



Univerzitet u Beogradu – Građevinski fakultet
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Studijski program: **Građevinarstvo**
Modul: Konstrukcije
Godina/Semestar: **III godina / V semestar**

Naziv predmeta (šifra): **Teorija betonskih konstrukcija 1
(b2k3b1)**

Nastavnik: **Ivan Ignjatović**

Naslov vežbi: **Smicanje – 2.deo**

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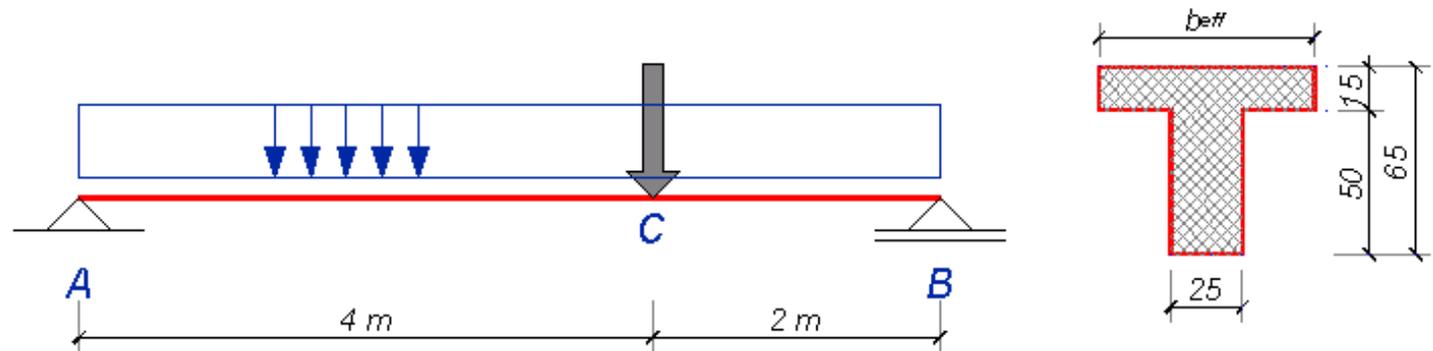


Zadatak 25 – SMICANJE

Za nosač prikazan na skici, čiji su dijagrami proračunskih vrednosti statičkih uticaja (M_{Ed} i V_{Ed}) prikazani na sledećem slajdu, izvršiti osiguranje nosača prema glavnim naponima zatezanja.

C25/30 B500B

XC2



C25/30

$$f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 \text{ MPa} = 1.42 \text{ kN/cm}^2$$

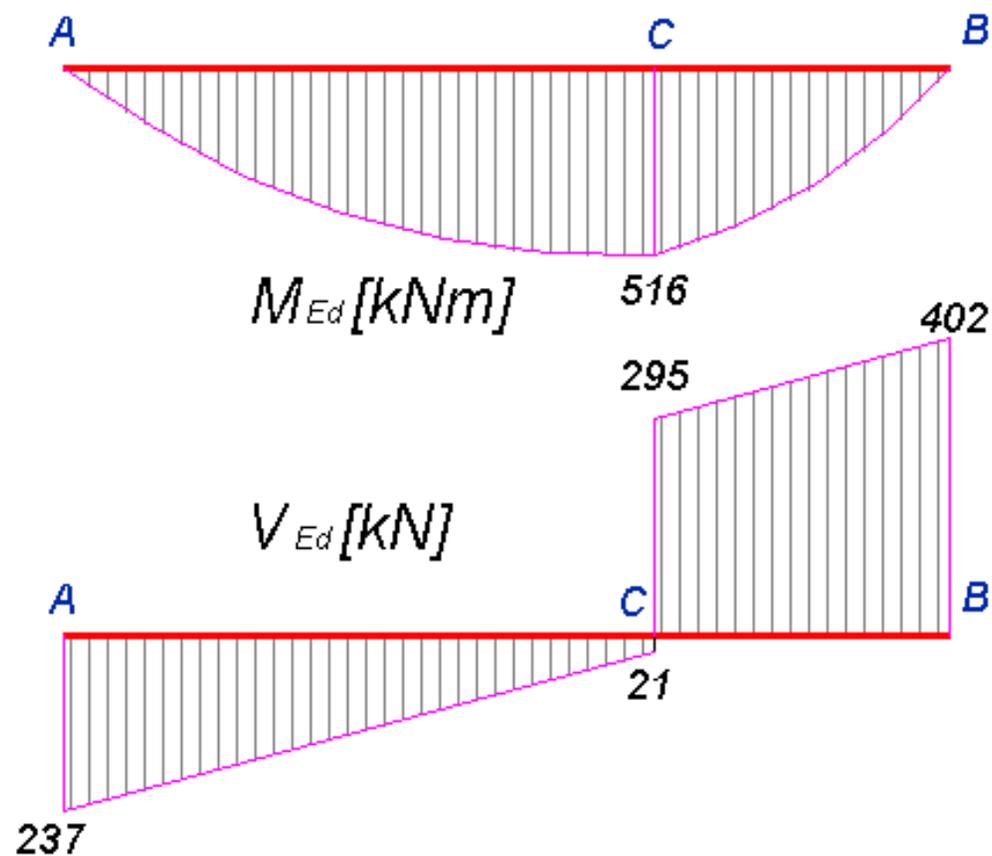
B500 B

$$f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$$



Zadatak 25 – SMICANJE

1. Zadati dijagrami statičkih uticaja



Zadatak 25 – SMICANJE

2. Dimenzionisanje prema momentima savijanja

1. $M_{Ed}=516 \text{ kNm}$

2. pretp. $d_1 = 7 \text{ cm}$

$$d = h - d_1 = 65 - 7 = 58 \text{ cm}$$

3. Efektivna širina: $b_{eff}=0.4 \cdot l_0 + b_w = 0.4 \cdot 600 + 25 = 265 \text{ cm}$

4. Računa se: $k = \frac{d}{\sqrt{\frac{M_{Ed}}{b_{eff} \cdot f_{cd}}}} = \frac{58}{\sqrt{\frac{516 \cdot 10^2}{265 \cdot 1.42}}} = 4.953$ $\varepsilon_c / \varepsilon_{s1} = 3.5/65\text{‰}$

$$A_{s1} = \omega_1 b_{eff} d \frac{f_{cd}}{f_{yd}} = 4.086 \cdot \frac{265 \cdot 58}{100} \cdot \frac{1.42}{43.5} = 20.5 \text{ cm}^2$$

