



Univerzitet u Beogradu – Građevinski fakultet

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Studijski program: **Građevinarstvo**

Modul: Konstrukcije

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Naziv predmeta (šifra): **Teorija betonskih konstrukcija 1  
(b2k3b1)**

Nastavnik: **Ivan Ignjatović**

Naslov vežbi: **Centrična naprezanje.**

**Mali ekscentricitet- sila zatezanja**

Datum : **03.11.2022.**

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Beograd, 2020.

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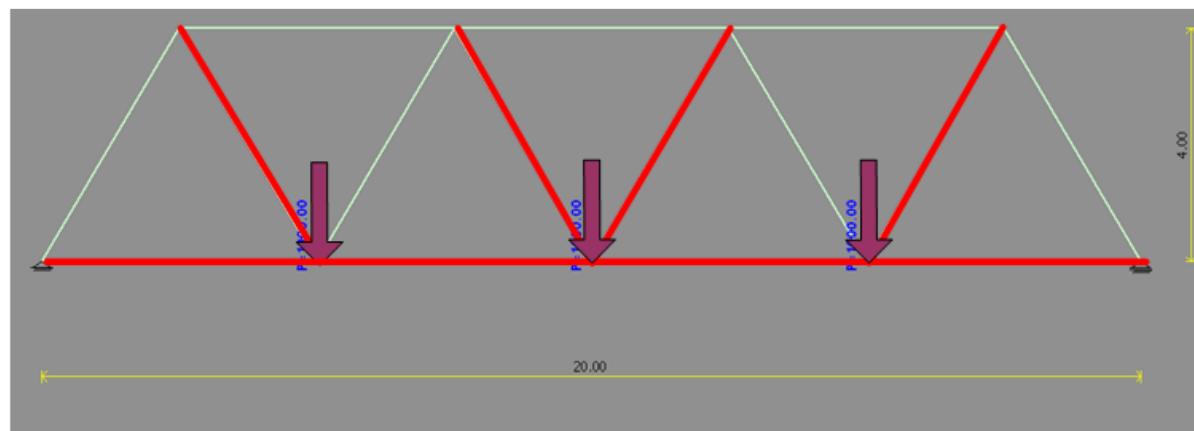


GF Beograd

materijala.

Teorija betonskih konstrukcija 1

# Centrično zatezanje



## Zadatak 17 – CENTRIČNO ZATEZANJE

Odrediti potrebnu površinu armature i oblikovati poprečni presek, pravougaonog oblika, centrično zategnutog elementa.  
Podaci za proračun:

$$N_{G,k} = -400 \text{ kN}$$

C25/30

XD1

$$N_{Q,k} = -500 \text{ kN}$$

B500 B

Treba ?

$$\text{C25/30} \rightarrow f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 \text{ MPa} = 1.42 \text{ kN/cm}^2$$

$$\text{B500 B} \rightarrow f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$$



## Zadatak 17 – **CENTRIČNO ZATEZANJE**

Granična sila **ZATEZANJA**:

$$N_{Ed} = 1.35 \cdot (-400) + 1.5 \cdot (-500) = -1290\text{kN}$$

Proračun površine armature:

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} = \frac{1290}{43.5} = 29.7\text{cm}^2$$

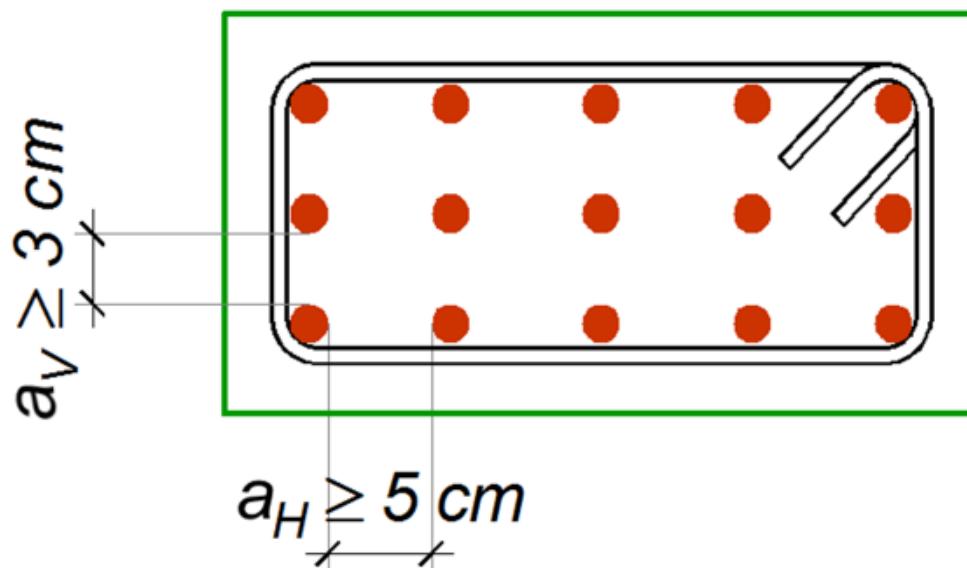
Usvojeno: **15 Ø16** ( $30.15\text{ cm}^2$ )



