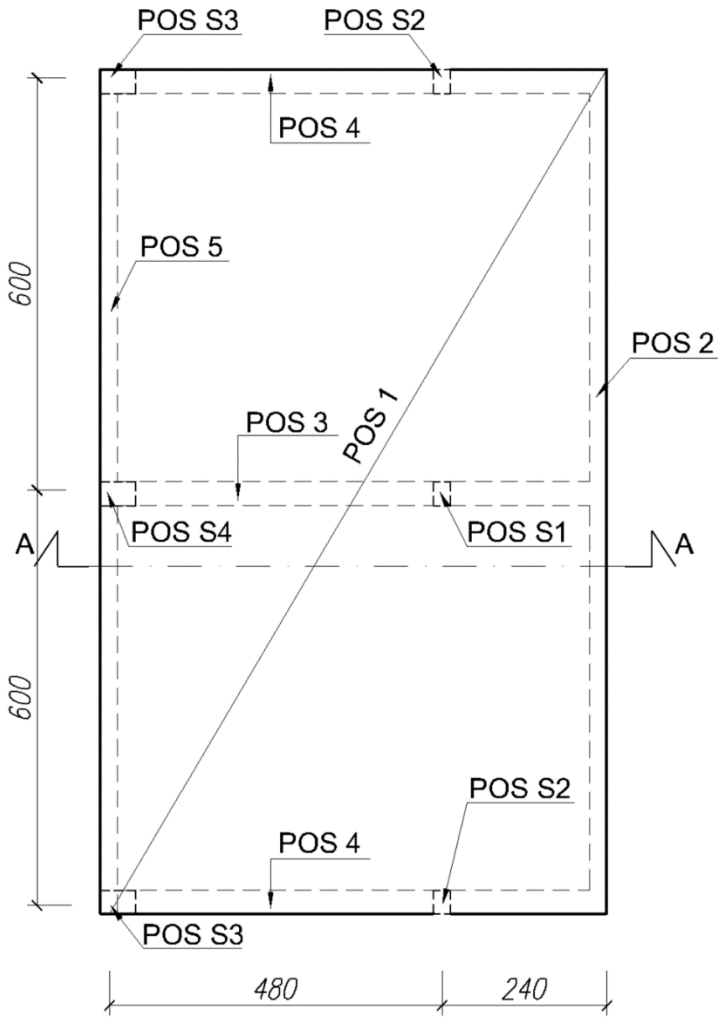
03.07.2012.


S1
POS 5a
POS 1
POS 4
POS 5
POS 2
POS 3
POS 4a
POS 1
POS 5a
POS 2
600
250
009
009

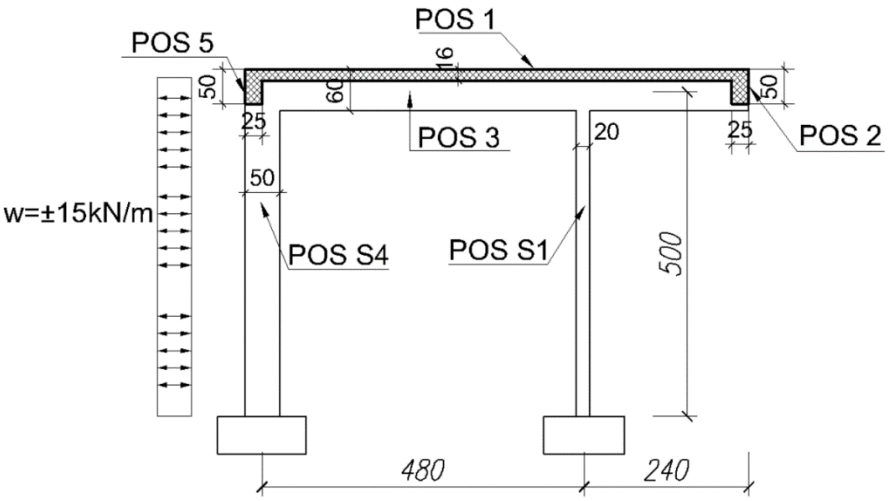
23.08.2012.


POS 1
POS 4
POS 4
POS 1
POS 3
POS 2
POS 3
S1
S1
POS 1
POS 1
720
720
600
300
600

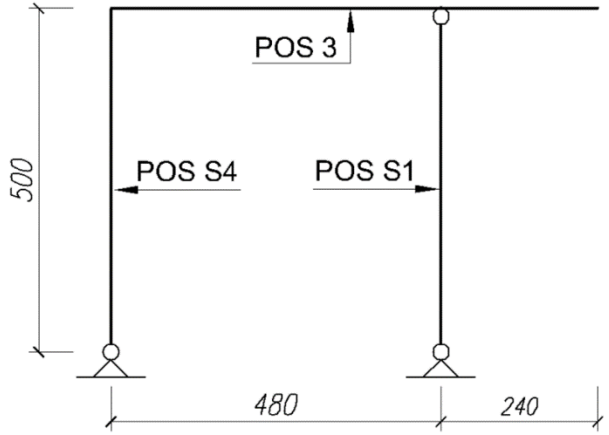
15.06.2011.