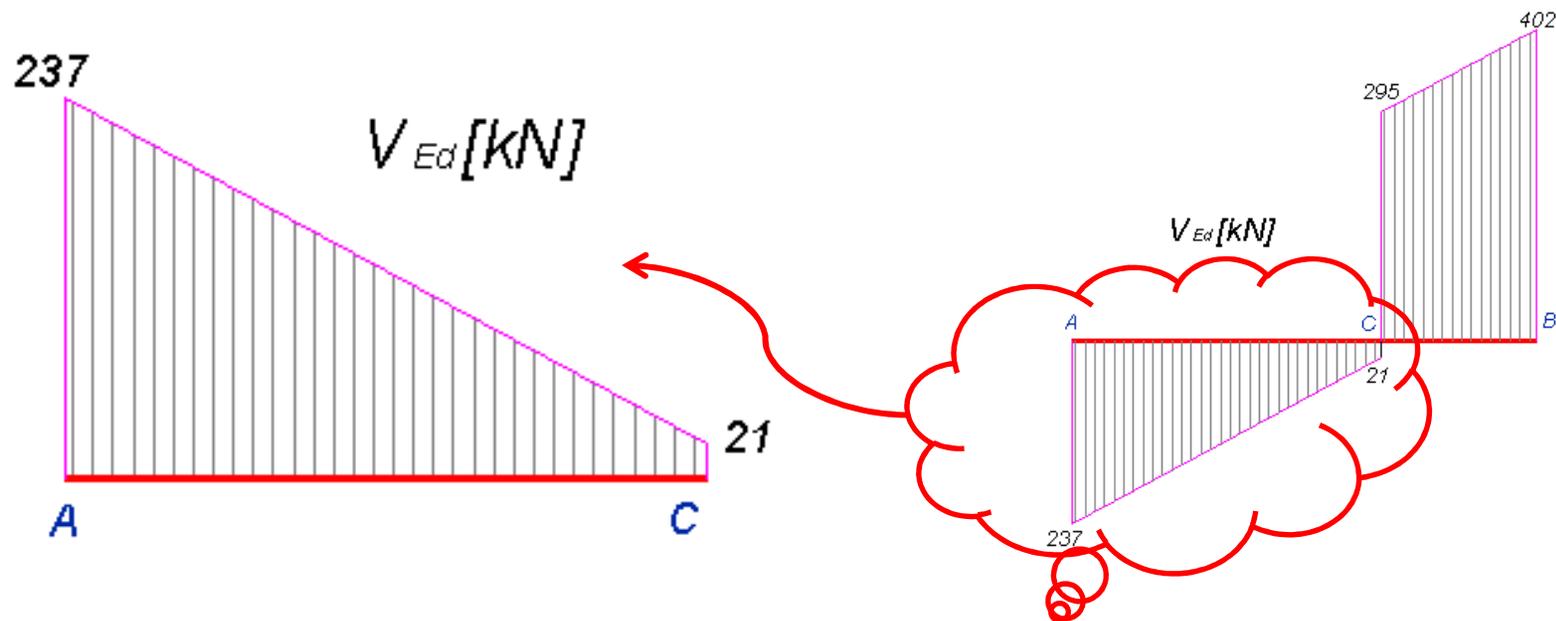
Usvojeno: **5Ø25** (24.55 cm²)



Zadatak 25 – SMICANJE

3. Dimenzionisanje prema smičućim silama: **DEO A-C**

1. Proračunske vrednosti uticaja na posmatranom segmentu:



Zadatak 25 – SMICANJE

2. Proračun nosivosti betona na smicanje:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \rho_i f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{580}} = 1.587$$

$$\rho_i = \frac{A_{sl}}{b_w \cdot d} = \frac{2 \cdot 4.91}{30 \cdot 58} = 0.0056 < 0.02$$

Vidi plan armature!

$$V_{Rd,c} = \left[0.12 \cdot 1.587 \cdot (100 \cdot 0.0056 \cdot 25)^{1/3} \right] \cdot \frac{25 \cdot 58}{10} = 66.5 \text{ kN}$$



Zadatak 25 – SMICANJE

3. Minimalna nosivost betona na smicanje:

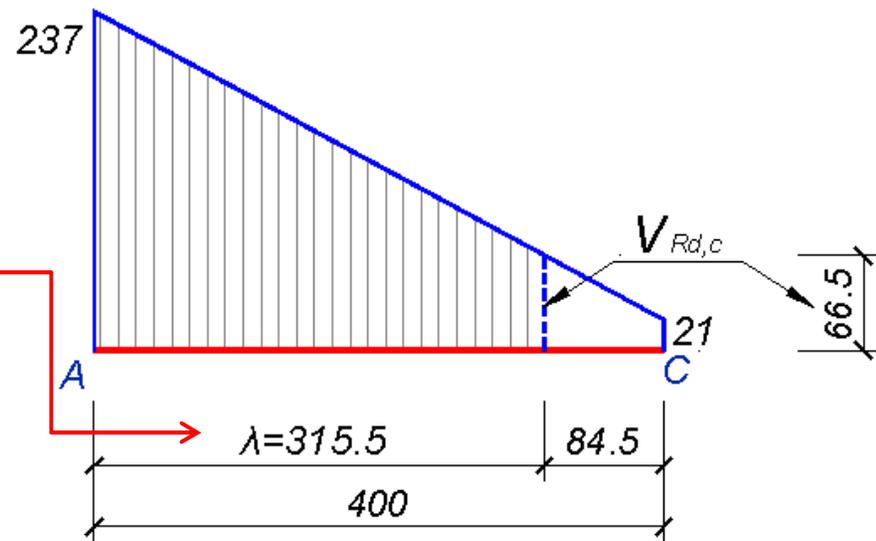
$$V_{\min} = [v_{\min} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d = [0.0035 \cdot k^{3/2} \cdot f_{ck}^{1/2}] \cdot b_w \cdot d$$

$$V_{\min} = [0.0035 \cdot 1.587^{3/2} \cdot 25^{1/2}] \cdot 25 \cdot 58 = 50.7 \text{ kN}$$

4. Nosivost betona na smicanje:

$$V_{Rd,c} = \max \left\{ \begin{array}{l} 66.5 \text{ kN} \\ 50.7 \text{ kN} \end{array} \right\} 66.5 \text{ kN}$$

Na delu nosača – **dužini osiguranja (λ)** potrebno osiguranje poprečnom armaturom!