## Zadatak 17 – CENTRIČNO ZATEZANJE

$$b \geq 2c_{\text{nom}} + 2\emptyset_s + m \times \emptyset + (m-1) \times a_H$$

$$h \geq 2c_{\text{nom}} + 2\emptyset_s + n \times \emptyset + (n-1) \times a_V$$



$$XD1 \Rightarrow c_{\text{nom}} = 35 + 10 = 45 \text{ mm}$$

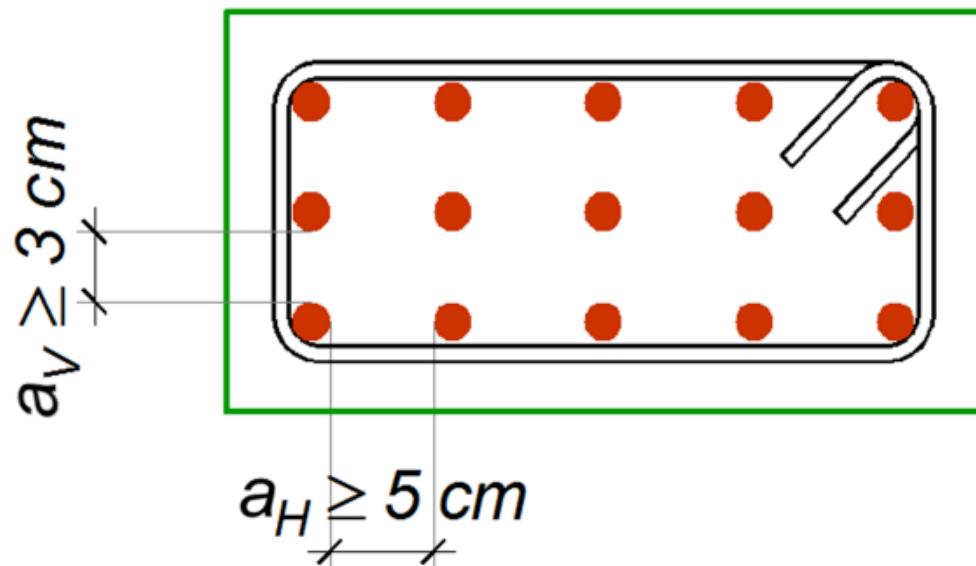
$$\text{Pretp.} \Rightarrow \emptyset_s = 8 \text{ mm}$$

Oznaka klase izloženosti	Minimalni zaštitni slojevi iz uslova trajnosti, $c_{\text{min dur}}$ , za klasu konstrukcija S4									
	10	15	20	25	30	35	40	45	50	55
X0	10	15	20	25	30	35	40	45	50	55
XC1	15	18	22	25	28	32	36	40	45	50
XC2	20	22	25	28	30	32	36	40	45	50
XC3	25	28	30	32	34	36	38	40	45	50
XC4	30	32	35	38	40	42	45	48	50	55
XD1	35	38	40	42	45	48	50	52	55	58
XD2	38	40	42	45	48	50	52	55	58	60
XD3	40	42	45	48	50	52	55	58	60	62
XS1	35	38	40	42	45	48	50	52	55	58
XS2	38	40	42	45	48	50	52	55	58	60
XS3	40	42	45	48	50	52	55	58	60	62



