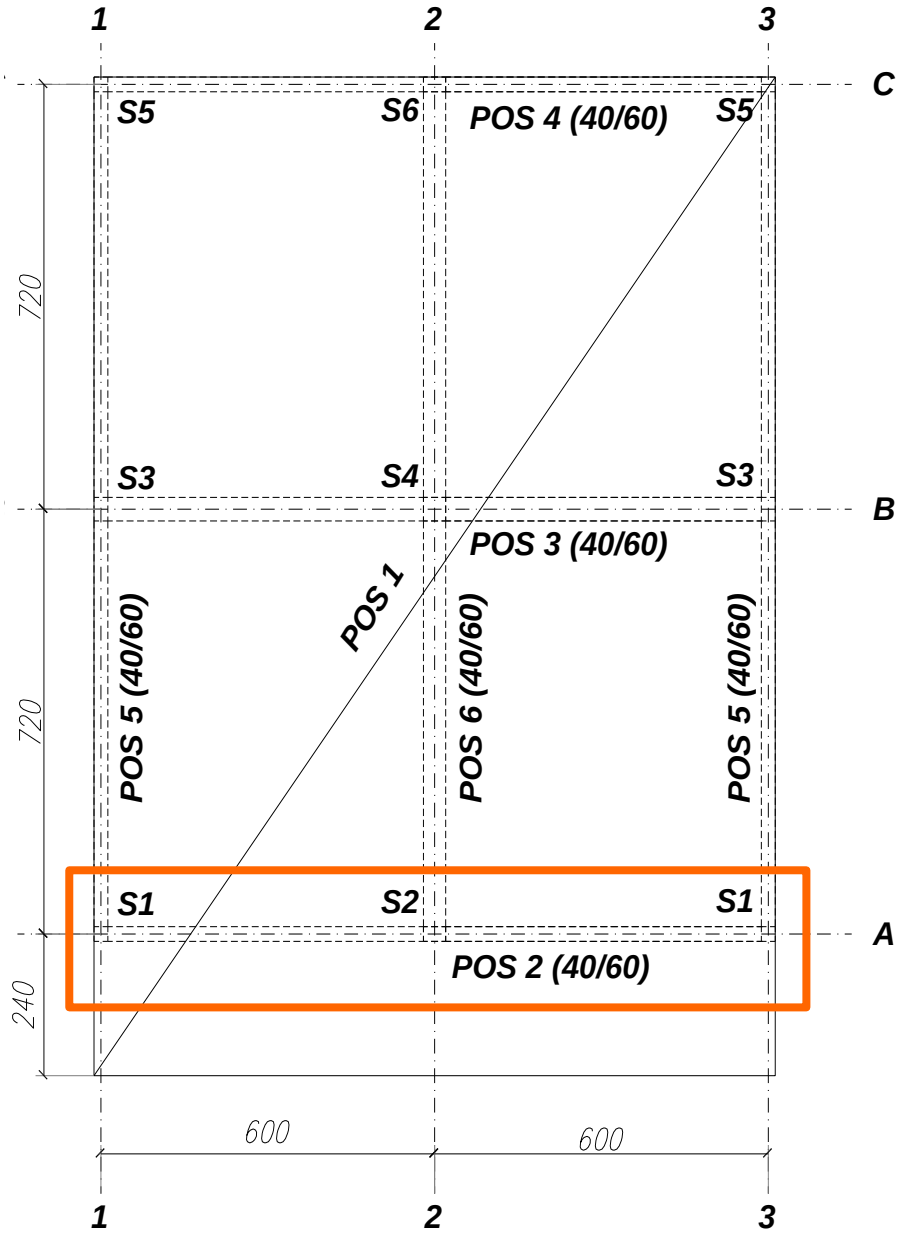
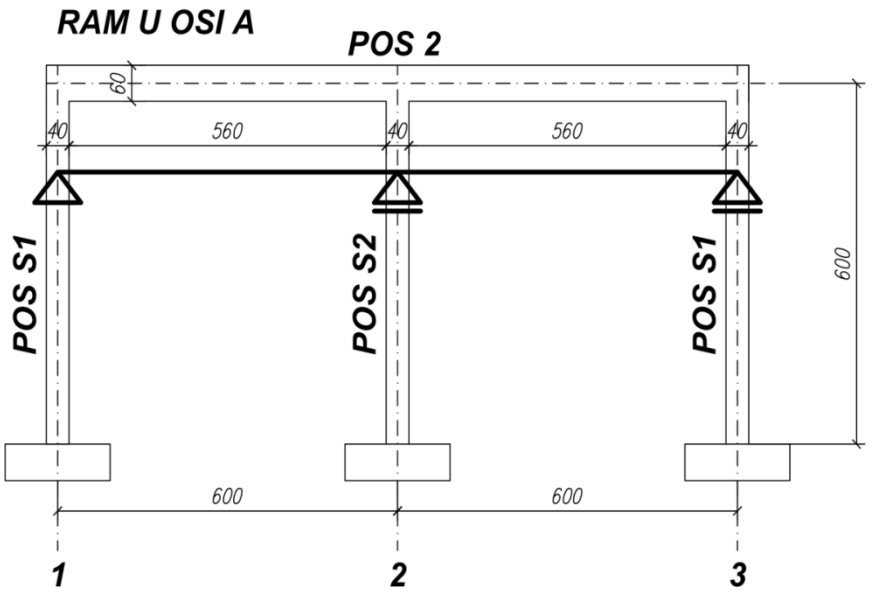
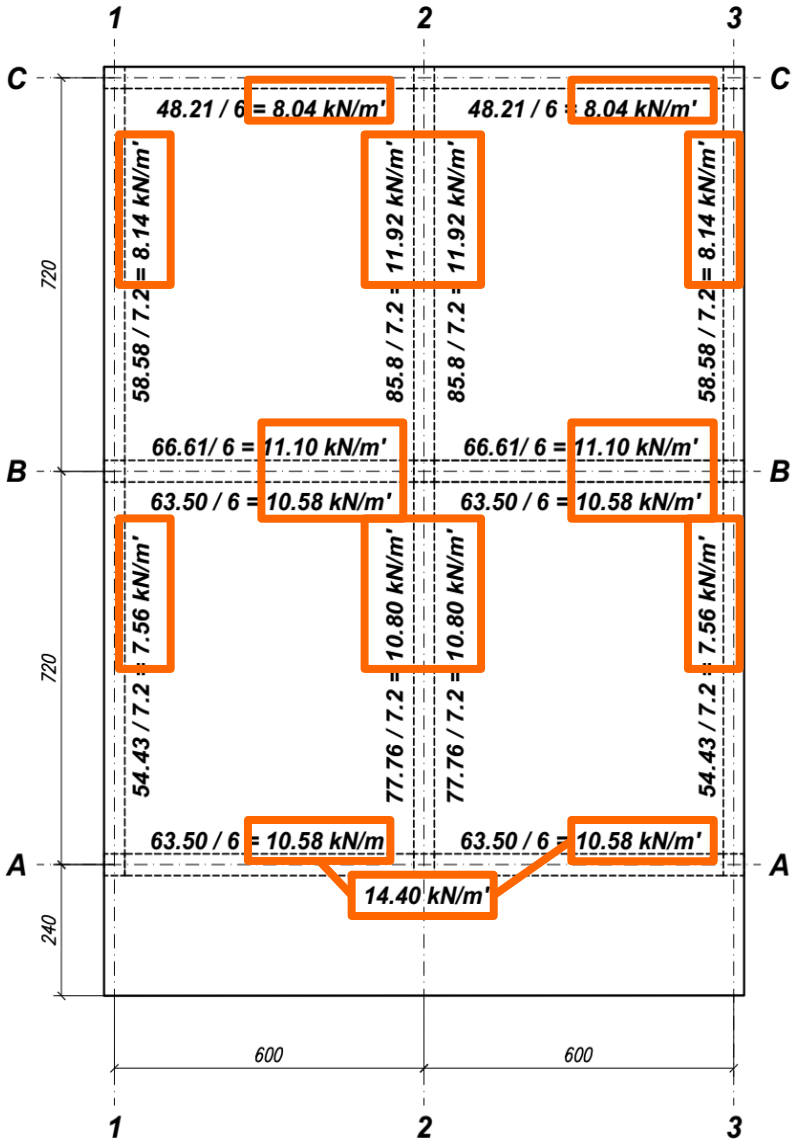
Statički sistem greda

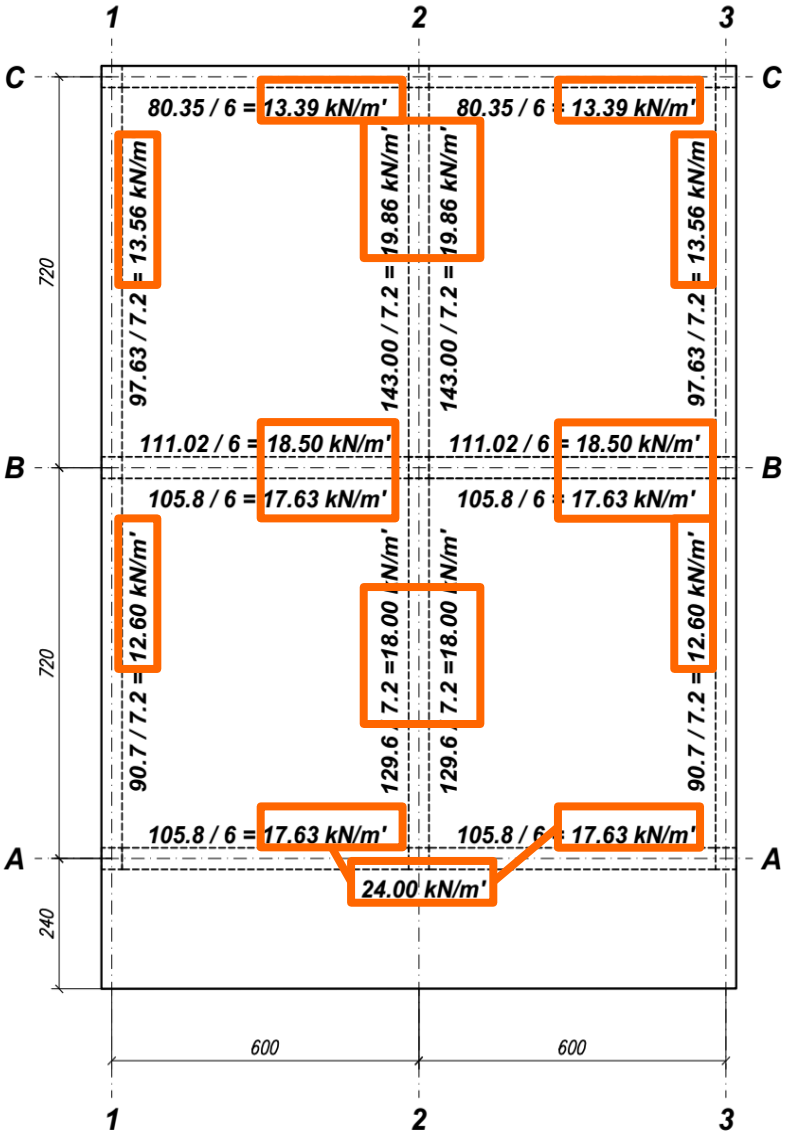




Reakcije ploče - stalno opterećenje g (kN/m')