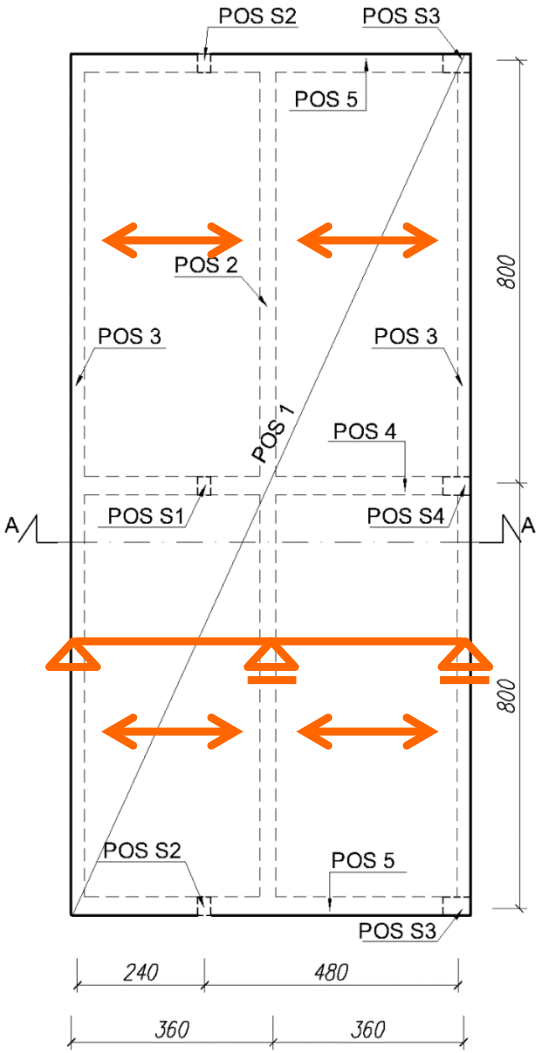
PRESEK A-A



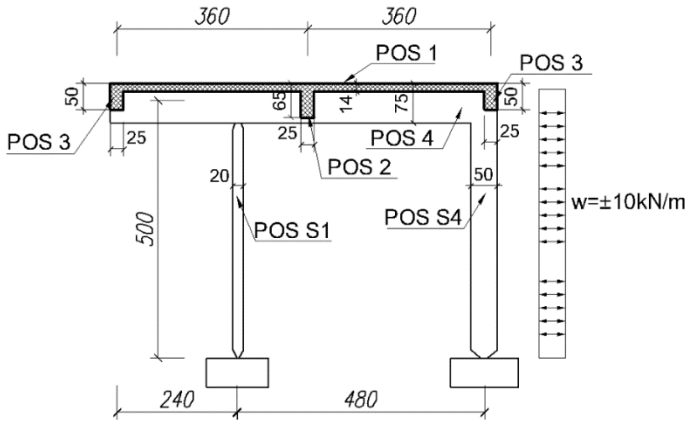
Statički sistem za ram



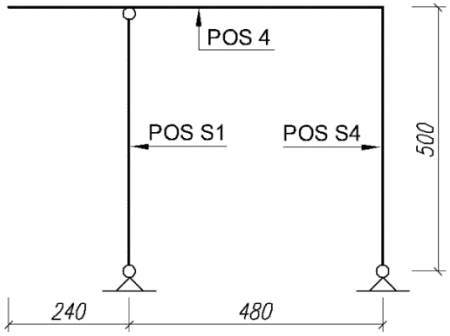
06.07.2011.