## Zadatak 17 – CENTRIČNO ZATEZANJE

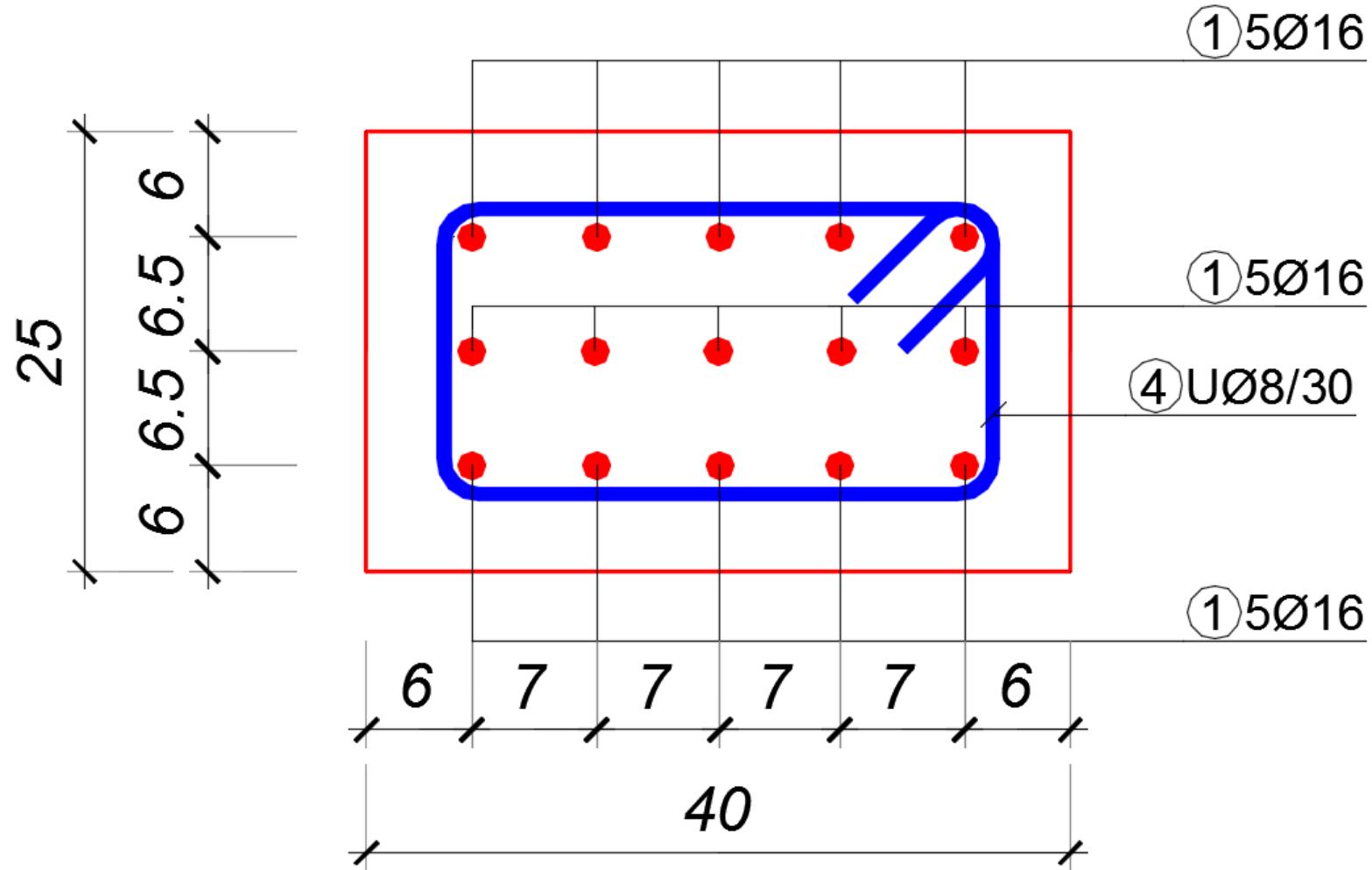
$$b \geq 2c_{\text{nom}} + 2\emptyset_s + m \times \emptyset + (m-1) \times a_H$$
$$h \geq 2c_{\text{nom}} + 2\emptyset_s + n \times \emptyset + (n-1) \times a_V$$