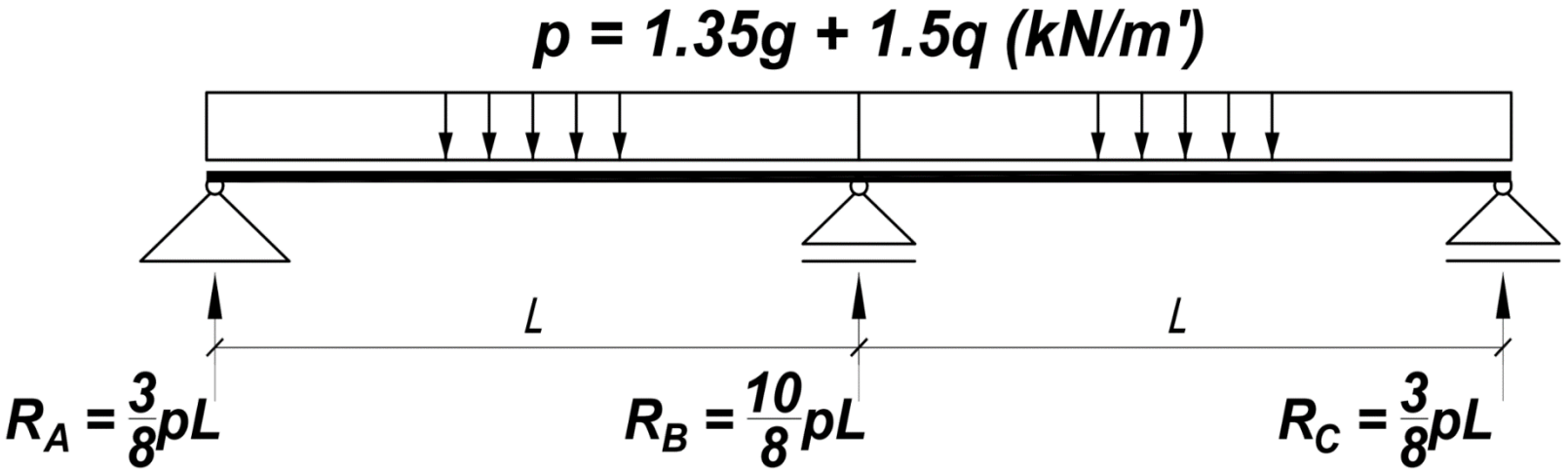


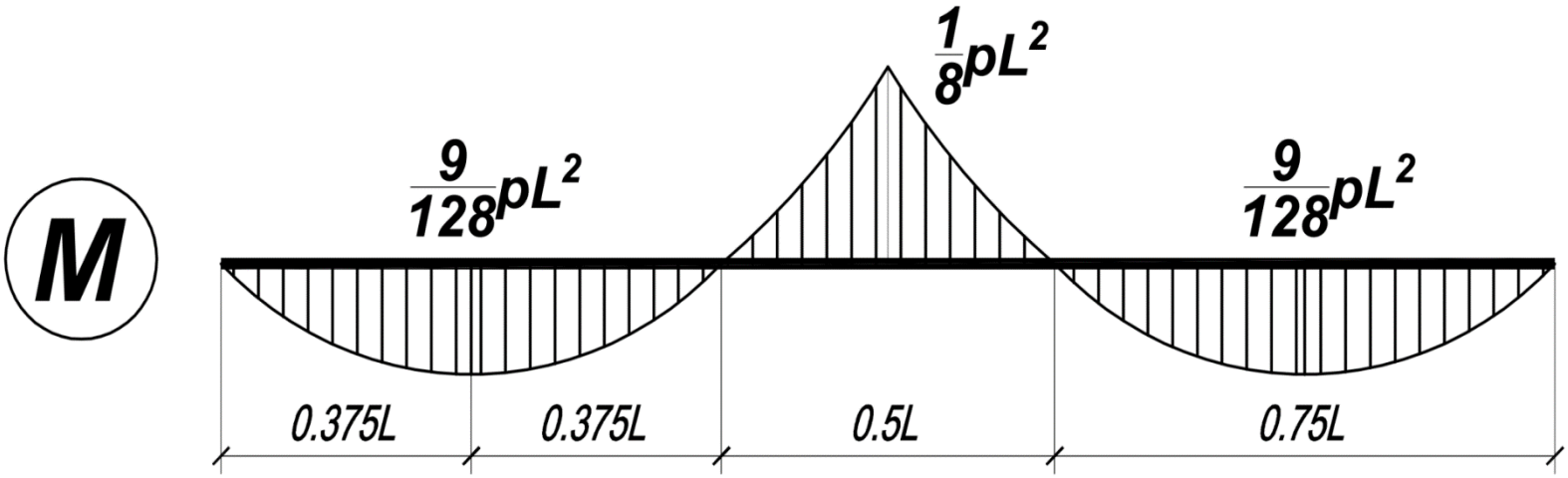
Reakcije ploče - promenljivo opterećenje q (kN/m')