Zadatak 25 – SMICANJE

5. Određivanje potrebne armature za smicanje

Na dužini osiguranja:

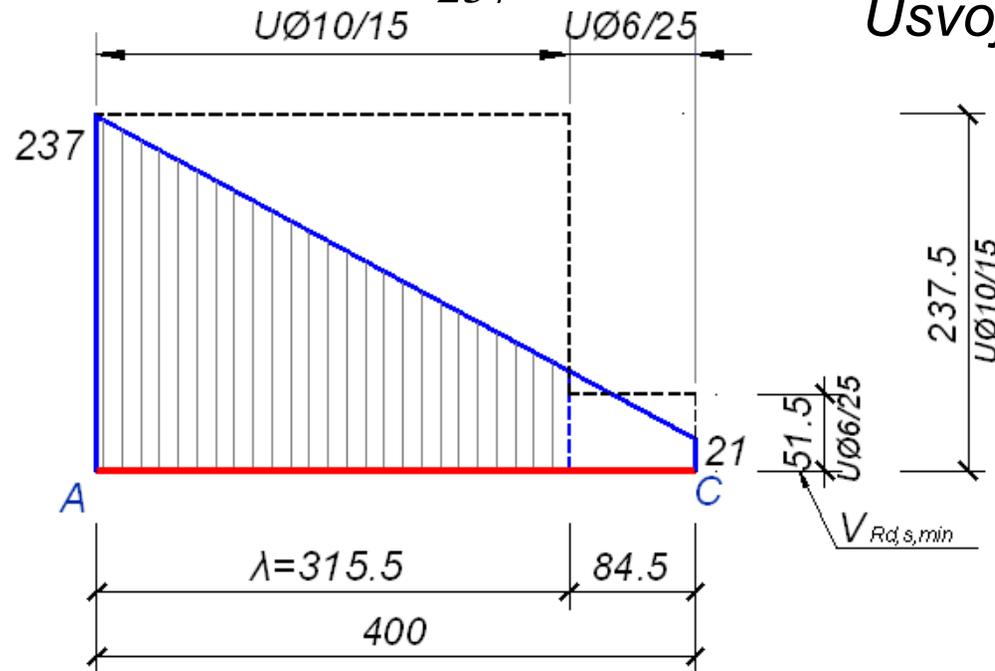
$$V_{Ed} = V_{Rd,s}$$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = V_{Ed}$$

Usvojeno: $\theta=45^\circ$, $ctg\theta=1$, $m=2$, $\mathbf{\text{Ø}10}$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$

$$s_{rač} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{237} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 15.04 \text{ cm}$$

Usvojeno: $\mathbf{\text{UØ}10/15}$

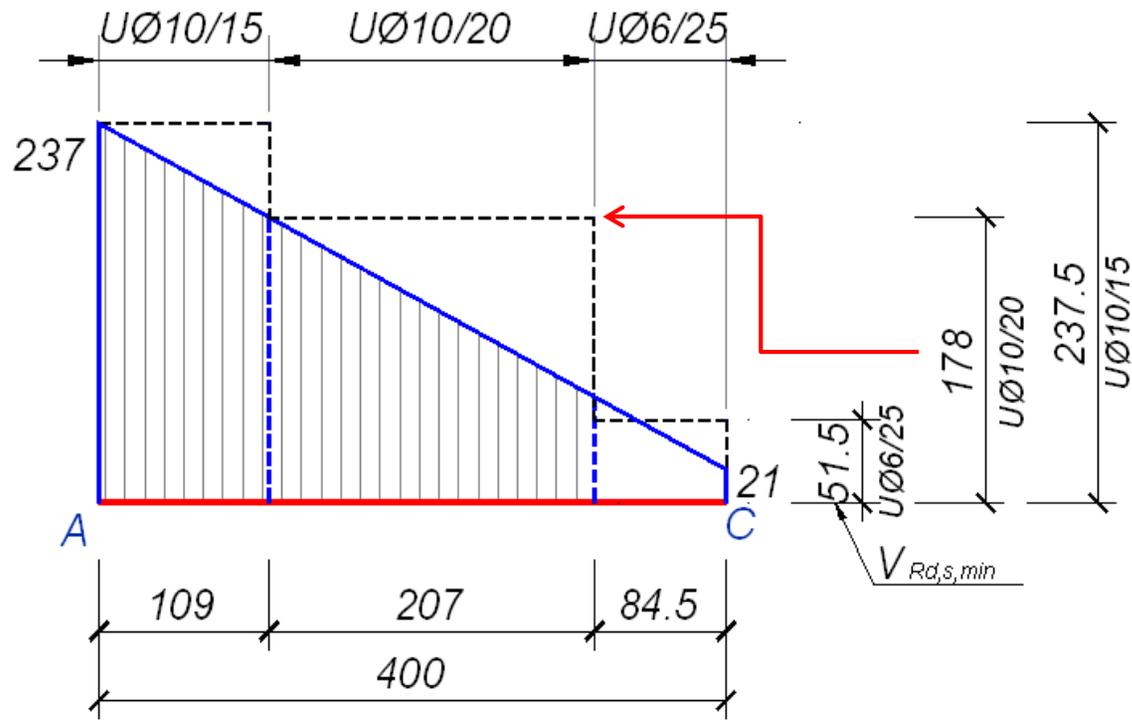


Zadatak 25 – SMICANJE

5a. Varijantno rešenje – optimizacija količine uzengija:

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$ **$s=20 \text{ cm}$**

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{20} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 178.2 \text{ kN}$$



Zadatak 25 – SMICANJE

6. Kontrola minimalne armature za smicanje

$$\rho_{w,\min} = 0.08 \cdot \frac{\sqrt{f_{ck}}}{f_{yk}} = 0.08 \cdot \frac{\sqrt{25}}{500} = 0.0008 = 0.08\%$$

$$\rho_w = \frac{m \cdot a_{sw}^{(1)}}{s \cdot b_w} \rightarrow S_{\rho,\min} = \frac{m \cdot a_{sw}^{(1)}}{\rho_{w,\min} \cdot b_w} = \frac{2 \cdot 0.785}{0.0008 \cdot 25} = 78.5 \text{ cm}$$

Usvojeno: **UØ6/25**

$$S_{\rho,\min} = \frac{m \cdot a_{sw}^{(1)}}{\rho_{w,\min} \cdot b_w} = \frac{2 \cdot 0.283}{0.0008 \cdot 25} = 28.3 \text{ cm}$$

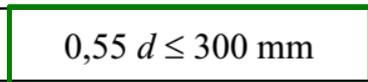
7. Kontrola maksimalne smičuće nosivosti:

$$V_{Rd,\max} = \frac{\alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd}}{ctg\theta + tg\theta} = \frac{1.0 \cdot 25 \cdot 0.9 \cdot 58 \cdot 0.54 \cdot 1.42}{1+1} = 500.3 \text{ kN}$$

Zadatak 25 – SMICANJE

8. Maksimalno podužno rastojanje armature s_{\max}

Табела 1 – Највеће подужно растојање између елемената арматуре за смицање $s_{l,\max}$

	Прорачунска вредност силе смицања V_{Ed}^*	Класе чврстоће бетона	
		$\leq C 50/60$	$> C 50/60$
1.	$V_{Ed} \leq 0,3V_{Rd,\max}$	$0,75 d^{**}) \leq 300 \text{ mm}$	$0,75 d \leq 200 \text{ mm}$
2.	$0,3V_{Rd,\max} \leq V_{Ed} \leq 0,6V_{Rd,\max}$ 	$0,55 d \leq 300 \text{ mm}$ 	$0,55 d \leq 200 \text{ mm}$
3.	$V_{Ed} > 0,6V_{Rd,\max}$	$0,3 d \leq 200 \text{ mm}$	

* $V_{Rd,\max}$ може да се одреди поједностављено са $\theta = 40^\circ$ ($\cot \theta = 1,2$).