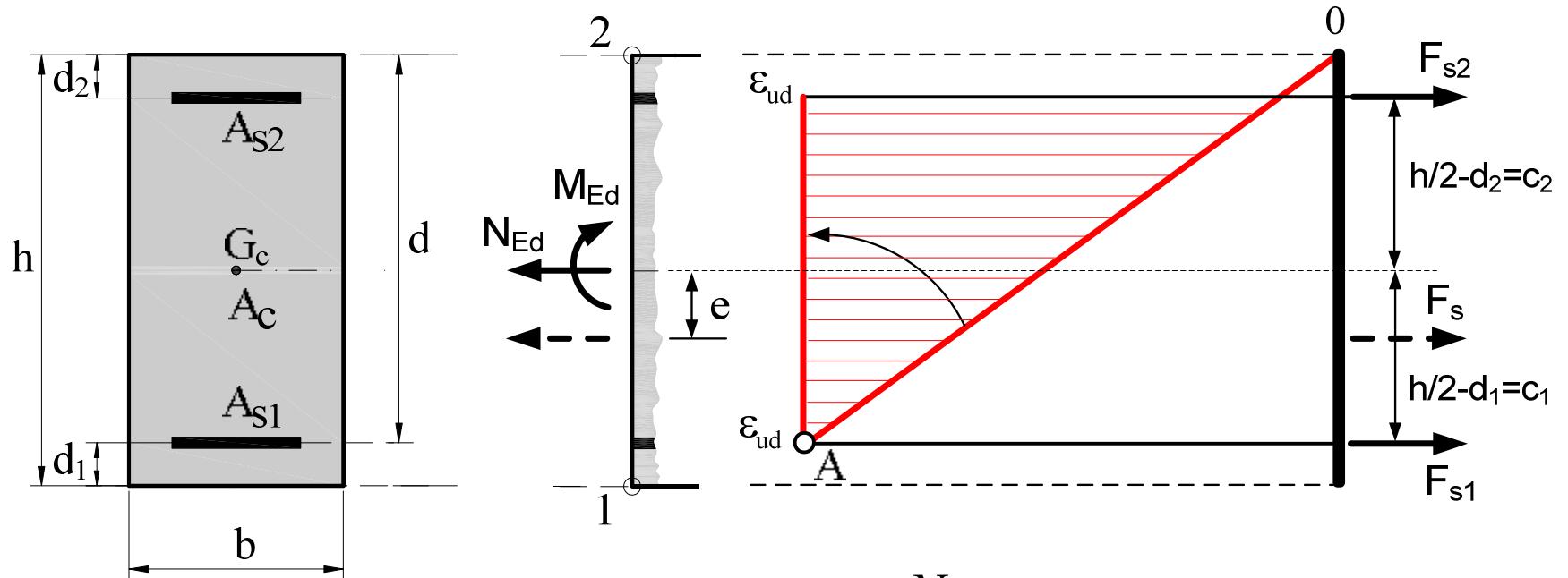
$$b \geq 2 \times 4.5 + 2 \times 0.8 + 5 \times 1.6 + (5-1) \times 5.0 = 38.6 \text{ cm} \Rightarrow b = 40 \text{ cm}$$
$$d \geq 2 \times 4.5 + 2 \times 0.8 + 3 \times 1.6 + (3-1) \times 3.0 = 21.4 \text{ cm} \Rightarrow d = 25 \text{ cm}$$



## Zadatak 17 – CENTRIČNO ZATEZANJE



# ULS – MALI EKSCENTRICITET, SILA ZATEZANJA



$$A_s = A_{s1} + A_{s2} = \frac{N_{Ed}}{f_{yd}}$$

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2}$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2}$$



## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

Dimenzionisati pravougaoni poprečni presek zadatih dimenzija, opterećen silom zatezanja i momentom savijanja u fazi malog ekscentriciteta.

Podaci za proračun:

$$N_{G,k} = -400 \text{ kN}$$

$$M_{G,k} = 10 \text{ kNm}$$

$$b = 40 \text{ cm}$$

$$N_{Q,k} = -500 \text{ kN}$$

$$h = 25 \text{ cm}$$

B500 B

$$f_{yd} = 500/1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$$



## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

$$N_{Ed} = 1.35 \cdot (-400) + 1.5 \cdot (-500) = -1290 \text{ kN}$$

$$M_{Ed} = 1.35 \cdot 10 = 13.5 \text{ kNm}$$

$$e = \frac{M_{Ed}}{N_{Ed}} = \frac{13.5 \times 10^2}{1290} = 1.05 \text{ cm}$$

pretp.  $d_1 = d_2 = 6 \text{ cm}$

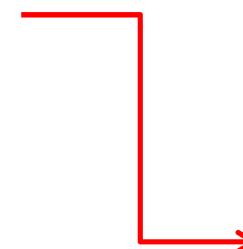
$$c_1 = c_2 = h/2 - d_1 = 25/2 - 6 = 6.5 \text{ cm}$$

$$A_s = A_{s1} + A_{s2} = \frac{N_{Ed}}{f_{yd}} = \frac{1290}{43.5} = 29.7 \text{ cm}^2$$