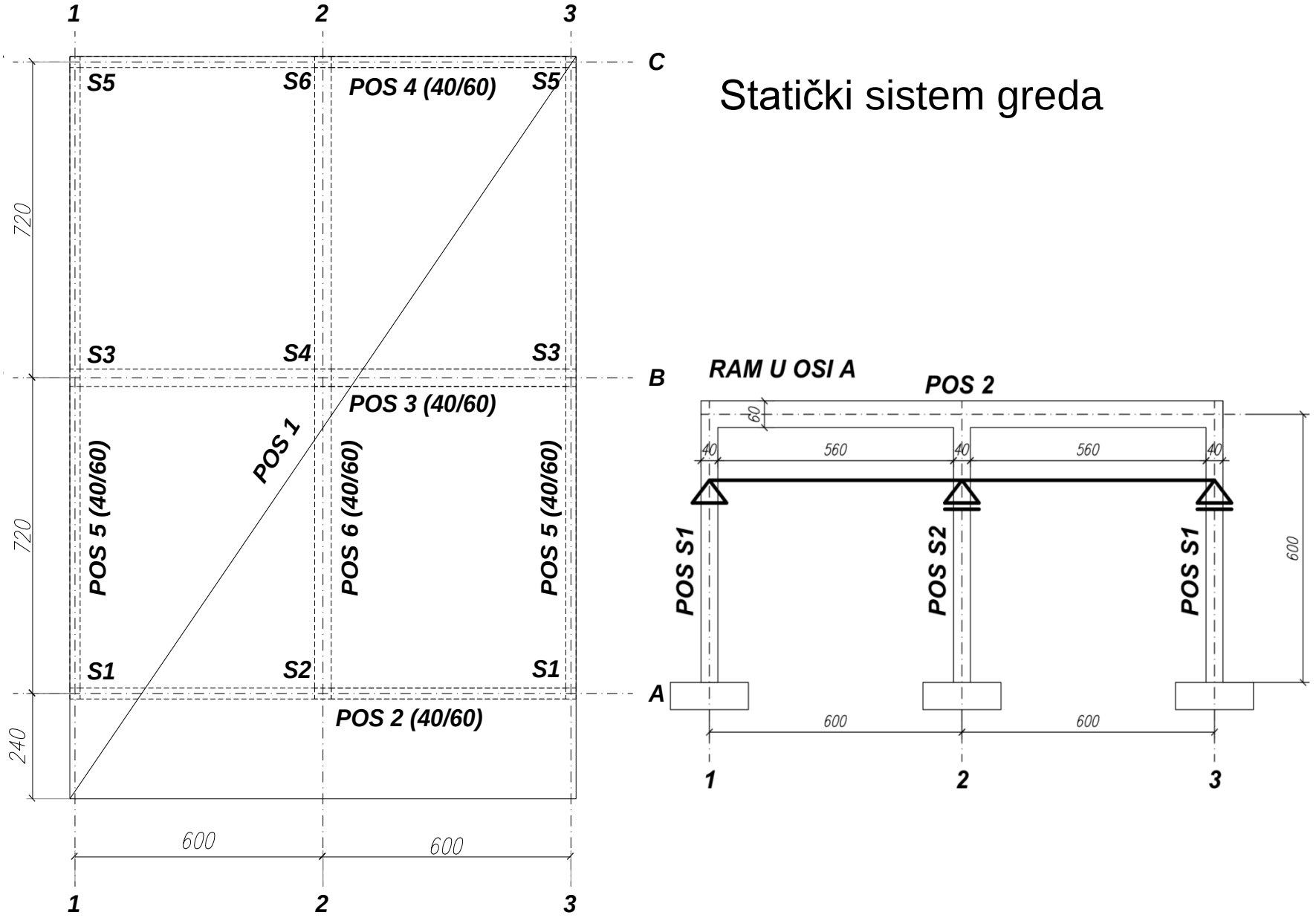
6. Analiza opterećenja greda

Grede u X –pravcu



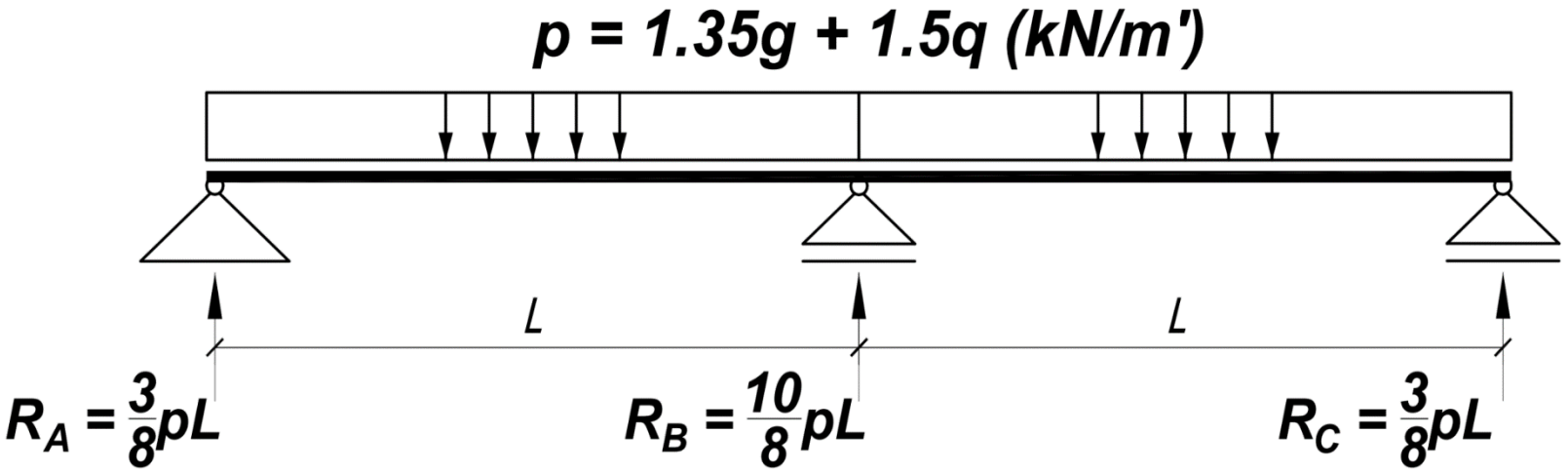


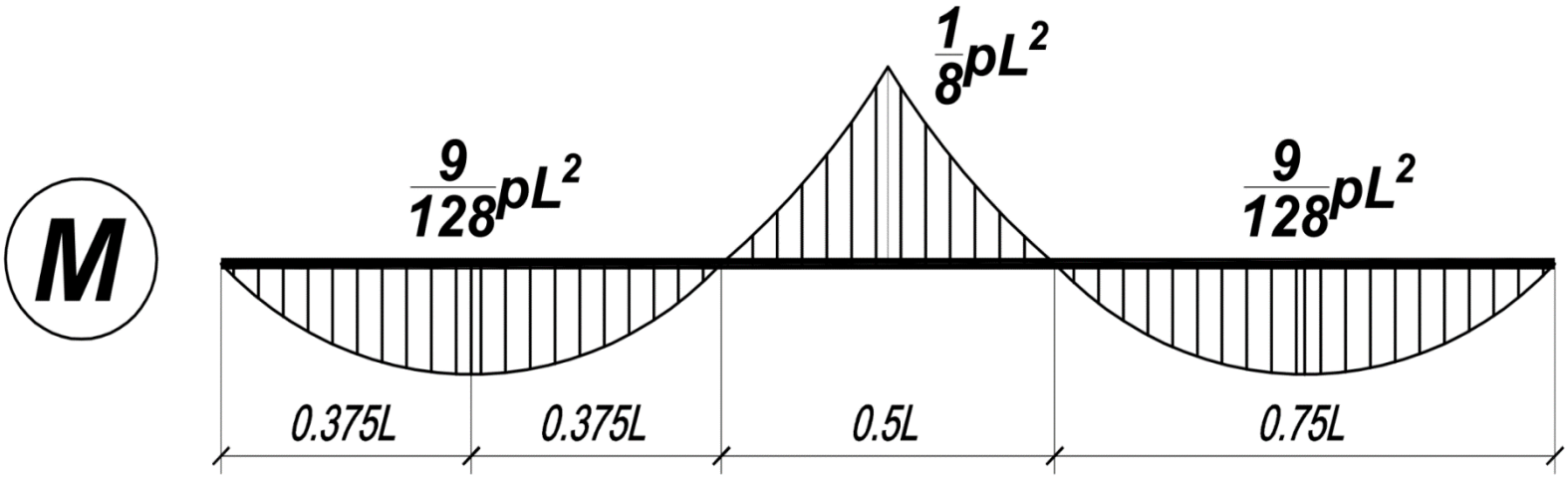
Statički sistem greda



6. Analiza opterećenja greda

Grede u X –pravcu





Dimenzionisanje greda

6.1. Greda POS 2

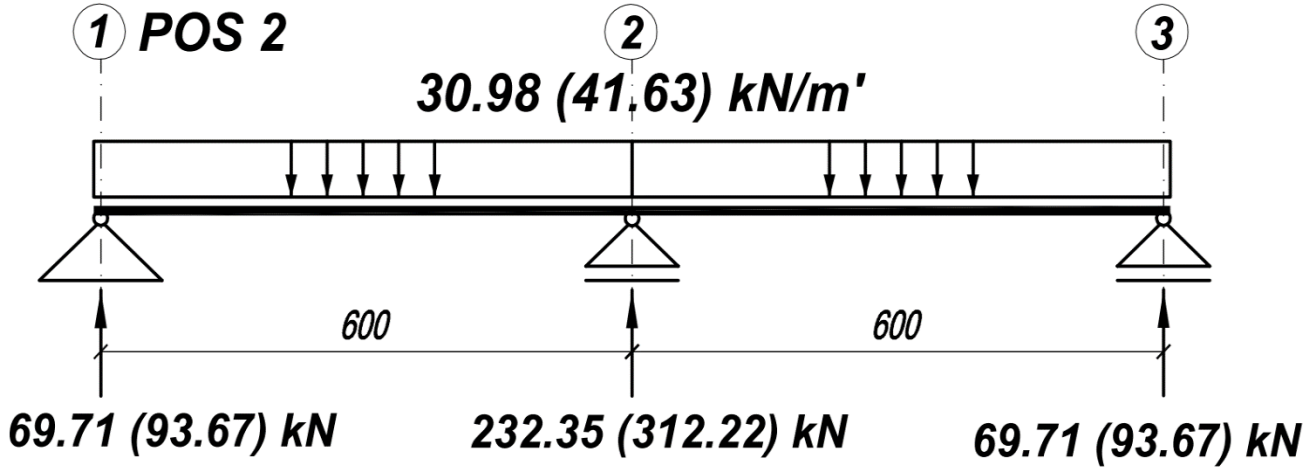
Stalno opterećenje

sopstvena težina grede $g_{st} = h \times b \times \gamma_c = 0.6 \times 0.4 \times 25.0 = 6.0 \text{ kN/m}$

reakcija sa ploče $g_p = 10.58 + 14.40 = 24.98 \text{ kN/m}$

$$\boldsymbol{g = 30.98 \text{ kN/m}}$$

Promenljivo opterećenje $\boldsymbol{q = 17.63 + 24.00 = 41.63 \text{ kN/m}}$



6.2. Greda POS 3

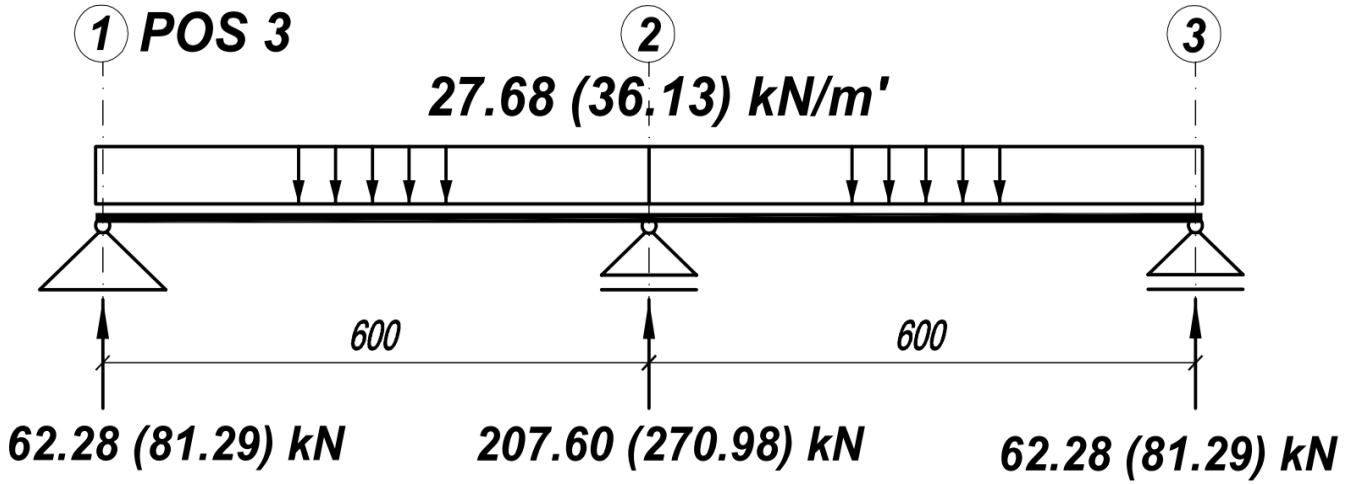
Stalno opterećenje

sopstvena težina grede $g_{st} = h \times b \times \gamma_c = 0.6 \times 0.4 \times 25.0 = 6.0 \text{ kN/m}$

reakcija sa ploče $g_p = 10.58 + 11.10 = 21.68 \text{ kN/m}$

$$\boldsymbol{g = 27.68 \text{ kN/m}}$$

Promenljivo opterećenje $\boldsymbol{q} = 17.63 + 18.50 = \boldsymbol{36.13 \text{ kN/m}}$



6.3. Greda POS 4

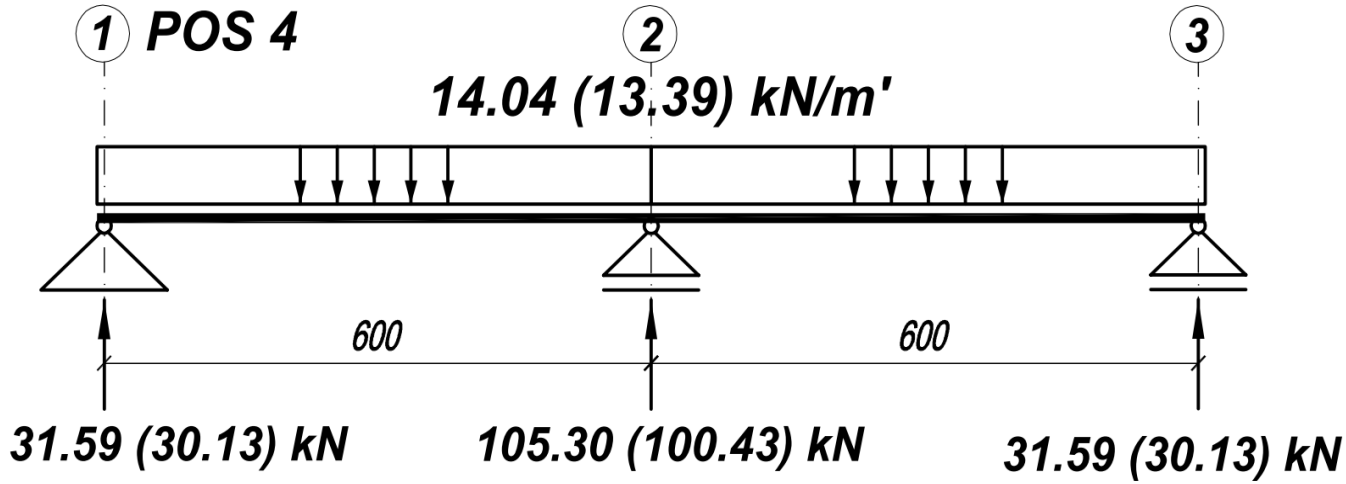
Stalno opterećenje

sopstvena težina grede $g_{st} = {}^{h} \times b \times \gamma_c = 0.6 \times 0.4 \times 25.0 = 6.0 \text{ kN/m}$

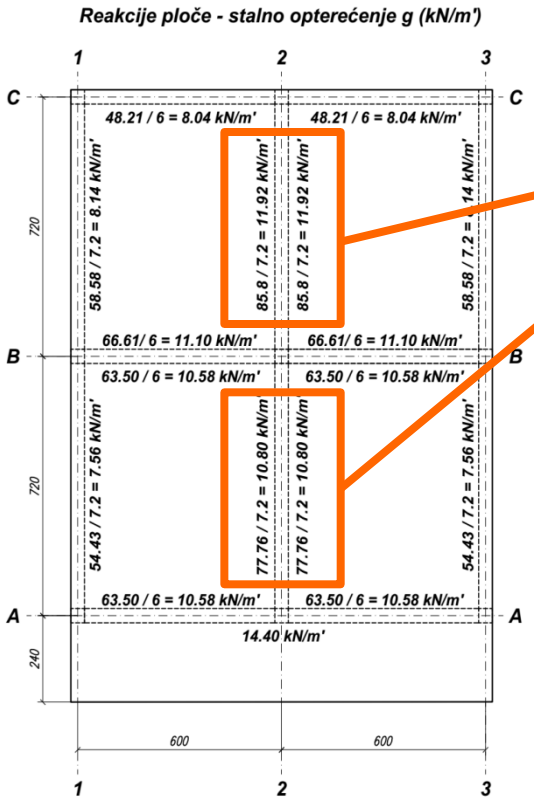
reakcija sa ploče $g_p = 8.04 \text{ kN/m}$

$$\boldsymbol{g = 14.04 \text{ kN/m}}$$

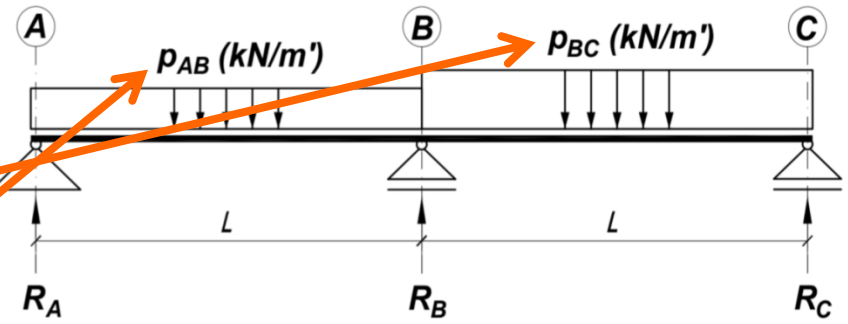
Promenljivo opterećenje $\boldsymbol{q = 13.39 \text{ kN/m}}$



Grede u Y - pravcu

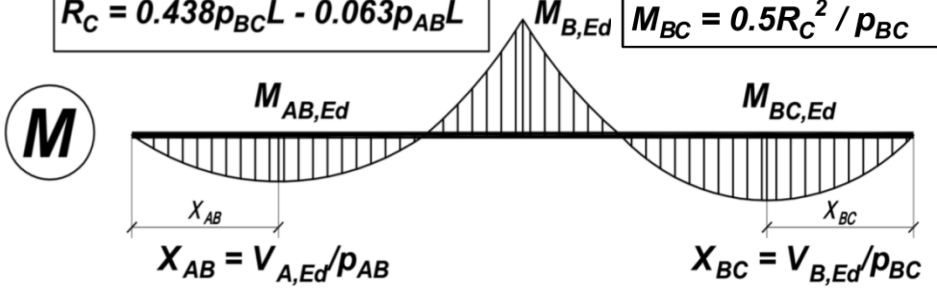


$p = 1.35g + 1.5q$ (kN/m')