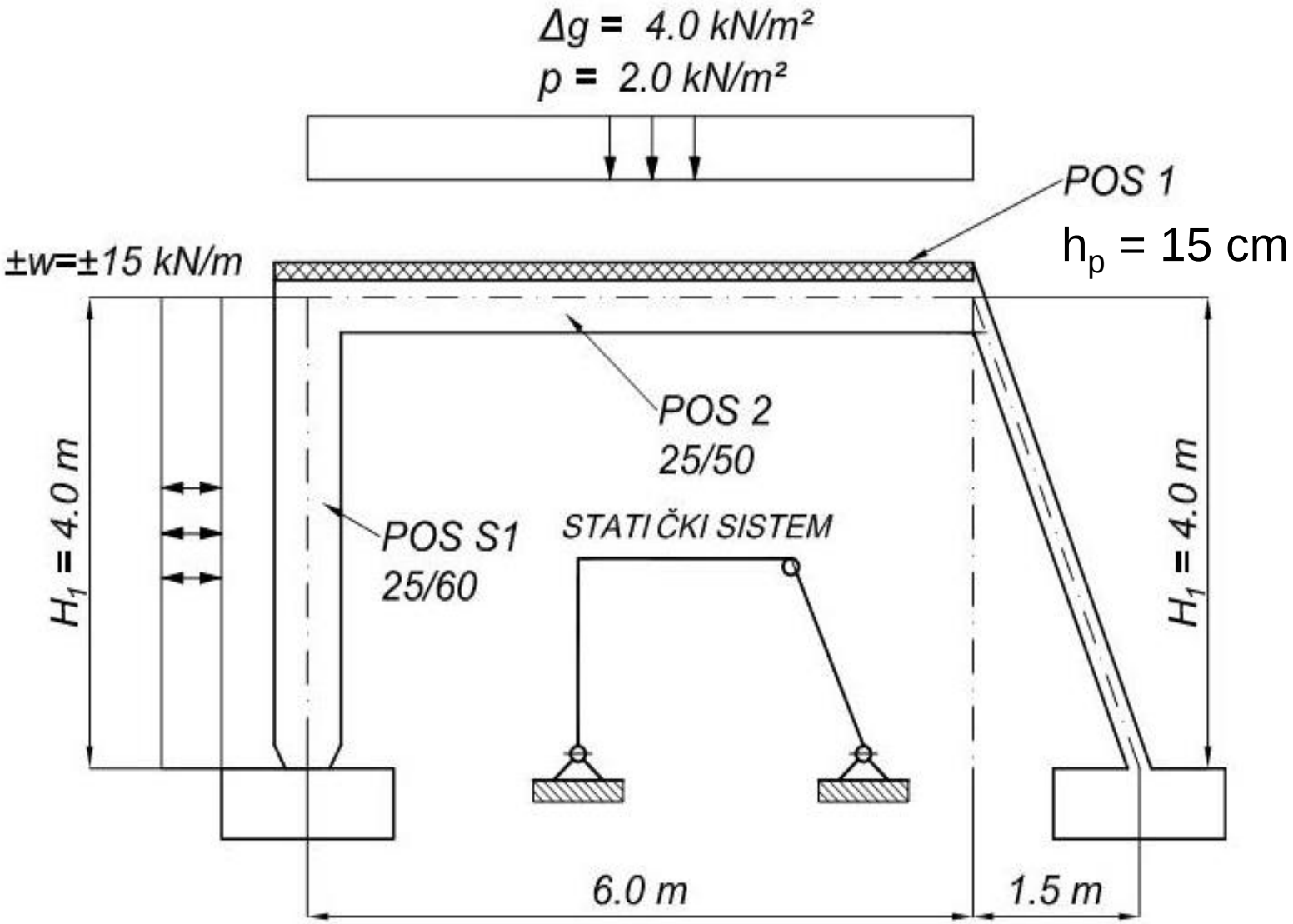


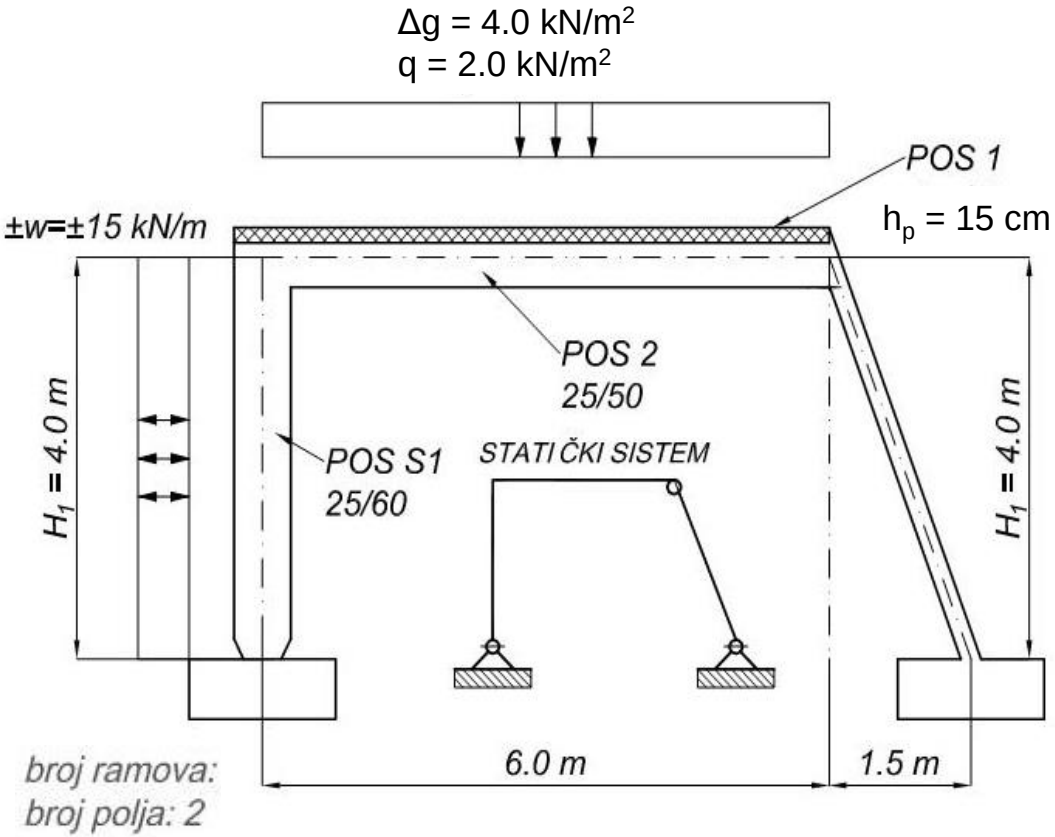
PRESEK A-A

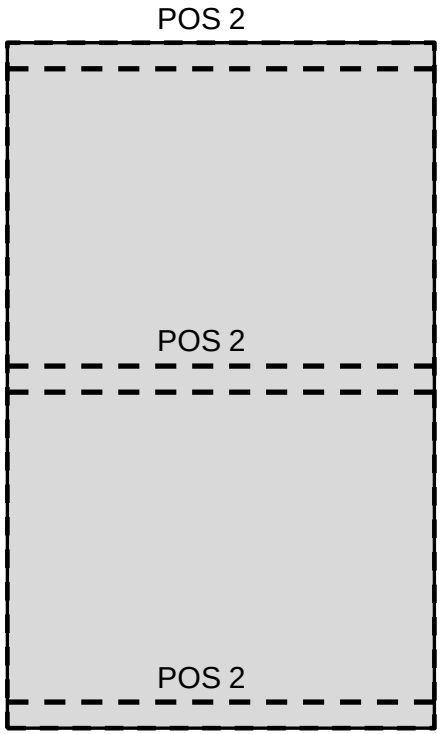