** За греде код којих је $h < 200 \text{ mm}$ и $V_{Ed} \leq V_{Rd,\max}$ растојање не мора да буде мање од 150 mm .

$$0,3 \cdot V_{Rd,\max} = 0,3 \cdot 500,3 = 150,1 \text{ kN} < V_{Ed} < 0,6 \cdot V_{Rd,\max} = 0,6 \cdot 500,3 = 300,2 \text{ kN}$$

$$s_{\max} = \min \{0,55d; 30 \text{ cm}\} = \min \{0,55 \cdot 58 = 31,9; 30 \text{ cm}\} = 30 \text{ cm}$$

Zadatak 25 – SMICANJE

9. Usvajanje armature za smicanje

Usvojeni prečnik: $\text{Ø}10$

Usvojeno rastojanje:

$$s = \min \left\{ \begin{array}{l} s_{rač} \\ s_{\rho, \min} \\ s_{\max} \end{array} \right\} = \left\{ \begin{array}{l} 15.04 \\ 78.5 \\ 30.0 \end{array} \right\} = 15.04 \text{ cm}$$

Usvojeno: $\text{UØ}10/15$

10. Nosivost usvojene armature za smicanje:

$$V_{Rd,s} = \frac{2 \cdot 0.785}{15} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 237.7 \text{ kN}$$

$$V_{Rd,s} = 237.7 \text{ kN} \left\{ \begin{array}{l} > 237 \text{ kN} = V_{Ed} \\ < 500.3 \text{ kN} = V_{Rd, \max} \end{array} \right. \begin{array}{l} \longrightarrow \text{Nosivost uzengija veća od proračunske sile smicanja} \\ \longrightarrow \text{Osigurano od loma pritiskute betonske dijagonale} \end{array}$$



Zadatak 25– SMICANJE

11. Dužina osiguranja: dužina $\lambda=315.5 \text{ cm}$

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \left\{ \begin{array}{l} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot \text{ctg}\theta, V_{Ed} > V_{Rd,c} \end{array} \right\} = 0.5 \cdot 237 \cdot 1.0 = 118.5 \text{ kN}$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{118.8}{43.5} = 2.72 \text{ cm}^2 < 2 \cdot 4.91 = 9.82 \text{ cm}^2$$

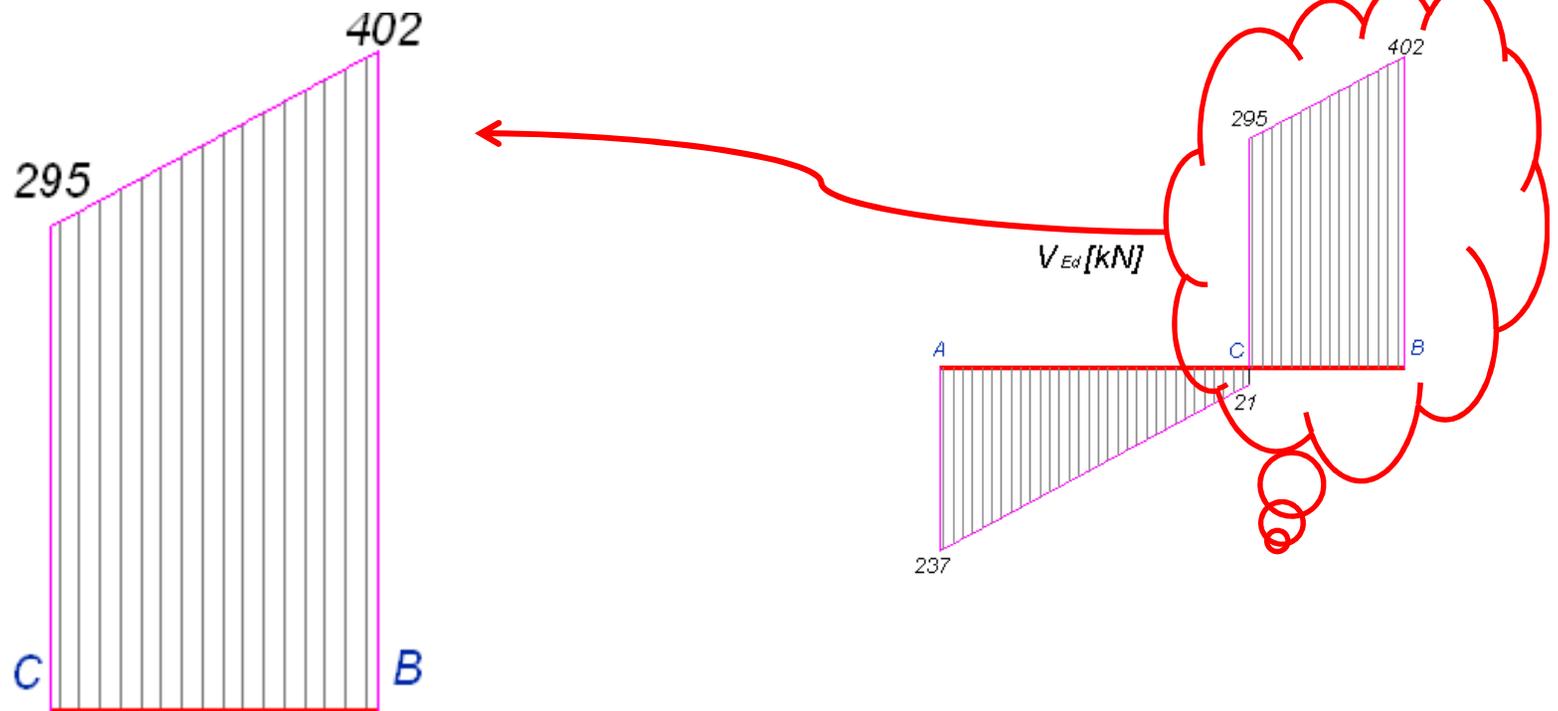
Armatura koju treba obezbediti u donjoj zoni na mestu slobodnog oslonca A

Armatura iz dimenzionisanja prema momentima savijanja – vidi plan armature

Zadatak 25 – SMICANJE

3. Dimenzionisanje prema smičućim silama: **DEO B-C**

1. Proračunske vrednosti uticaja na posmatranom segmentu:



Zadatak 25 – SMICANJE

2. Proračun nosivosti betona na smicanje:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \rho_i f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{580}} = 1.587$$

$$\rho_i = \frac{A_{sl}}{b_w \cdot d} = \frac{2 \cdot 4.91}{30 \cdot 58} = 0.0056 < 0.02$$

Vidi plan armature!