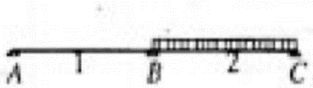
$$R_A = 0.438 p_{AB} L - 0.063 p_{BC} L$$
$$R_B = 0.625 (p_{AB} + p_{BC}) L$$
$$R_C = 0.438 p_{BC} L - 0.063 p_{AB} L$$

$$M_{AB} = 0.5 R_A^2 / p_{AB}$$
$$M_B = -0.063 (p_{AB} + p_{BC}) L^2$$
$$M_{BC} = 0.5 R_C^2 / p_{BC}$$



$X_{AB} = V_{A,Ed} / p_{AB}$ $\qquad$ $X_{BC} = V_{B,Ed} / p_{BC}$

$M_{11(max)}$	0.096	pl^2
$M_{12(max)}$	–	
M_b	–0.063	pl^2
$A = T_{1a(max)}$	0.438	pl



$M_{11(max)}$	–	
$M_{12(max)}$	–	
$A = T_{1a(min)}$	–0.063	pl

6.4. Greda POS 5

Stalno opterećenje

sopstvena težina grede $g_{st} = h \times b \times \gamma_c = 0.6 \times 0.4 \times 25.0 = 6.0$ kN/m

reakcija sa ploče u polju AB $g_{AB} = 7.56$ kN/m

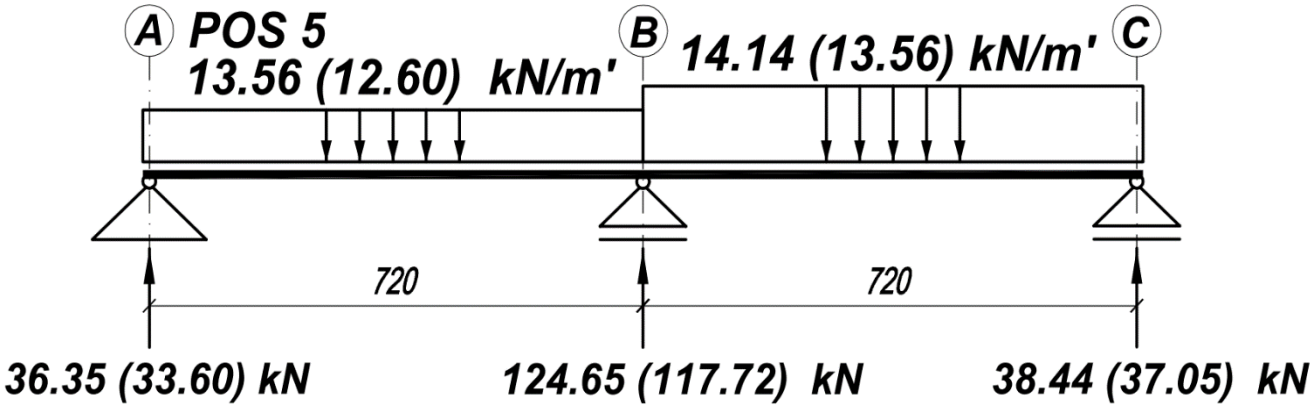
reakcija sa ploče u polju BC $g_{BC} = 8.14$ kN/m

$\boldsymbol{g_{AB} = 13.56}$ **kN/m**

$\boldsymbol{g_{BC} = 14.14}$ **kN/m**

Promenljivo opterećenje $\boldsymbol{q_{AB} = 12.60}$ **kN/m**

$\boldsymbol{q_{BC} = 13.56}$ **kN/m**



6.5. Greda POS 6

Stalno opterećenje

sopstvena težina grede $g_{st} = h \times b \times \gamma_c = 0.6 \times 0.4 \times 25.0 = 6.0$ kN/m

reakcija sa ploče u polju AB $g_{AB} = 2 \times 10.80 = 21.60$ kN/m

reakcija sa ploče u polju BC $g_{BC} = 2 \times 11.92 = 23.84$ kN/m

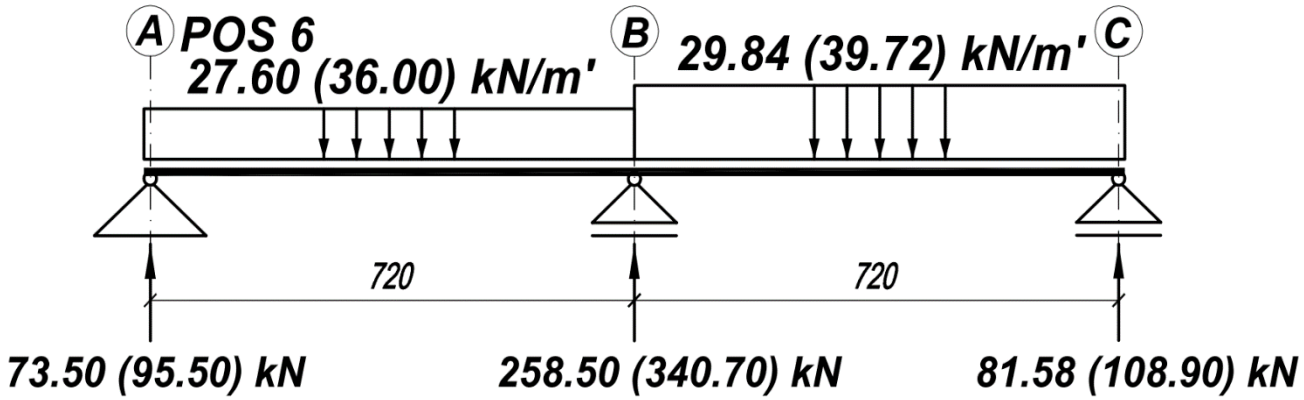
---

$$\boldsymbol{g_{AB} = 27.60\ \text{kN/m}}$$

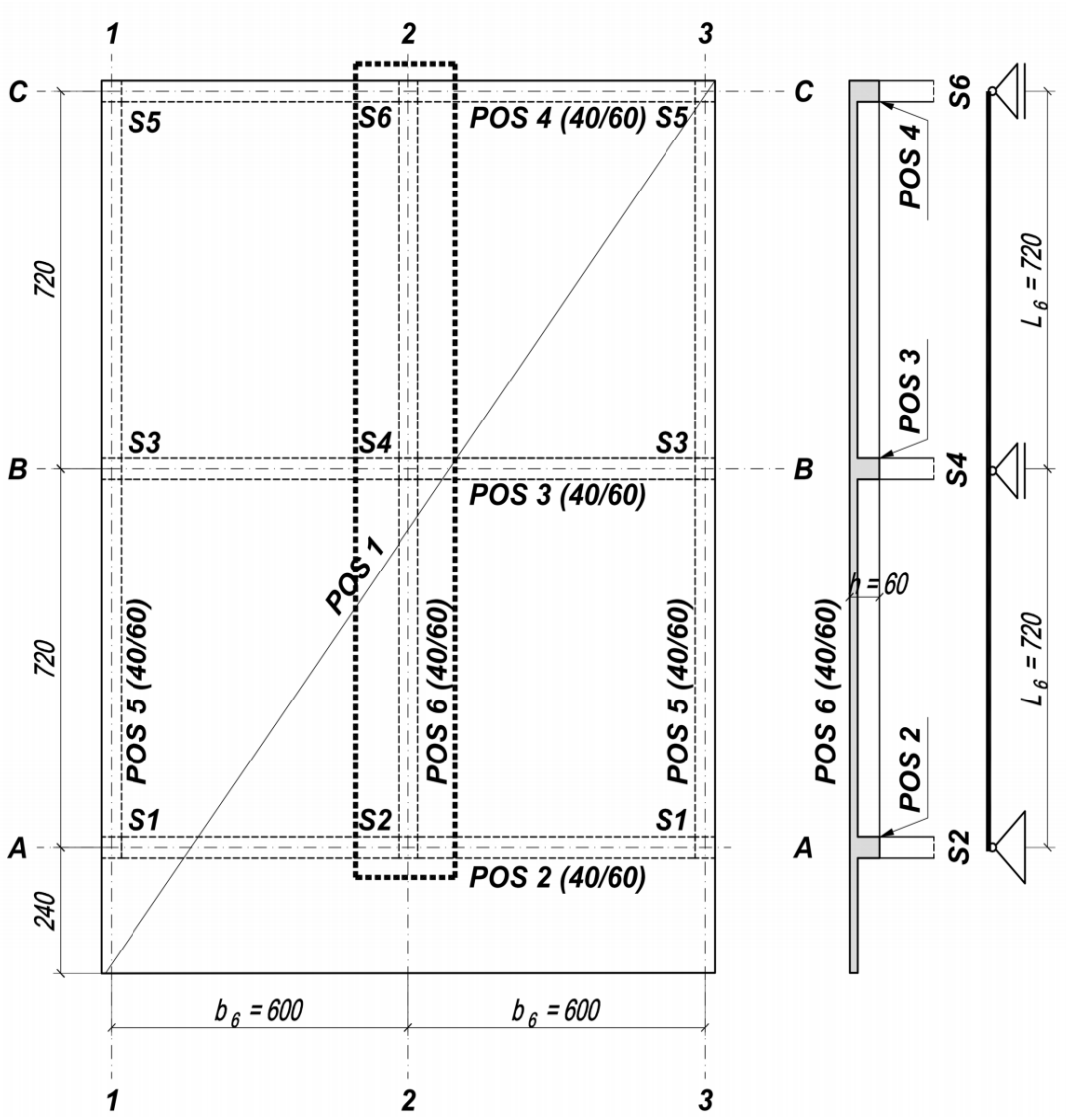
$$\boldsymbol{g_{BC} = 29.84\ \text{kN/m}}$$

Promenljivo opterećenje $\boldsymbol{q_{AB} = 2 \times 18.00 = 36.00\,\text{kN/m}}$

$\boldsymbol{q_{BC} = 2 \times 19.86 = 39.72\,\text{kN/m}}$



6.5.1. Greda POS 6 - Dimenzionisanje prema momentima savijanja



POS 6 b/h = 40/60 cm

C30/37 $f_{cd} = 1.7$ kN/cm^2

B500B $f_{yd} = 43.48$ kN/cm^2

XC3 $C_{nom} = 3.5$ cm

Maksimalni moment u polju AB:

$$M_{AB,g} = \frac{{R_{A,g}}^2}{2 \times g_{AB}} = \frac{73.5^2}{2 \times 27.6} = 97.9\ kNm$$

$$M_{AB,q} = \frac{{R_{A,q}}^2}{2 \times q_{AB}} = \frac{95.5^2}{2 \times 36.0} = 126.7\ kNm$$

Maksimalni moment u polju BC:

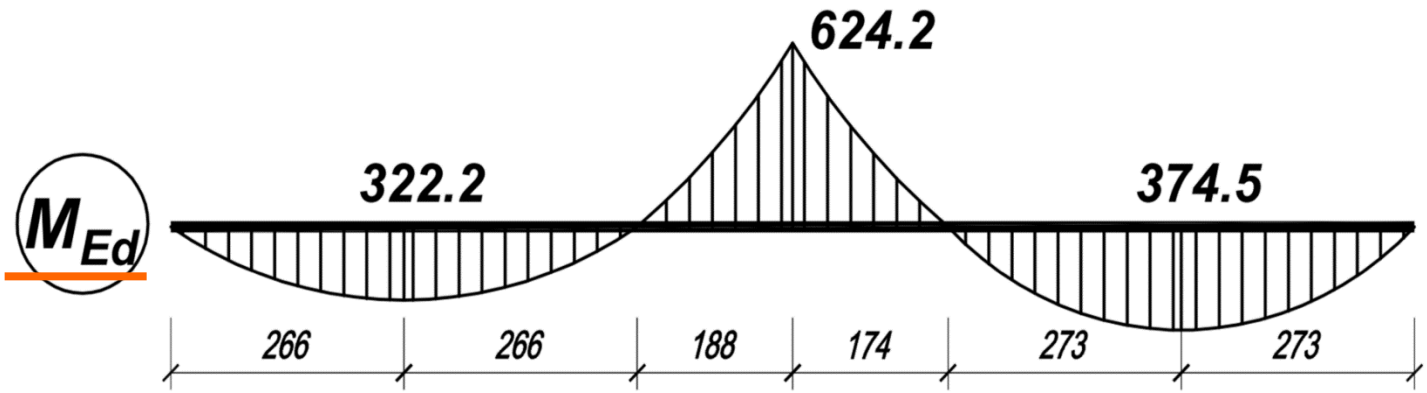
$$M_{BC,g} = \frac{{R_{C,g}}^2}{2 \times g_{BC}} = \frac{81.58^2}{2 \times 29.84} = 111.52\ kNm$$

$$M_{BC,q} = \frac{{R_{B,q}}^2}{2 \times q_{BC}} = \frac{108.9^2}{2 \times 39.72} = 149.29\ kNm$$