Statički sistem za ram

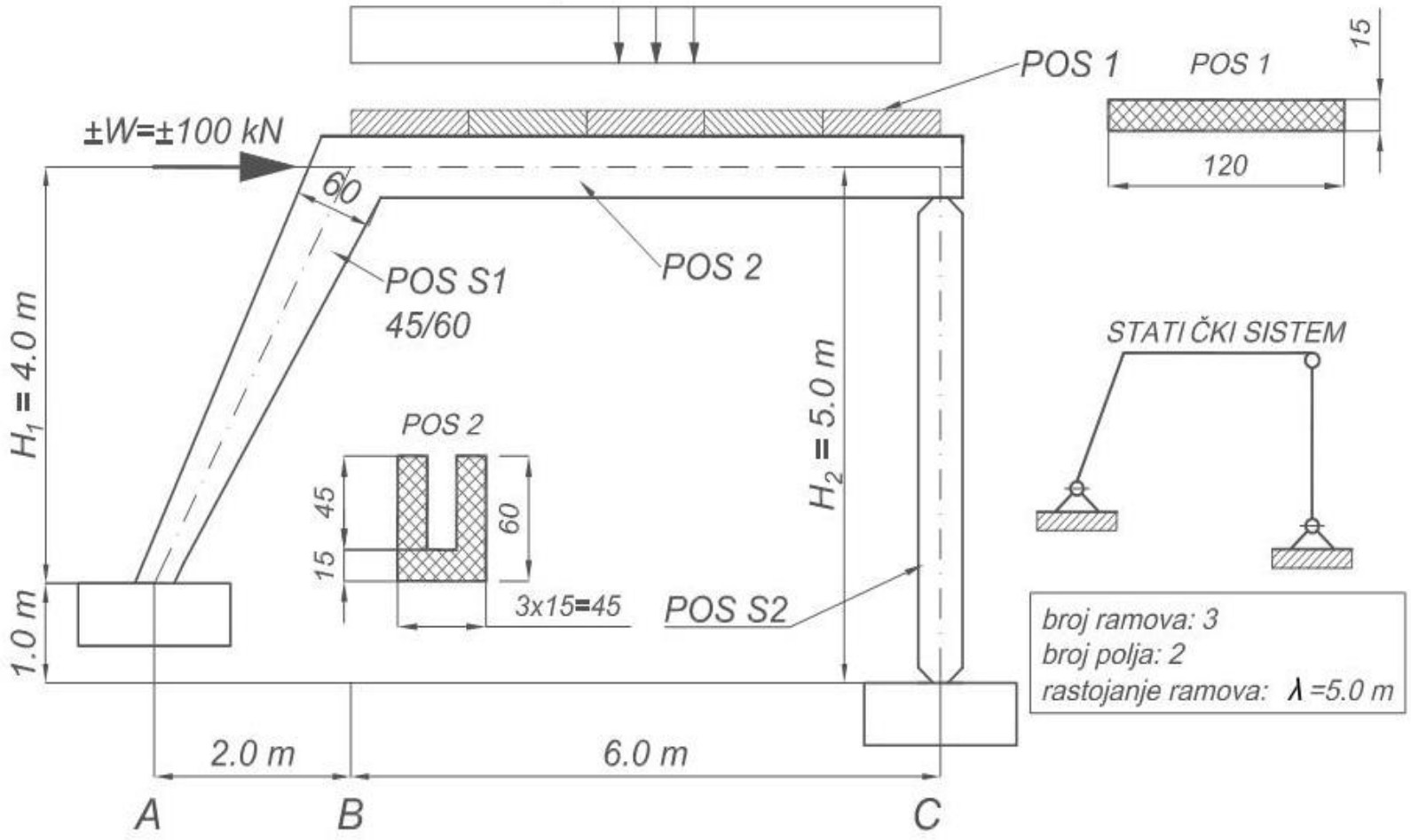


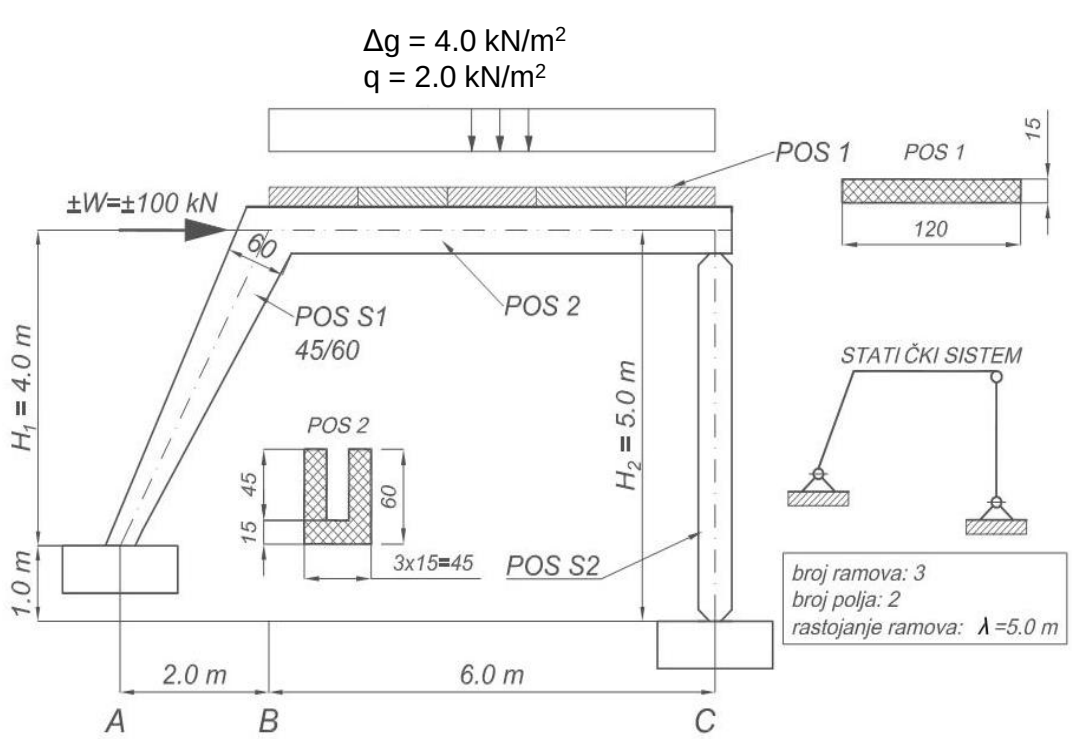

Δg = 4.0 kN/m²
p = 2.0 kN/m²
POS 1
h_p = 15 cm
±w=±15 kN/m
POS 2
25/50
POS S1
25/60
STATIČKI SISTEM
H1 = 4.0 m
H1 = 4.0 m
6.0 m
1.5 m