$$V_{Rd,c} = \left[0.12 \cdot 1.587 \cdot (100 \cdot 0.0056 \cdot 25)^{1/3} \right] \cdot \frac{25 \cdot 58}{10} = 66.5 \text{ kN}$$



Zadatak 25 – SMICANJE

3. Minimalna nosivost betona na smicanje:

$$V_{\min} = [v_{\min} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d = [0.0035 \cdot k^{3/2} \cdot f_{ck}^{1/2}] \cdot b_w \cdot d$$

$$V_{\min} = [0.0035 \cdot 1.587^{3/2} \cdot 25^{1/2}] \cdot 25 \cdot 58 = 50.7 \text{ kN}$$

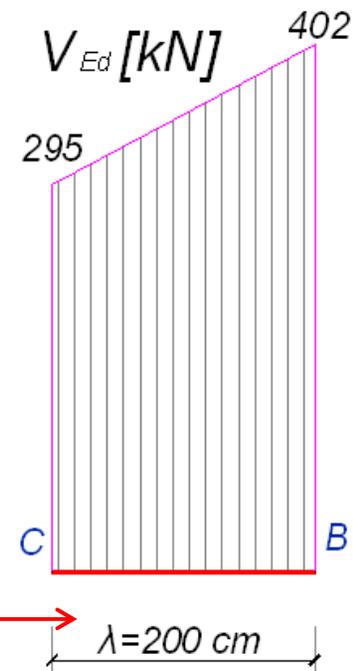
4. Nosivost betona na smicanje:

$$V_{Rd,c} = \max \left\{ \begin{array}{l} 66.5 \text{ kN} \\ 50.7 \text{ kN} \end{array} \right\} 66.5 \text{ kN} < 295 \text{ kN}$$

Ceo segment nosača B-C potrebno je osigurati poprečnom armaturom.



Dužina osiguranja (λ) jednaka je dužini segmenta B-C



Zadatak 25 – SMICANJE

5. Određivanje potrebne armature za smicanje

Na dužini osiguranja:

$$V_{Ed} = V_{Rd,s}$$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = V_{Ed}$$

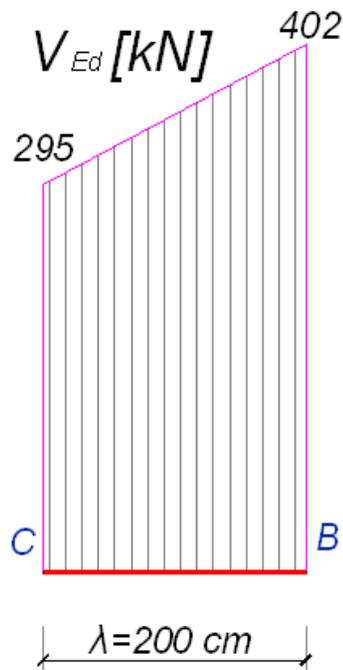
Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$

$$s_{rač} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{402} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 8.9 \text{ cm}$$

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}12$, $a_{sw}^{(1)}=1.13 \text{ cm}^2$

$$s_{rač} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 1.13}{402} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 12.7 \text{ cm}$$

Usvojeno: **UØ12/12.5**



Zadatak 25– SMICANJE

11. Dužina osiguranja: dužina $\lambda=200$ cm

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \left\{ \begin{array}{l} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot \text{ctg}\theta, V_{Ed} > V_{Rd,c} \end{array} \right\} = 0.5 \cdot 402 \cdot 1.0 = 201 \text{ kN}$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{201}{43.5} = 4.62 \text{ cm}^2 < 2 \cdot 4.91 = 9.82 \text{ cm}^2$$

Armatura koju treba obezbediti u donjoj zoni na mestu slobodnog oslonca B

Armatura iz dimenzionisanja prema momentima savijanja – vidi plan armature

Zadatak 25 – SMICANJE

5a. Varijantno rešenje: **uzengije+kosa armatura**

Usvojeno: $\theta=45^\circ$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$ **$s=20\text{cm}$**

Deo smicanja koji se poverava uzengijama:

$$V_{Rd,s} = \frac{2 \cdot 0.785}{20} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 178.2 \text{ kN}$$

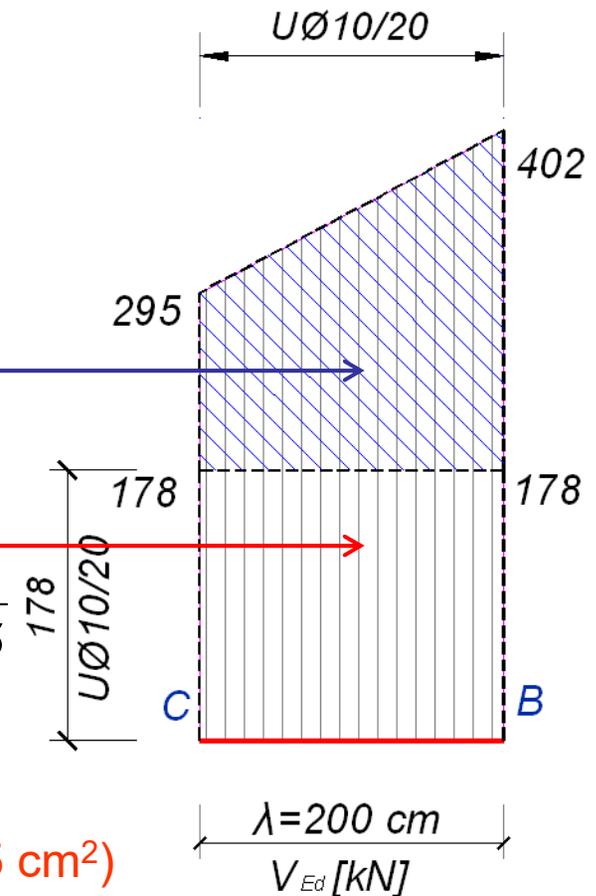
Deo smicanja koji se poverava kosoj armaturi:

$$V_{Rd,s,k} = \frac{(402 - 178) + (295 - 178)}{2} \cdot 200 \cdot \frac{1}{0.9 \cdot 58} = 653.2 \text{ kN}$$

$$A_{sw,k} = \frac{V_{Rd,s,k}}{f_{ykw} \cdot (ctg\theta + ctg\alpha) \cdot \sin\alpha} = \frac{653.2 \text{ kN}}{43.5 \cdot (ctg45 + ctg45) \cdot \sin45}$$

$$A_{sw,k} = \frac{653.2 \text{ kN}}{43.5 \cdot (1+1) \cdot \frac{\sqrt{2}}{2}} = \frac{653.2 \text{ kN}}{43.5 \cdot \sqrt{2}} = 10.6 \text{ cm}^2$$