Maksimalni moment iznad oslonca B:

$$M_{B,g} = -0.063 \times (g_{AB} + g_{BC}) \times l^2 = -0.063 \times (27.6 + 29.84) \times 7.2^2 = 187.56\ kNm$$

$$M_{B,q} = -0.063 \times (q_{AB} + q_{BC}) \times l^2 = -0.063 \times (36.0 + 39.72) \times 7.2^2 = 247.30\ kNm$$



Dimenzionisanje prema maksimalnom momentu u polju AB

$$M_{AB,Ed} = 1.35 \times 97.9 + 1.5 \times 126.7 = 322.2\ kNm$$

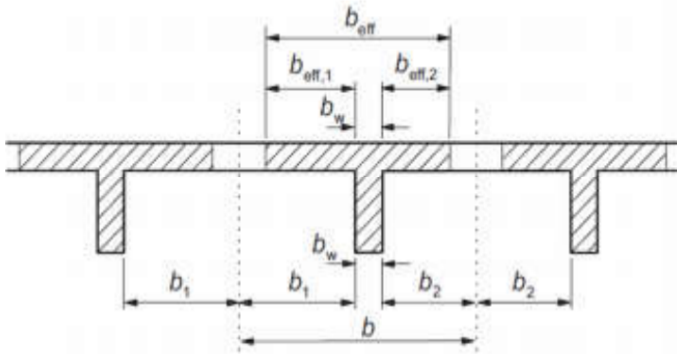
pret. $d_1 = 6\ cm \rightarrow d = 60 - 6 = 54\ cm$

$$k = \frac{54}{\sqrt{\frac{322.2 \times 100}{252.8 \times 1.7}}} = 6.237$$

$$\rightarrow \zeta = 0.987, \varepsilon_{s1} = 108.36‰$$

$$A_{s1} = \frac{322.2 \times 100}{0.987 \times 54 \times 43.48} = 13.90\ cm^2$$

Usvaja se: **5∅20 (15.7 cm^2)**



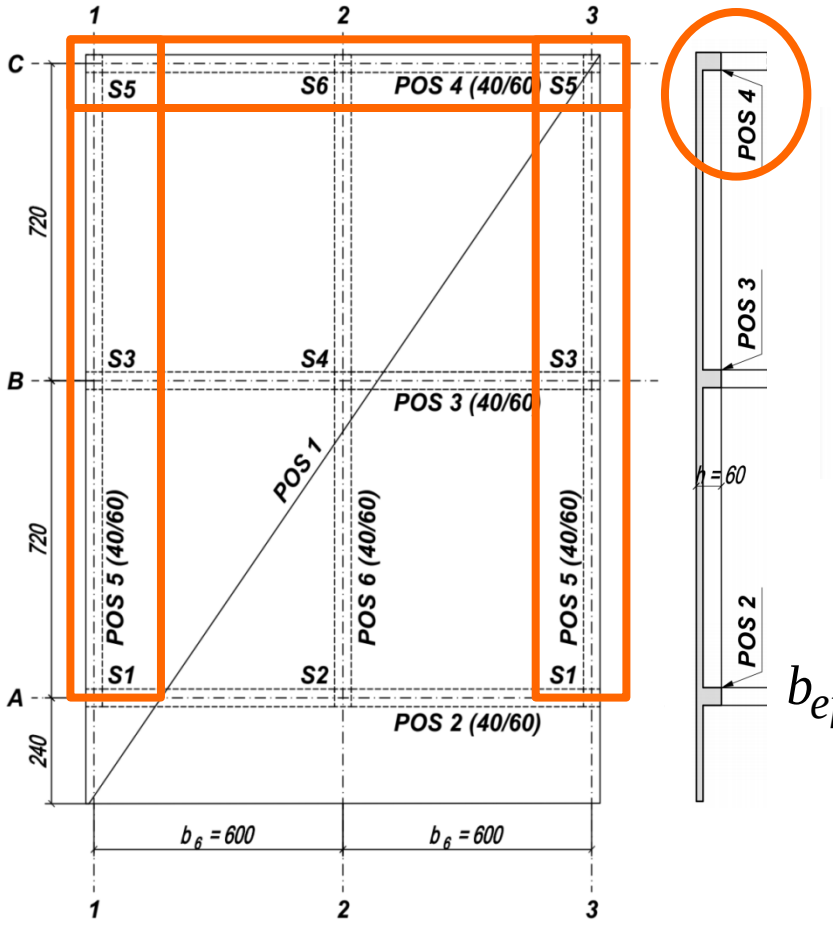
$$b_6 = 600\ cm$$

$$x_{AB} = \frac{73.5}{27.6} = 2.66\ m$$

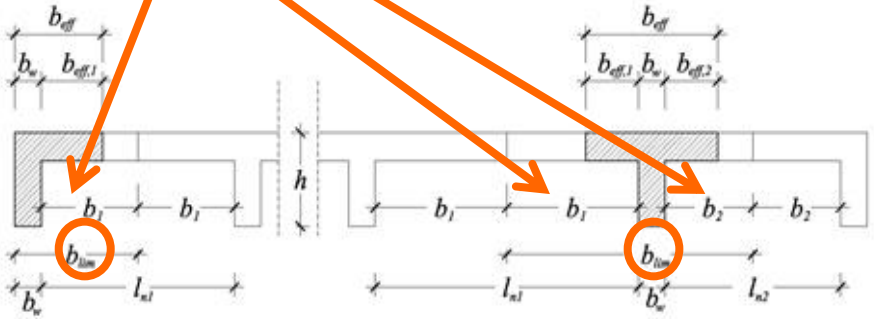
$$\rightarrow \quad l_0 = 2 \times 266 = 532\ cm$$

$$b_{eff} = \begin{cases} 0.4 \times l_0 + b_w, b \geq l_0 \\ 0.2 \times l_0 + 0.2 \times b, b < l_0 \end{cases}$$

$$b_{eff,AB} = 0.4 \times 532 + 40 = 252.8cm, \quad 600 \geq 532$$



$$b_{\lim} = b = \begin{cases} b_1 + b_w, \text{"}\Gamma\text{"} \ \ presek \\ b_1 + b_2 + b_w, \text{"}T\text{"} \ \ presek \end{cases}$$



"T" presek – grede POS 2, 3, 6

$$b_{eff} = \begin{cases} 0.4 \times l_0 + b_w, b \geq l_0 \\ 0.2 \times l_0 + 0.2 \times b, b < l_0 \end{cases} \leq (b_1 + b_2 + b_w)$$

"Γ" presek – grede POS 4 i 5

$$b_{eff} = \begin{cases} 0.2 l_0 + b_w, b \geq l_0 \\ 0.1 \times l_0 + 0.2 \times b_1 + b_w, b < l_0 \end{cases} \leq (b_1 + b_w)$$

Dimenzionisanje prema maksimalnom momentu u polju BC

$$M_{BC,Ed} = 1.35 \times 111.52 + 1.5 \times 149.29 = 374.49\ kNm$$

pret. $d_1 = 6\ cm \rightarrow d = 60 - 7 = 53\ cm$

$$x_{BC} = \frac{81.58}{29.84} = 2.73\ m \rightarrow l_0 = 2 \times 273 = 546\ cm$$

$$b_{eff,BC} = 0.4 \times 546 + 40 = 258.4 cm,\ 600 \geq 532$$

$$k = \frac{53}{\sqrt{\frac{374.49 \times 100}{258.4 \times 1.7}}} = 5.740 \rightarrow \zeta = 0.984, \varepsilon_{s1} = 89.47‰$$

$$A_{s1} = \frac{374.49 \times 100}{0.984 \times 53 \times 43.48} = 16.51\ cm^2$$

Usvaja se: **6Ø20 ($18.84\ cm^2$)**

Dimenzionisanje prema maksimalnom momentu iznad oslonca B

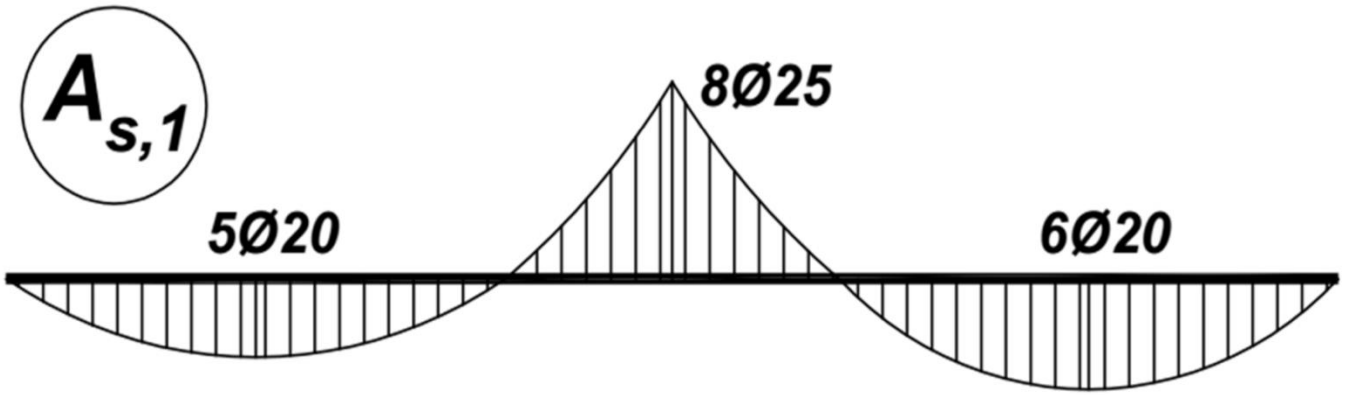
$$M_{B,Ed} = 1.35 \times 187.56 + 1.5 \times 247.30 = 624.16\ kNm$$

pret. $d_1 = 8\ cm \rightarrow d = 60 - 8 = 52\ cm$

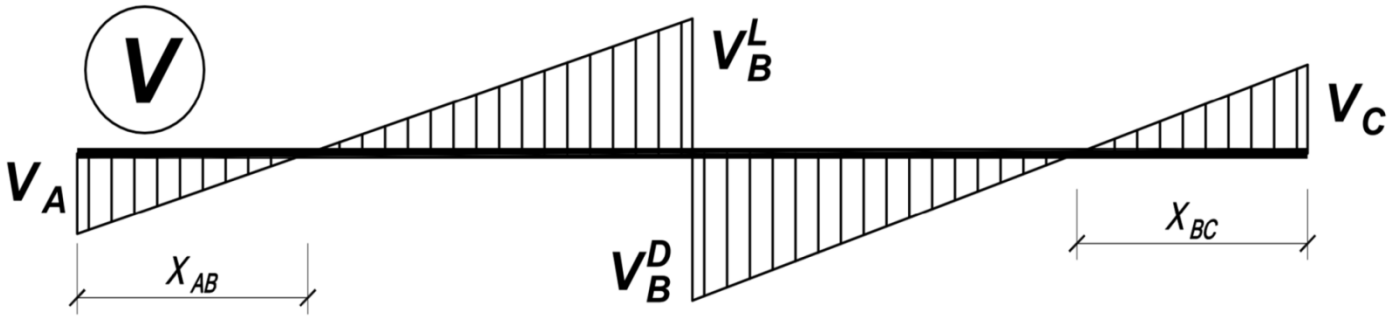
$$k = \frac{52}{\sqrt{\frac{624.16 \times 100}{40 \times 1.7}}} = 1.716 \rightarrow \zeta = 0.774, \varepsilon_{s1} = 2.953‰$$

$$A_{s1} = \frac{624.16 \times 100}{0.774 \times 52 \times 43.48} = 35.67 cm^2$$ Usvaja se: **8Ø25 ($39.28\ cm^2$)**