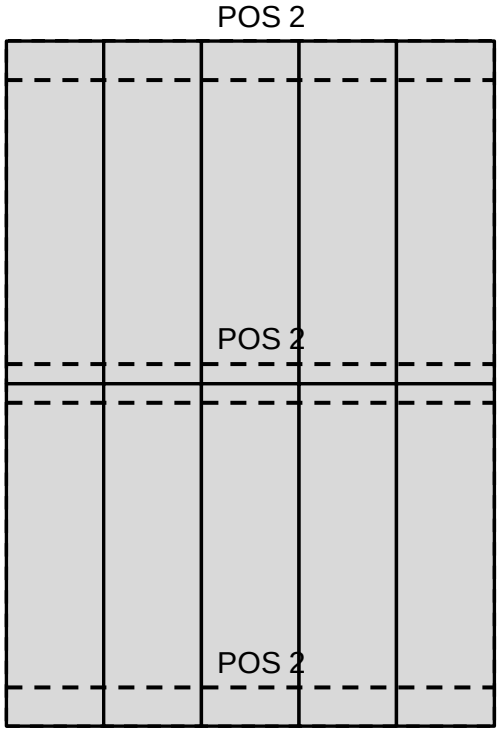

Δg = 4.0 kN/m2
q = 2.0 kN/m2
POS 1
±w=±15 kN/m
hp = 15 cm
POS 2
25/50
POS S1
25/60
STATIČKI SISTEM
H1 = 4.0 m
H1 = 4.0 m
broj ramova:
broj polja: 2
6.0 m
1.5 m


POS 2
POS 2
POS 2

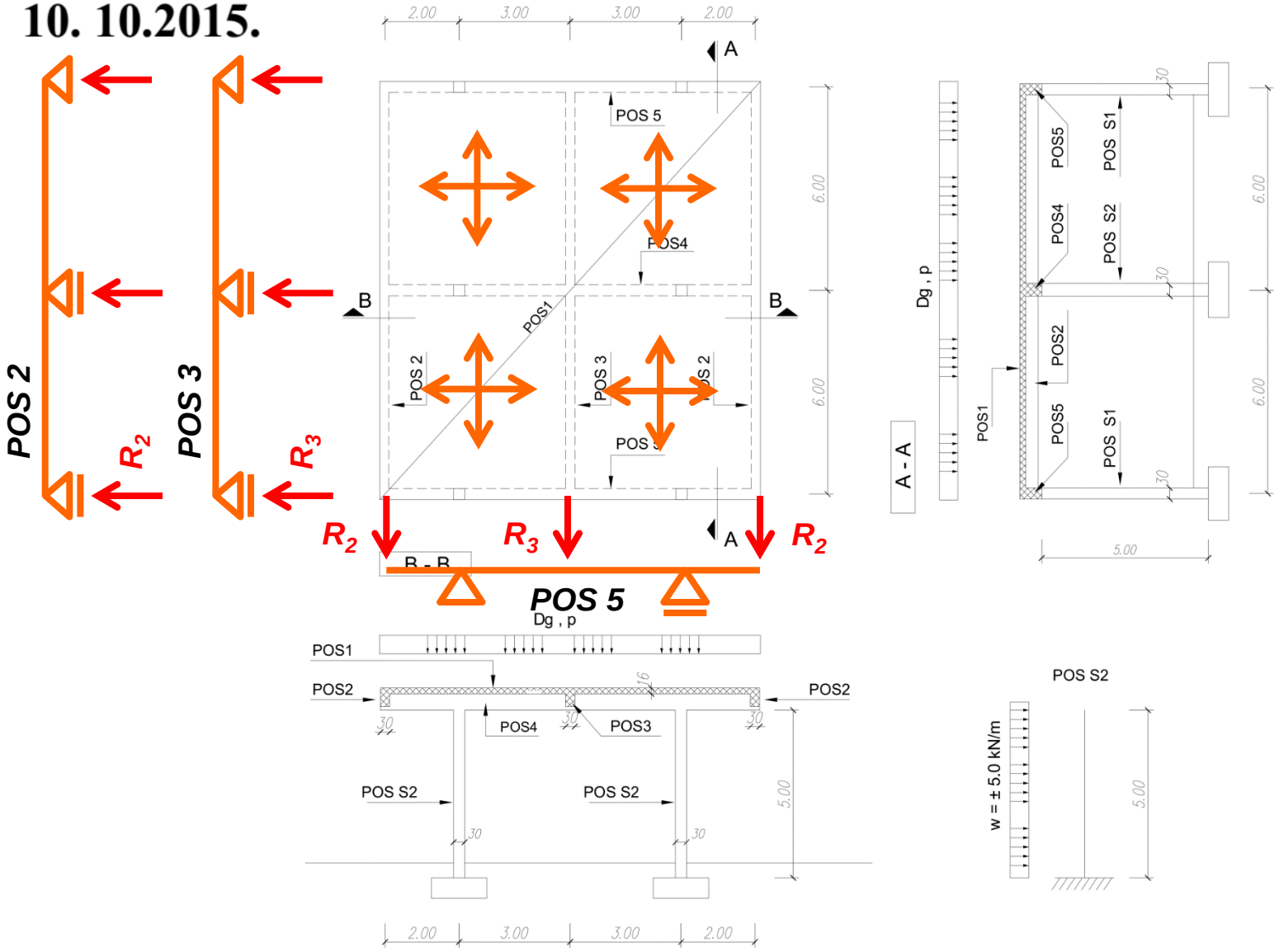
$\Delta g = 4.0\ kN/m^2$
$q = 2.0\ kN/m^2$