Usvojeno: **$3\text{Ø}25$ (14.85 cm^2)**



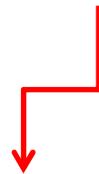
Zadatak 25 – SMICANJE

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \left\{ \begin{array}{l} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot (\operatorname{ctg}\theta - \operatorname{ctg}\alpha), V_{Ed} > V_{Rd,c} \end{array} \right\} = 0.5 \cdot (V_{Rd,s} + V_{Rd,s,k}) \cdot (\operatorname{ctg}\theta - \operatorname{ctg}\alpha) = 0.5 \cdot V_{Rd,s} \cdot \operatorname{ctg}\theta$$

$$\Delta F_{td} = 0.5 \cdot 178.2 \cdot 1.0 = 89.1 \text{ kN}$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{89.1}{43.5} = 2.05 \text{ cm}^2 < 2 \cdot 4.91 = 9.82 \text{ cm}^2$$



Armatura koju treba obezbediti u donjoj zoni na mestu slobodnog oslonca B

Armatura iz dimenzionisanja prema momentima savijanja – vidi plan armature 

Zadatak 25 – SMICANJE

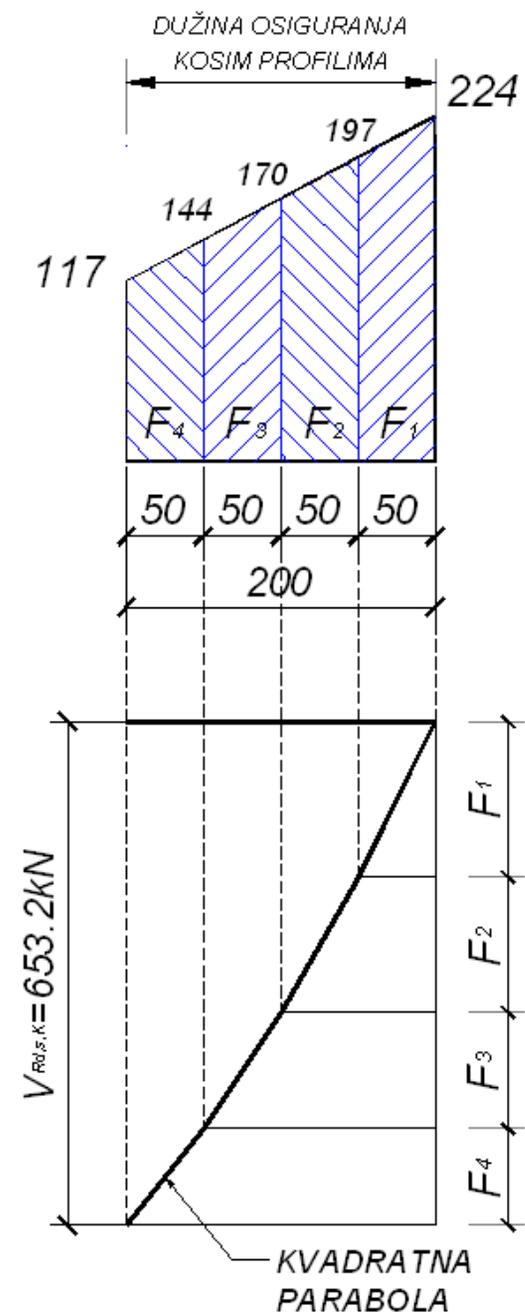
13. Određivanje mesta povijanja kosih profila

$$F_1 = \frac{224+197}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 201.6kN$$

$$F_2 = \frac{197+170}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 175.8kN$$

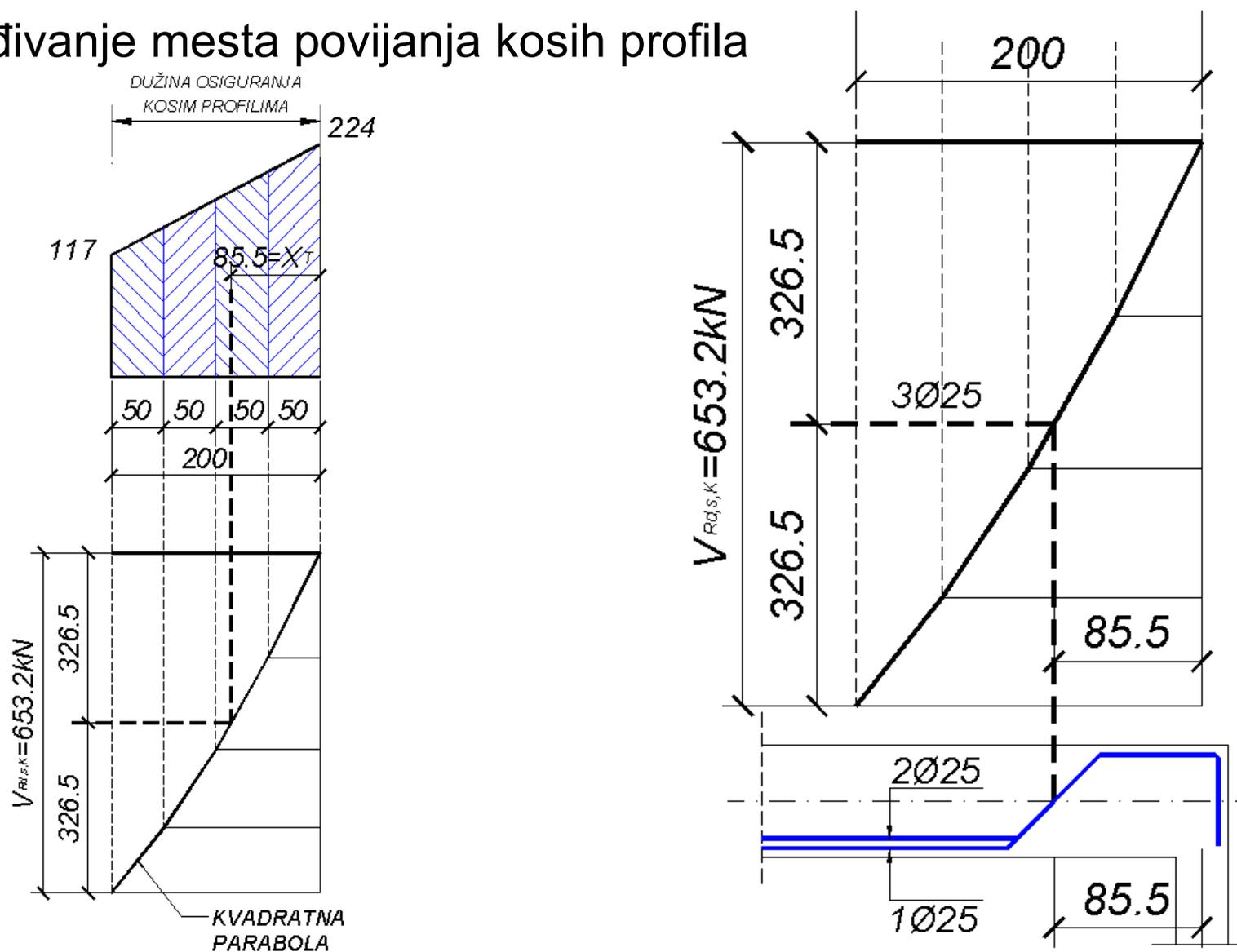
$$F_3 = \frac{170+144}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 150.4kN$$

$$F_4 = \frac{144+117}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 125kN$$