Usvojena armatura



6.5.2. Greda POS 6 - dimenzionisanje prema transverzalnim silama



$V_{A,g} = 73.50\ kN$

$V_{B,g}{}^{L} = 73.5 - 27.60 \times 7.2 = 125.22\ kN$

$V_{B,g}{}^{D} = 125.22 - 258.50 = 133.28\ kN$

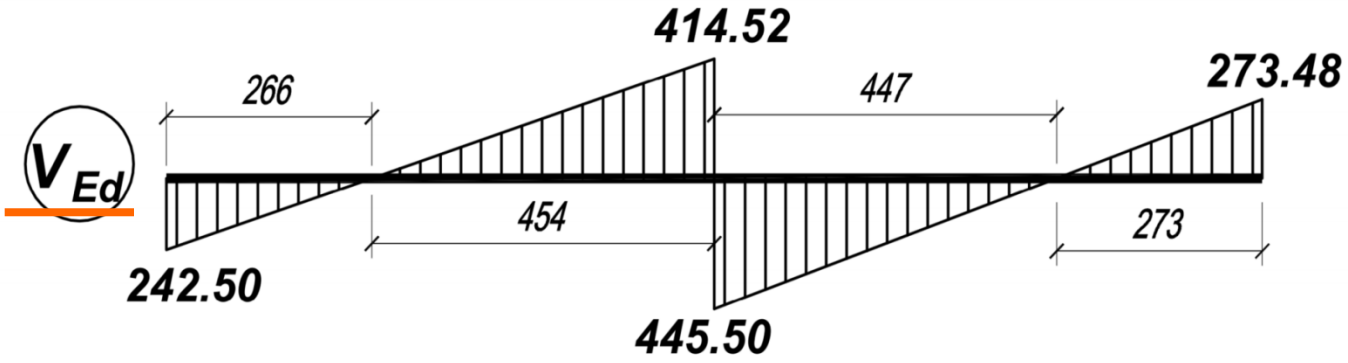
$V_{C,g} = 81.58\ kN$

$V_{A,q} = 95.50\ kN$

$V_{B,q}{}^{L} = 95.50 - 36.00 \times 7.2 = 163.65\ kN$

$V_{B,q}{}^{D} = 163.65 - 340.70 = 177.05\ kN$

$V_{C,q} = 108.90\ kN$



Oslonac A:

$$V_{A,Ed} = 1.35 \times 73.50 + 1.50 \times 95.50 = 242.50\ kN$$

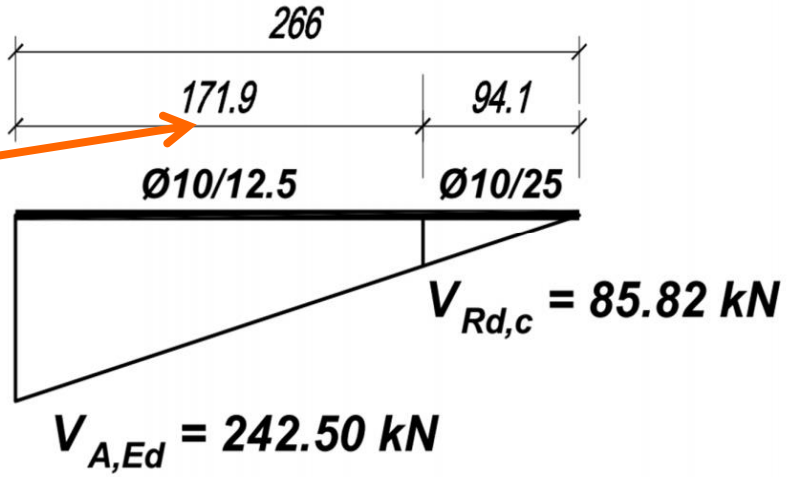
2Ø20 na mestu krajnjeg oslonca A

$$V_{Rd,c} = \left[\frac{0.18}{1.5} \times (1 + \sqrt{\frac{200}{540}}) \times \sqrt[3]{(100 \times \frac{6.28}{40 \times 54} \times 30)}\right] \times \frac{40 \times 54}{10} = 85.82\ kN$$

$V_{Rd,c} < V_{A,Ed}$ $\rightarrow$ *potrebno je osiguranje uzengijama*

<u>*Dužina osiguranja:*</u>

$$\lambda = 266 \times (1 - \frac{85.82}{242.5}) = 171.86\ cm$$



$$\nu_1 = 0.6\left[1 - \frac{30}{250}\right] = 0.528$$

Usvaja se: $\boldsymbol{\theta = 45°},\ \boldsymbol{\alpha = 90°},\ \boldsymbol{ctg\theta = 1}$

$$V_{Rd,max} = 40 \times 0.9 \times 54 \times 0.528 \times \frac{1.70}{(1+1)} = 872.47\ kN\ > 228.64\ kN$$

$0.3V_{Rd,max} > V_{Ed}$ $\rightarrow$ $s_{max} = min\{0.75d\,;30cm\} = 30cm$

Pret. ∅10, m=2

$$s \leq \frac{2 \times 0.785 \times 0.9 \times 54}{242.50} \times 43.5 \times 1 = 13.69\ cm < s_{max} = 30cm$$

Usvaja se: **∅10/12.5**

Dodatna zategnuta armature

$$\Delta F_{td} = 0.5 \times 242.50 \times 1 = 114.32\ kN$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{121.25}{43.5} = 2.79cm2$$

Usvaja se: **2∅20** (6.28 cm^2)

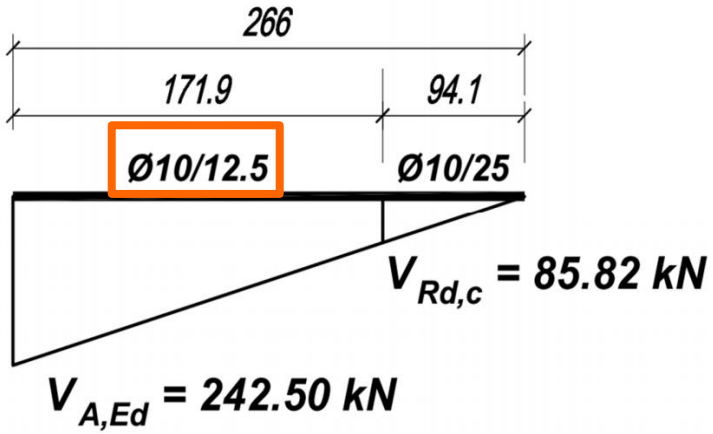
Izvan zone osiguranja usvaja se minimalni procenat armiranja

$$\rho_{w,min} = 0.08\frac{\sqrt{f_{ck}}}{f_{yk}} = 0.08\frac{\sqrt{30}}{500} = 0.00087 = 0.087\%$$

Pret. ∅10, m=2

$$s \leq \frac{2 \times 0.785}{0.00087 \times 40} = 45.10\ cm > s_{max} = 30cm$$

Usvaja se: **∅10/25**



Oslonac B levo:

$$V_{B,Ed}{}^{L} = 1.35 \times 125.22 + 1.50 \times 163.65 = 414.52\ kN$$

8Ø25 na mestu srednjeg oslonca

$$V_{Rd,c} = \left[\frac{0.18}{1.5} \times (1 + \sqrt{\frac{200}{520}}) \times \sqrt[3]{(100 \times \frac{39.28}{40 \times 52} \times 30)}\right] \times \frac{40 \times 52}{10} = 155.30\ kN$$

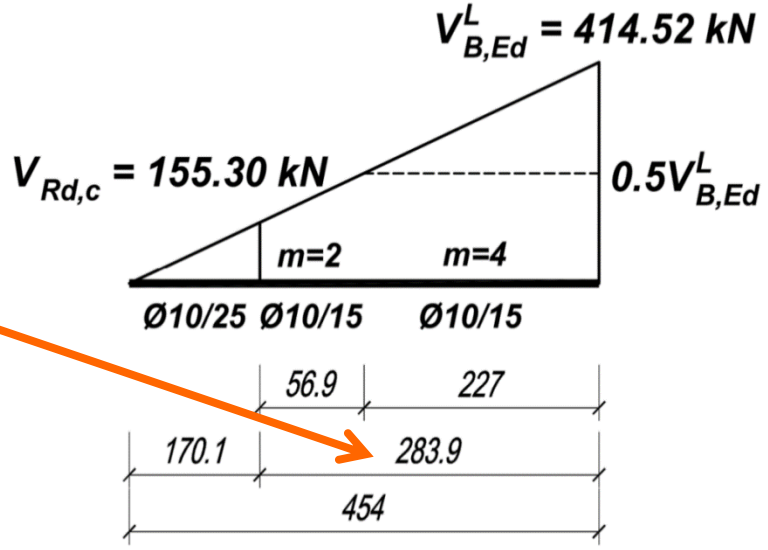
$V_{Rd,c} < V_{B,Ed}{}^{L} \rightarrow$ *potrebno je osiguranje uzengijama*

<u>*Dužina osiguranja:*</u>

$$\lambda = 454 \times \left(1 - \frac{155.30}{414.52}\right) = 283.91\ cm$$

$$\nu_1 = 0.6\left[1 - \frac{30}{250}\right] = 0.528$$

Usvaja se: $\boldsymbol{\theta = 45°},\ \boldsymbol{\alpha = 90°},\ \boldsymbol{ctg\theta = 1}$



$$V_{Rd,max} = 40 \times 0.9 \times 52 \times 0.528 \times \frac{1.70}{(1+1)} = 840.15\ kN\ > 414.52\ kN$$

$$0.3V_{Rd,max} < V_{Ed} < 0.6V_{Rd,max} \rightarrow s_{max} = min\{0.55d\,;30cm\} = 28.6cm$$

Pret. ∅10, m=4

$$s \leq \frac{4 \times 0.785 \times 0.9 \times 52}{414.52} \times 43.5 \times 1 = 15.41\ cm < s_{max} = 28.6\ cm$$

Usvaja se: **∅10/15 m=4** na dužini od 227cm

∅10/15 m=2 na dužini od 56.9 cm

!Dodatna zategnuta armature je jednaka nuli na mestu "špica" momenta (srednji oslonac)!

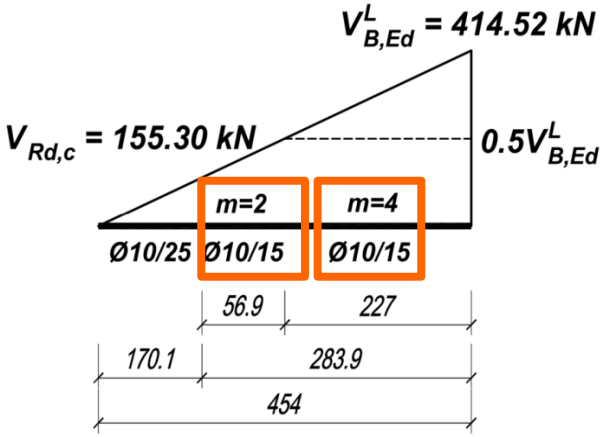
Izvan zone osiguranja usvaja se minimalni procenat armiranja

$$\rho_{w,min} = 0.08\frac{\sqrt{f_{ck}}}{f_{yk}} = 0.08\frac{\sqrt{30}}{500} = 0.00087 = 0.087\%$$

Pret. ∅10, m=2

$$s \leq \frac{2 \times 0.785}{0.00087 \times 40} = 45.10\ cm > s_{max} = 28.6\ \text{cm}$$

Usvaja se: **∅10/25**



Oslonac B desno:

$$V_{B,Ed}{}^{D} = 1.35 \times 133.28 + 1.50 \times 177.05 = 445.50\ kN$$

8Ø25 na mestu srednjeg oslonca

$$V_{Rd,c} = \left[\frac{0.18}{1.5} \times (1 + \sqrt{\frac{200}{520}}) \times \sqrt[3]{(100 \times \frac{39.28}{40 \times 52} \times 30)}\right] \times \frac{40 \times 52}{10} = 155.30\ kN$$