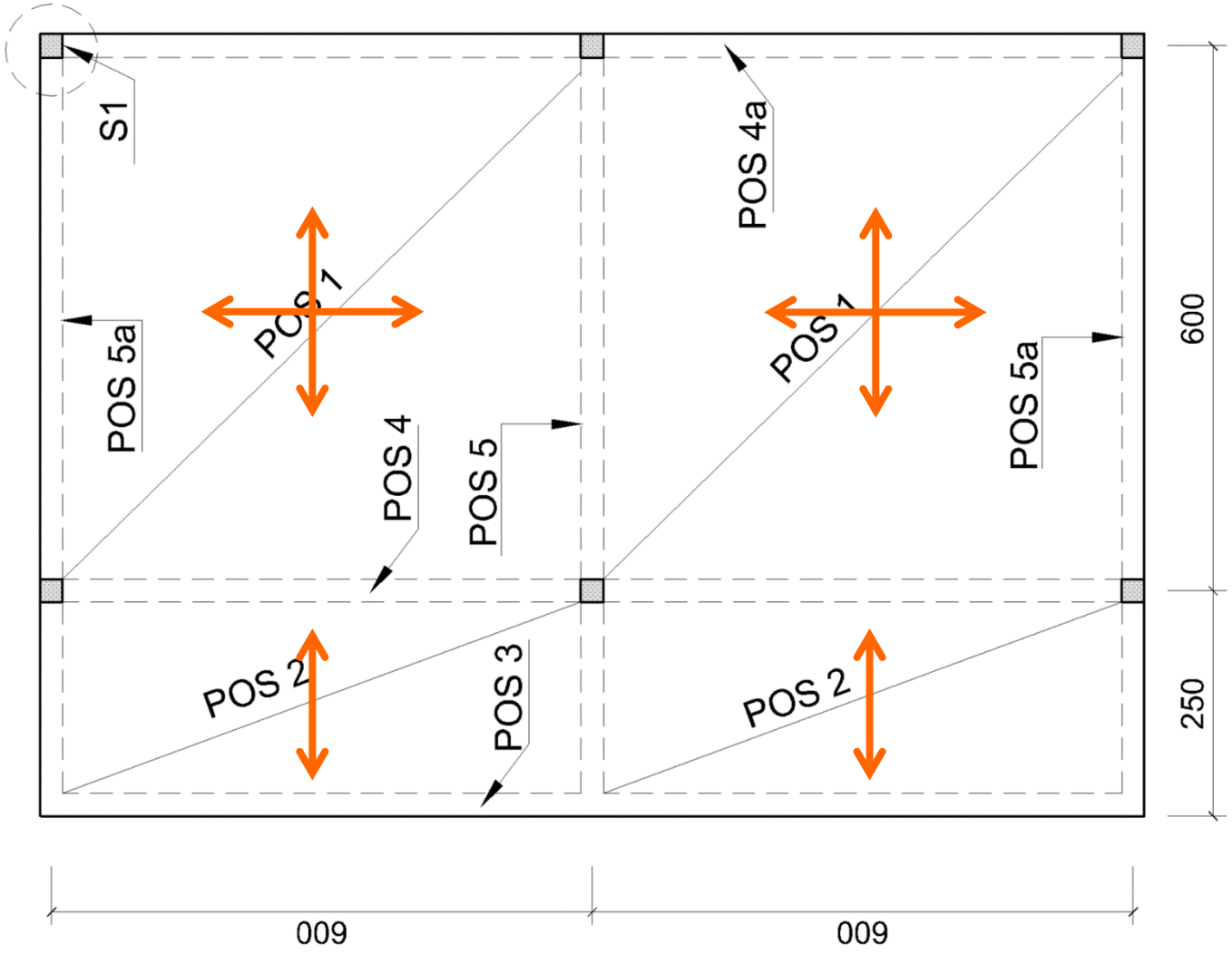

Δg = 4.0 kN/m²
q = 2.0 kN/m²
±W=±100 kN
60
POS S1
45/60
POS 2
POS 1
POS 1
15
120
H1 = 4.0 m
H2 = 5.0 m
1.0 m
POS 2
45
15
60
3x15=45
POS S2
STATIČKI SISTEM
broj ramova: 3
broj polja: 2
rastojanje ramova: λ =5.0 m
2.0 m
6.0 m
A
B
C


POS 2
POS 2
POS 2

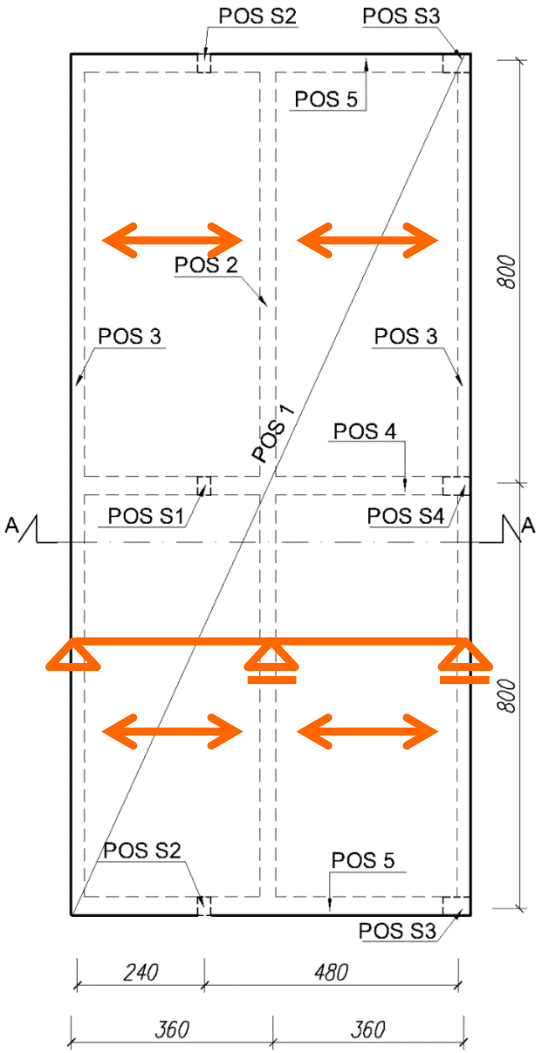
10. 10.2015.


POS 2
POS 3
R_2
R_3
2.00
3.00
3.00
2.00
A
B