Zadatak 25 – SMICANJE

13. Određivanje mesta povijanja kosih profila



Zadatak 25 – SMICANJE

5b. Varijantno rešenje: **uzengije+kosa armatura**

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$ **$s=10\text{cm}$**

Deo smicanja koji se poverava uzengijama:

$$V_{Rd,s} = \frac{2 \cdot 0.785}{10} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 356.4 \text{ kN}$$

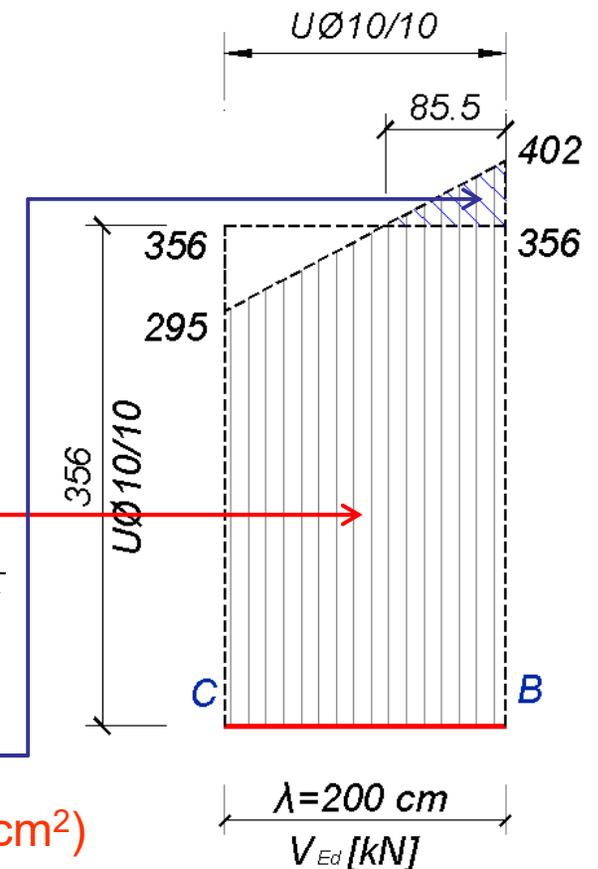
Deo smicanja koji se poverava kosoj armaturi:

$$V_{Rd,s,k} = \frac{(402 - 356)}{2} \cdot 85.5 \cdot \frac{1}{0.9 \cdot 58} = 37.7 \text{ kN}$$

$$A_{sw,k} = \frac{V_{Rd,s}}{f_{ykw} \cdot (ctg\theta + ctg\alpha) \cdot \sin\alpha} = \frac{37.7 \text{ kN}}{43.5 \cdot (ctg45 + ctg45) \cdot \sin45}$$

$$A_{sw,k} = \frac{37.7 \text{ kN}}{43.5 \cdot (1+1) \cdot \frac{\sqrt{2}}{2}} = \frac{37.7 \text{ kN}}{43.5 \cdot \sqrt{2}} = 0.6 \text{ cm}^2$$

Usvojeno: **1Ø25 (4.91 cm²)**



Zadatak 25 – SMICANJE

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \left\{ \begin{array}{l} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot (\text{ctg}\theta - \text{ctg}\alpha), V_{Ed} > V_{Rd,c} \end{array} \right\} = 0.5 \cdot (V_{Rd,s} + V_{Rd,s,K}) \cdot (\text{ctg}\theta - \text{ctg}\alpha) = 0.5 \cdot V_{Rd,s} \cdot \text{ctg}\theta$$

$$\Delta F_{td} = 0.5 \cdot 356 \cdot 1.0 = 178 \text{ kN}$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{178}{43.5} = 4.1 \text{ cm}^2 < 2 \cdot 4.91 = 9.82 \text{ cm}^2$$



Armatura koju treba obezbediti u donjoj zoni na mestu slobodnog oslonca B



Armatura iz dimenzionisanja prema momentima savijanja – vidi plan armature

Zadatak 25 – SMICANJE

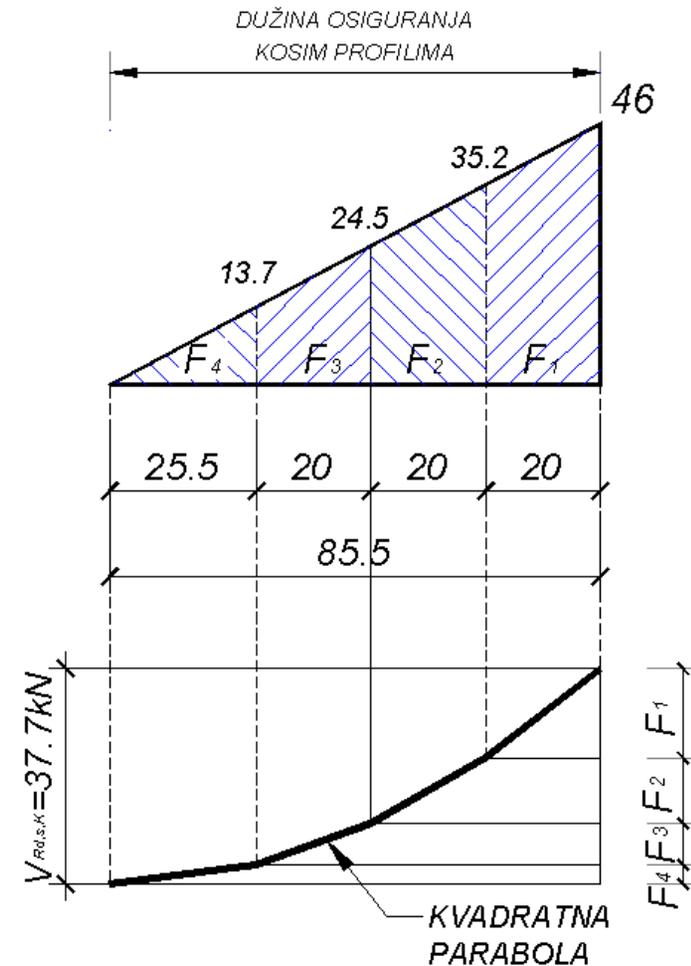
13. Određivanje mesta povijanja kosih profila

$$F_1 = \frac{46 + 35.2}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 15.6 \text{ kN}$$

$$F_2 = \frac{35.2 + 24.5}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 11.4 \text{ kN}$$