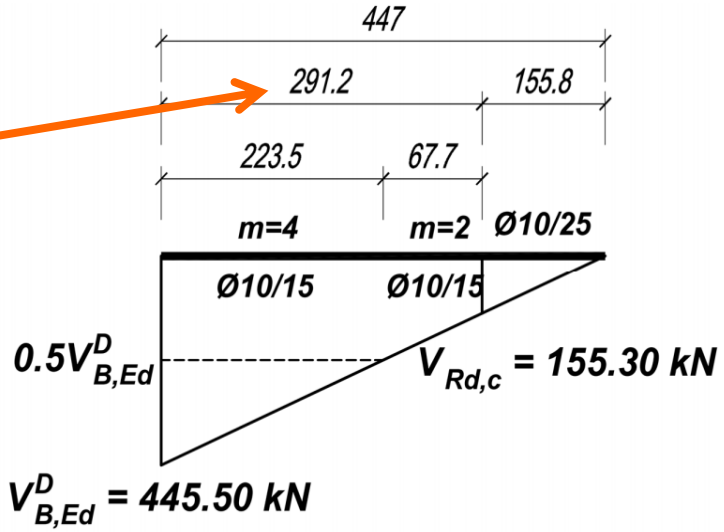
$$V_{Rd,c} < V_{B,Ed}{}^{D} \rightarrow potrebno\ je\ osiguranje\ uzengijama$$

<u>*Dužina osiguranja:*</u>

$$\lambda = 447 \times \left(1 - \frac{155.30}{445.50}\right) = 291.18\ cm$$

$$\nu_1 = 0.6\left[1 - \frac{30}{250}\right] = 0.528$$

Usvaja se: $\boldsymbol{\theta = 45°,\ \alpha = 90°,\ ctg\theta = 1}$



$$V_{Rd,max} = 40 \times 0.9 \times 52 \times 0.528 \times \frac{1.70}{(1+1)} = 840.15\ kN\ > 445.50\ kN$$

$$0.3V_{Rd,max} < V_{Ed} < 0.6V_{Rd,max} \rightarrow s_{max} = min\{0.55d\,;\,30cm\} = 28.6\ cm$$

Pret. ∅10, m=4

$$s \leq \frac{4 \times 0.785 \times 0.9 \times 52}{445.50} \times 43.5 \times 1 = 14.35\ cm < s_{max} = 28.6\ cm$$

Usvaja se: **∅10/15 m=4** na dužini od 223.5 cm

∅10/15 m=2 na dužini od 67.7 cm

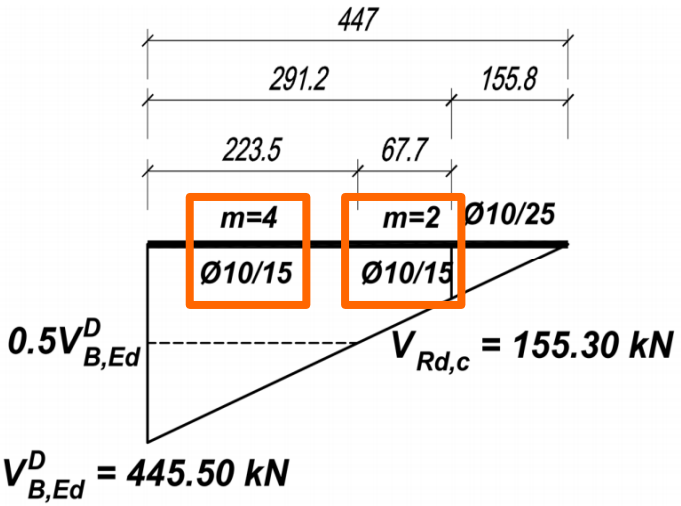
!Dodatna zategnuta armature je jednaka nuli na mestu "špica" momenta (srednji oslonac)!

Izvan zone osiguranja usvaja se minimalni procenat armiranja

Pret. ∅10, m=2

$$s \leq \frac{2 \times 0.785}{0.00087 \times 40} = 45.10\ cm > s_{max} = 28.6\ \text{cm}$$

Usvaja se: **∅10/25**



Oslonac C:

$$V_{C,Ed} = 1.35 \times 81.58 + 1.50 \times 108.90 = 273.48\ kN$$

2Ø20 na mestu krajnjeg oslonca C

$$V_{Rd,c} = \left[\frac{0.18}{1.5} \times (1 + \sqrt{\frac{200}{530}}) \times \sqrt[3]{(100 \times \frac{6.28}{40 \times 53} \times 30)}\right] \times \frac{40 \times 53}{10} = 85.06\ kN$$

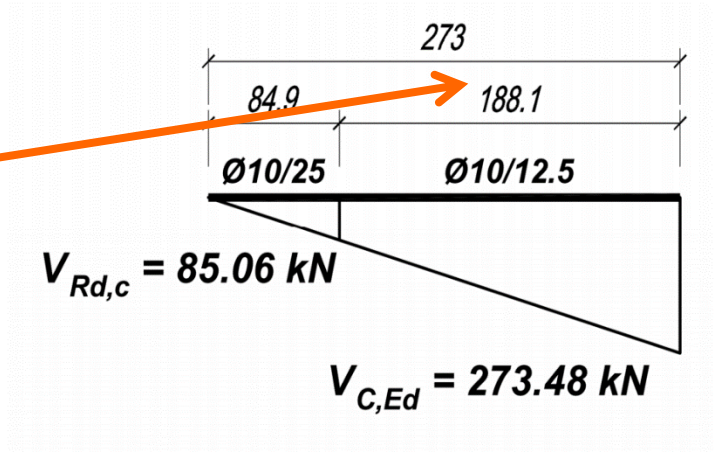
$$V_{Rd,c} < V_{B,Ed}{}^{C} \rightarrow \textit{potrebno je osiguranje uzengijama}$$

<u>*Dužina osiguranja:*</u>

$$\lambda = 273 \times \left(1 - \frac{85.06}{273.48}\right) = 188.09\ cm$$

$$\nu_1 = 0.6\left[1 - \frac{30}{250}\right] = 0.528$$

Usvaja se: $\boldsymbol{\theta = 45°,\ \alpha = 90°,\ ctg\theta = 1}$



$$V_{Rd,max} = 40 \times 0.9 \times 53 \times 0.528 \times \frac{1.70}{(1+1)} = 856.31\ kN > 247.03\ kN$$

$$0.3V_{Rd,max} > V_{Ed} \quad \rightarrow \quad s_{max} = min\{0.75d\ ; 30cm\} = 30cm$$

Pret. ∅10, m=2

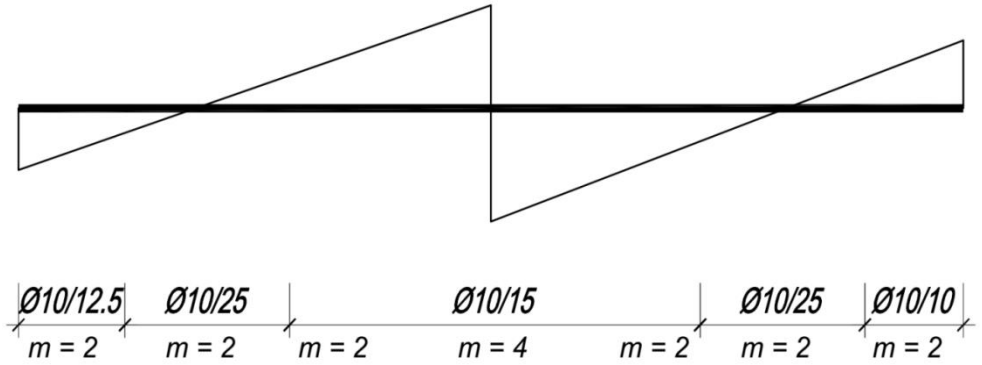
$$s \le \frac{2 \times 0.785 \times 0.9 \times 53}{273.48} \times 43.5 \times 1 = 11.91\ cm < s_{max} = 30cm$$

Usvaja se: **∅10/10**

Dodatna zategnuta armatura

$$\Delta F_{td} = 0.5 \times 273.48 \times 1 = 136.74\ kN$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{136.74}{43.5} = 3.14cm2$$



Usvaja se: **2∅20** (6.28 cm²)

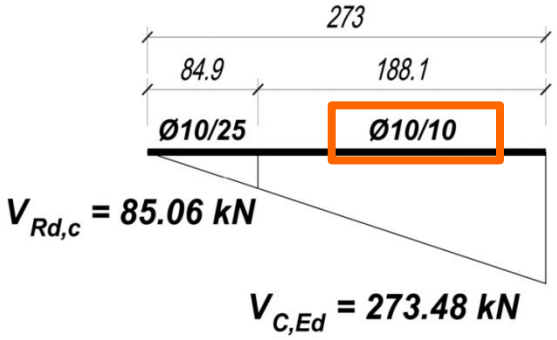
Izvan zone osiguranja usvaja se minimalni procenat armiranja

$$\rho_{w,min} = 0.08\frac{\sqrt{f_{ck}}}{f_{yk}} = 0.08\frac{\sqrt{30}}{500} = 0.00087 = 0.087\%$$

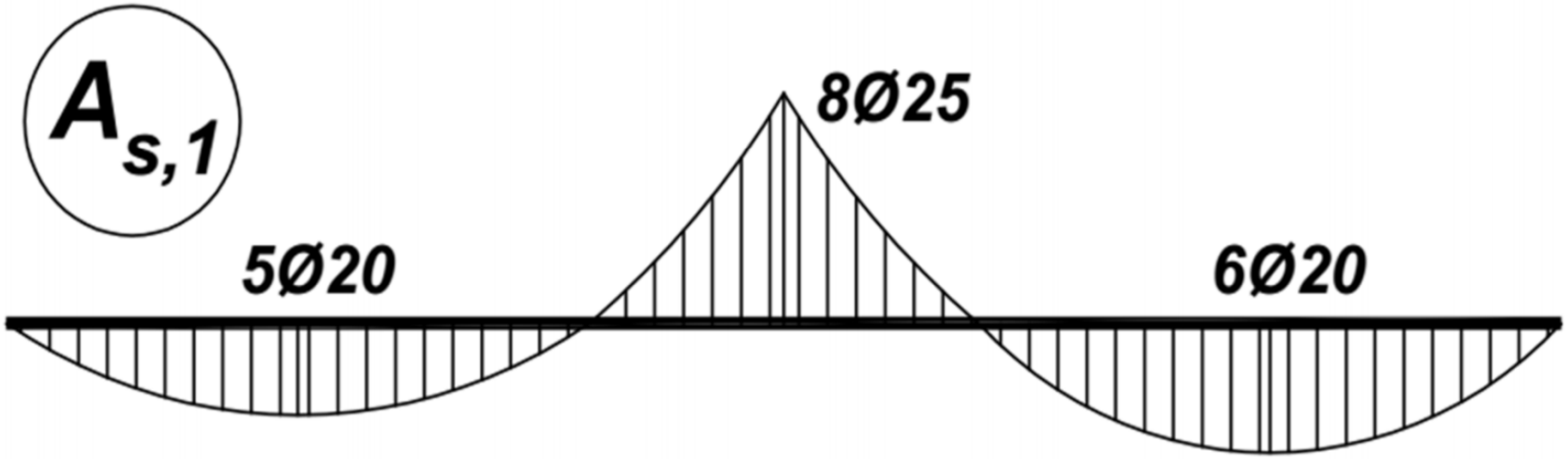
Pret. ∅10, m=2

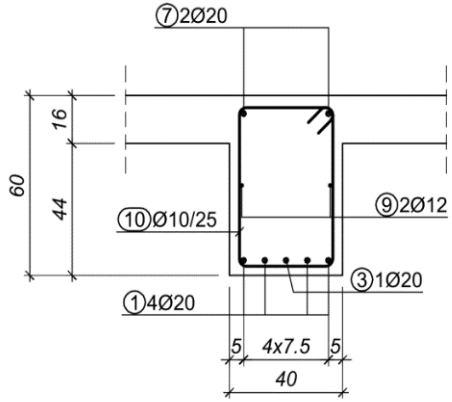
$$s \le \frac{2 \times 0.785}{0.00087 \times 40} = 45.10\ cm > s_{max} = 30cm$$