POS 5
POS4
POS1
POS 2
POS 3
6.00
R_2
R_3
R_2
B - B
POS 5
Dg , p
POS1
POS2
POS2
POS4
POS3
POS S2
POS S2
5.00
30
16
A - A
Dg , p
POS1
POS5
POS S1
POS4
POS S2
POS2
POS5
POS S1
5.00
POS S2
w = ± 5.0 kN/m
5.00

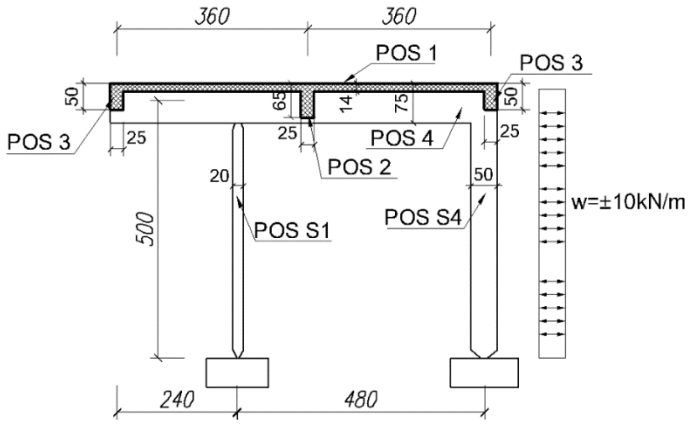
03.07.2012.


S1
POS 5a
POS 1
POS 4
POS 5
POS 2
POS 3
POS 4a
POS 1
POS 5a
POS 2
600
250
009
009

06.07.2011.



PRESEK A-A



Statički sistem za ram