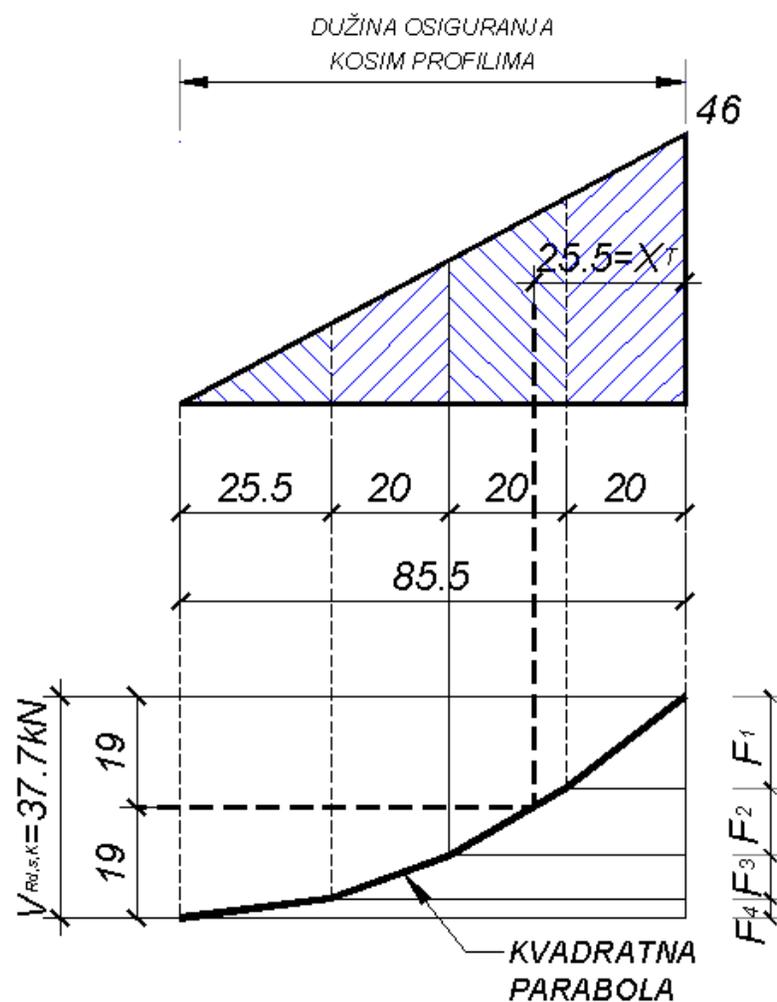
$$F_3 = \frac{24.5 + 13.7}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 7.3 \text{ kN}$$

$$F_4 = \frac{13.7 + 25.5}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 3.3 \text{ kN}$$



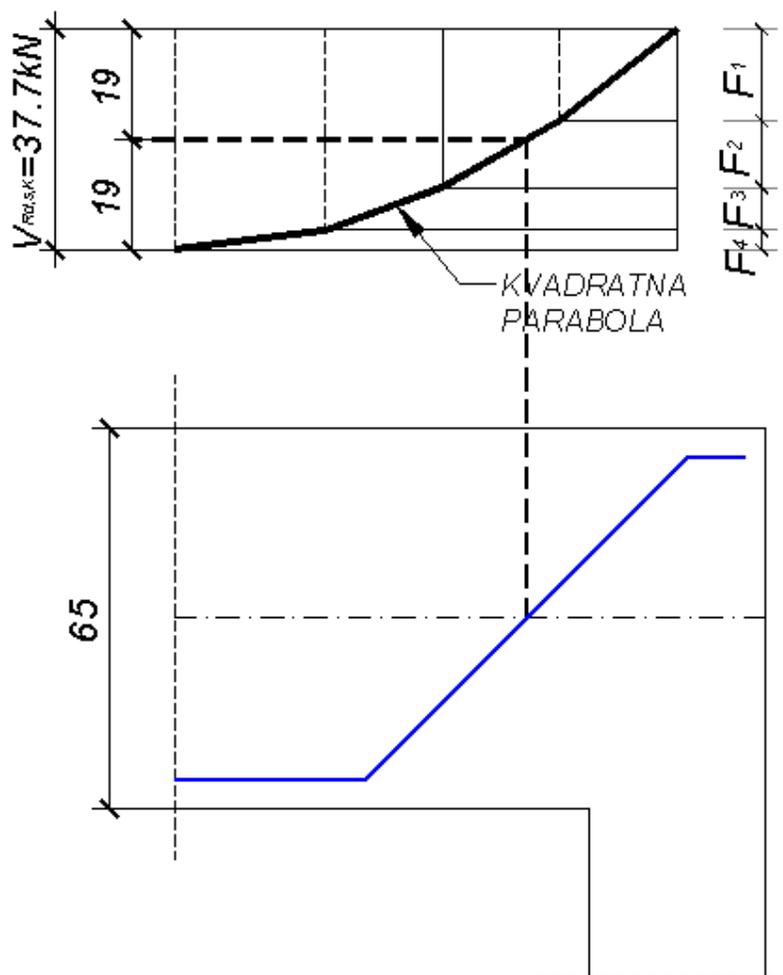
Zadatak 25 – SMICANJE

13. Određivanje mesta povijanja kosih profila

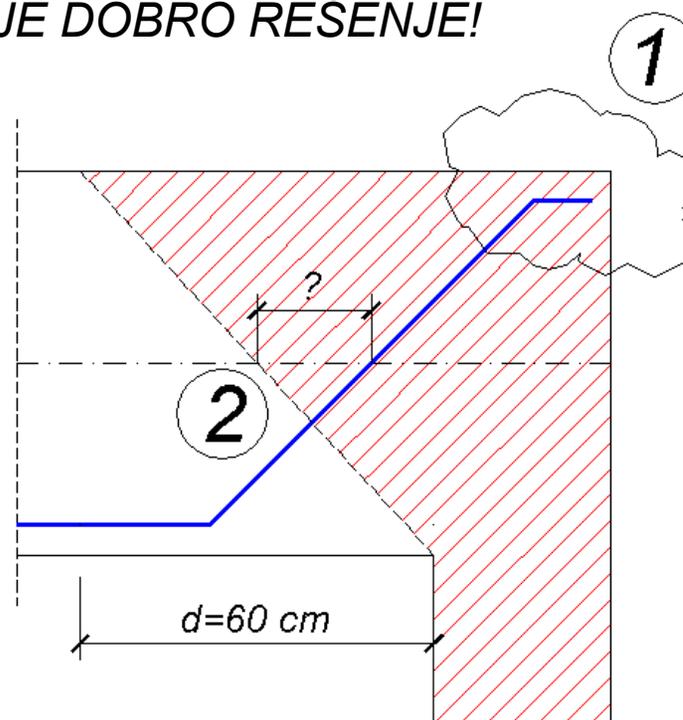


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13. Određivanje mesta povijanja kosih profila



NIJE DOBRO REŠENJE!



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PLAN ARMATURE