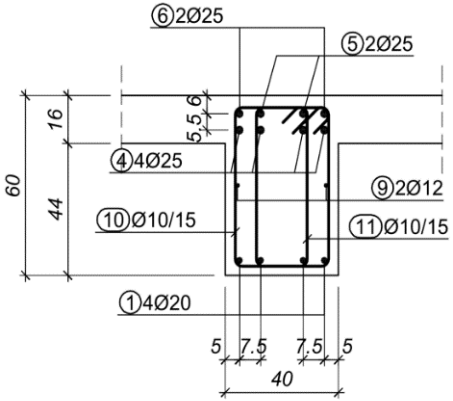
Usvaja se: **∅10/25**

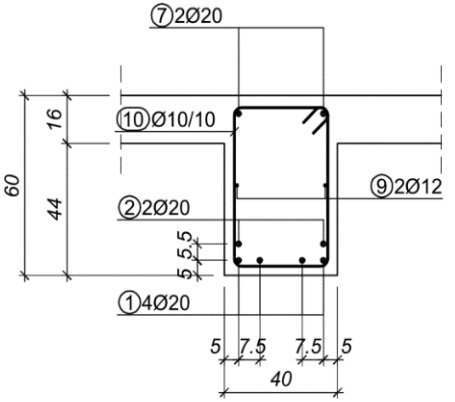


Usvojena armatura

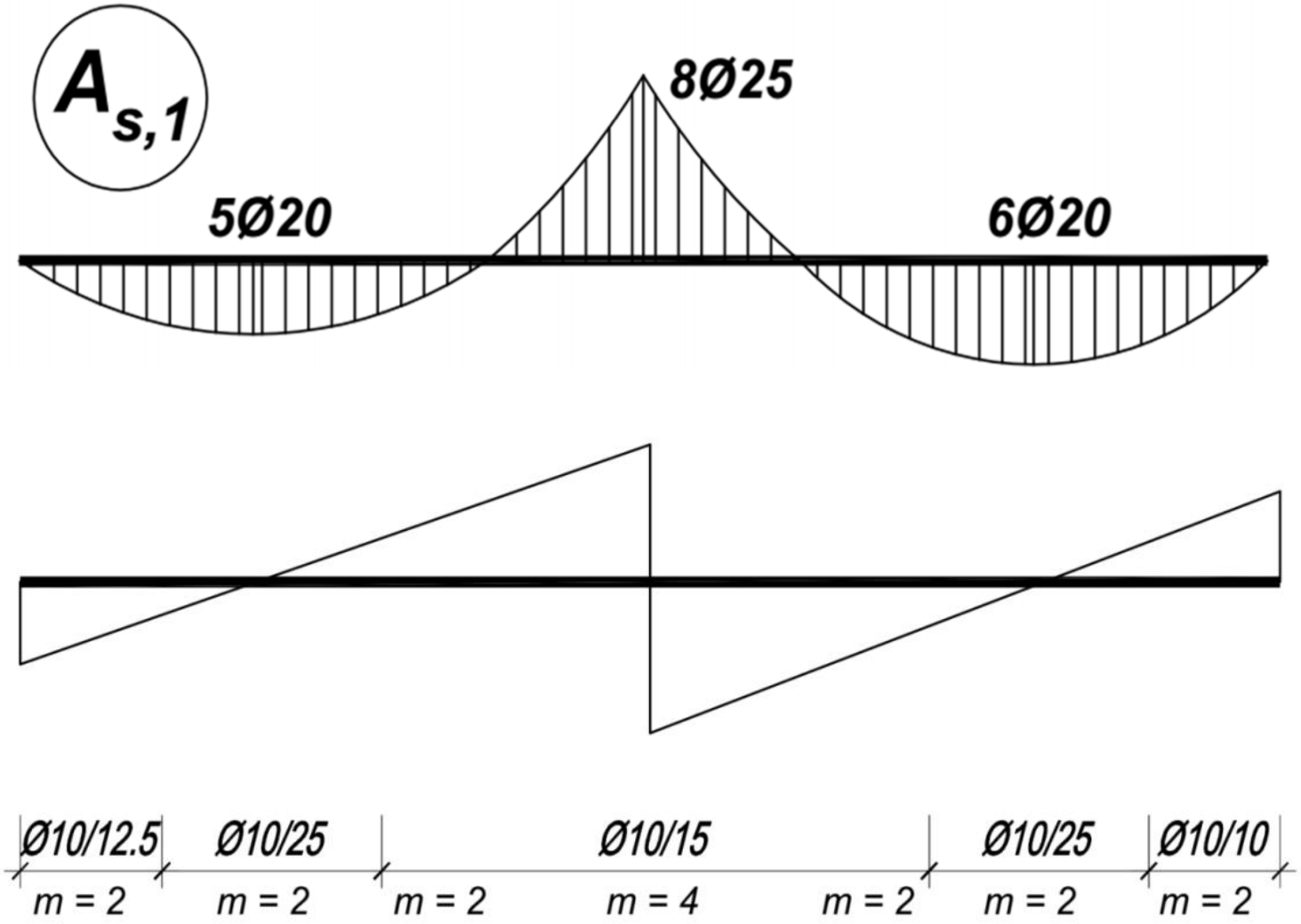


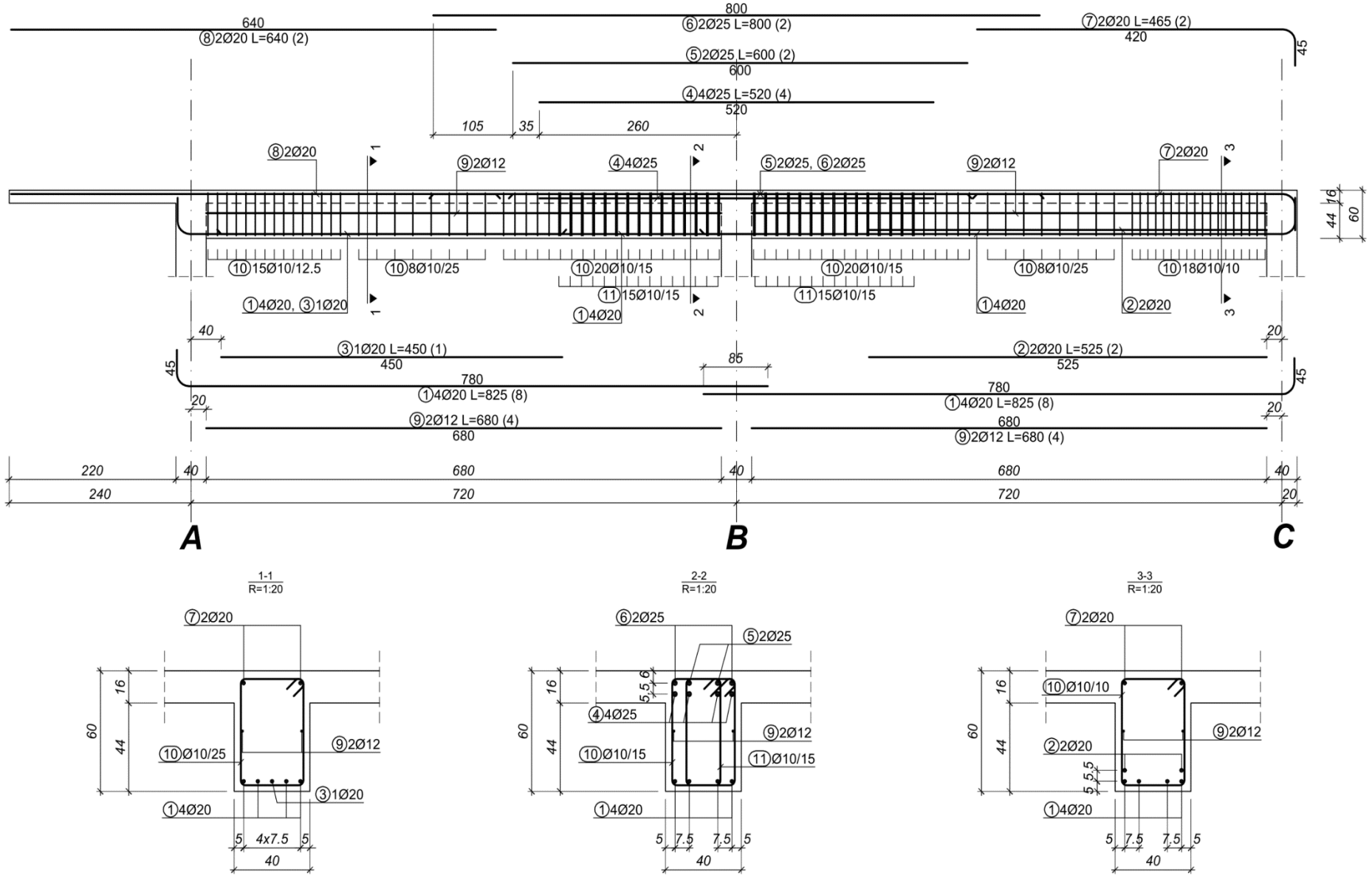




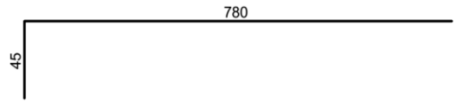
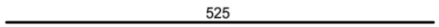
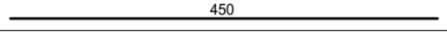
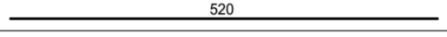
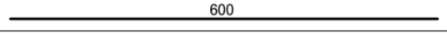
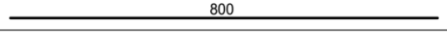
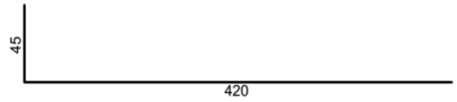
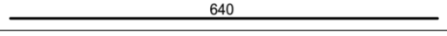
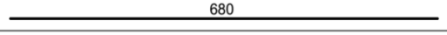
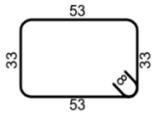
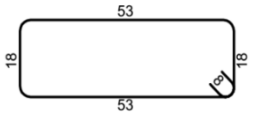


Usvojena armatura




640
⑧2Ø20 L=640 (2)
800
⑥2Ø25 L=800 (2)
⑦2Ø20 L=465 (2)
420
45
⑤2Ø25 L=600 (2)
600
④4Ø25 L=520 (4)
520
105
35
260
⑧2Ø20
⑨2Ø12
④4Ø25
⑤2Ø25, ⑥2Ø25
⑨2Ø12
⑦2Ø20
⑩15Ø10/12.5
⑩8Ø10/25
⑩20Ø10/15
⑩20Ø10/15
⑩8Ø10/25
⑩18Ø10/10
①4Ø20, ③1Ø20
⑪15Ø10/15
①4Ø20
⑪15Ø10/15
①4Ø20
②2Ø20
16
44
60
40
③1Ø20 L=450 (1)
450
85
②2Ø20 L=525 (2)
525
20
45
780
①4Ø20 L=825 (8)
780
①4Ø20 L=825 (8)
20
⑨2Ø12 L=680 (4)
680
680
⑨2Ø12 L=680 (4)
220
40
680
40
680
40
240
720
720
20
A
B
C
1-1
R=1:20
⑦2Ø20
16
60
44
⑩Ø10/25
⑨2Ø12
③1Ø20
①4Ø20
5
4x7.5
5
40
2-2
R=1:20
⑥2Ø25
⑤2Ø25
5,5,6
16
60
44
④4Ø25
⑨2Ø12
⑩Ø10/15
⑪Ø10/15
①4Ø20
5
7.5
7.5
5
40
3-3
R=1:20
⑦2Ø20
16
60
44
⑩Ø10/10
⑨2Ø12
②2Ø20
5,5,5
①4Ø20
5
7.5
7.5
5
40

Šipke - specifikacija

ozn.	oblik i mere [cm]	ozn.	Ø	lg [m]	n [kom]	lgn [m]
POS 6 (1 kom)						
1		B500B	20	8.25	8	66.00
2		B500B	20	5.25	2	10.50
3		B500B	20	4.50	1	4.50
4		B500B	25	5.20	4	20.80
5		B500B	25	6.00	2	12.00
6		B500B	25	8.00	2	16.00
7		B500B	20	4.65	2	9.30
8		B500B	20	6.40	2	12.80
9		B500B	12	6.80	4	27.20
10		B500B	10	1.88	89	167.32
11		B500B	10	1.58	30	47.40

Šipke - rekapitulacija

Ø [mm]	lgn [m]	Jedinična težina [kg/m']	Težina [kg]
B500B			
10	214.72	0.62	133.13
12	27.20	0.89	24.15
20	103.10	2.47	254.24
25	48.80	3.85	188.03
Ukupno			599.55