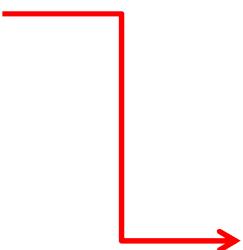
## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 + 1.05}{6.5 + 6.5} = 17.2 \text{ cm}^2$$



$$\approx 9\varnothing 16 \\ d_1 = 8 \text{ cm} \\ c_1 = 25/2 - 8 \\ = 4.5 \text{ cm}$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 - 1.05}{6.5 + 6.5} = 12.5 \text{ cm}^2$$

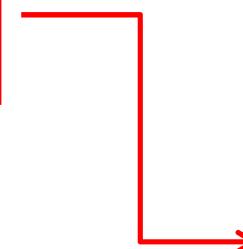


$$\approx 6\varnothing 16 \\ d_2 = 6 \text{ cm} \\ c_2 = 25/2 - 6 = \\ 6.5 \text{ cm}$$



## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

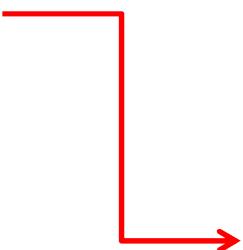
$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 + 1.05}{4.5 + 6.5} = 20.4 \text{ cm}^2$$



$\approx 10\varnothing 16$

$$\begin{aligned}d_1 &= 8.25 \text{ cm} \\c_1 &= 25/2 - 8.25 \\&= 4.25 \text{ cm}\end{aligned}$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2} = 29.7 \cdot \frac{4.5 - 1.05}{4.5 + 6.5} = 9.3 \text{ cm}^2$$



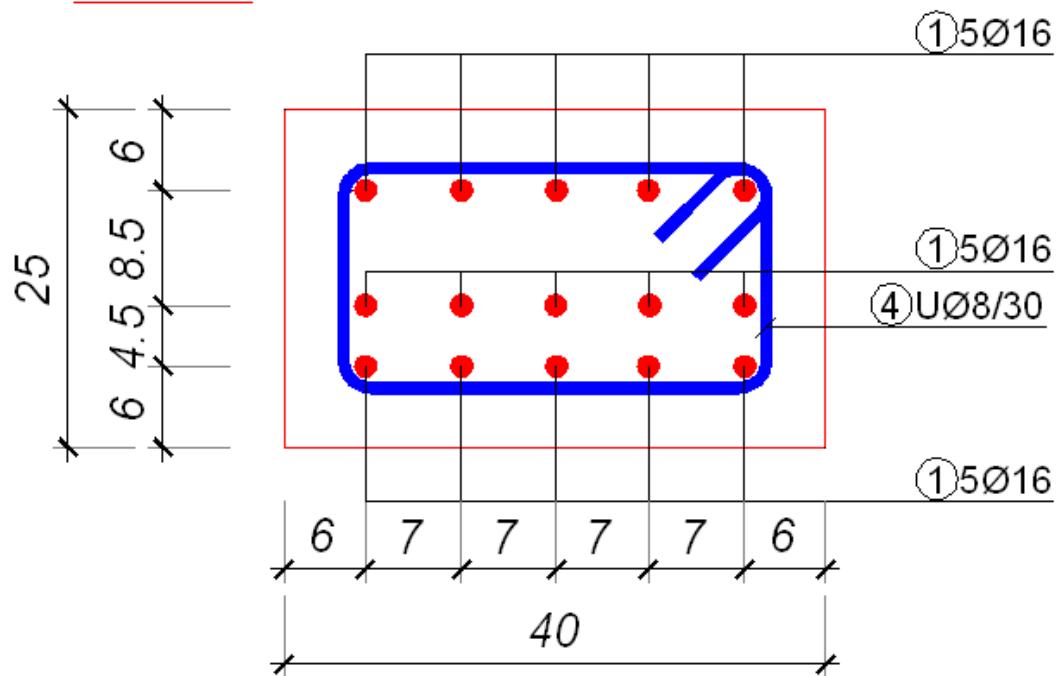
$\approx 5\varnothing 16$

$$\begin{aligned}d_2 &= 6 \text{ cm} \\c_2 &= 25/2 - 6 = \\&6.5 \text{ cm}\end{aligned}$$



## Zadatak 18 – EKSCENTRIČNO ZATEZANJE

XD1



$$d^I = c_{\text{nom}} + \emptyset_s + \emptyset/2$$

$$d^I = 4.5 + 0.8 + 1.6/2 = 6.1 \text{ cm}$$

usv.  $d^I = 6 \text{ cm}$

$$d^{II} = d^I + a_v + 2 \times \emptyset/2$$

$$d^{II} = 6 + 3.0 + 2 \times 1.6/2 = 10.6 \text{ cm}$$

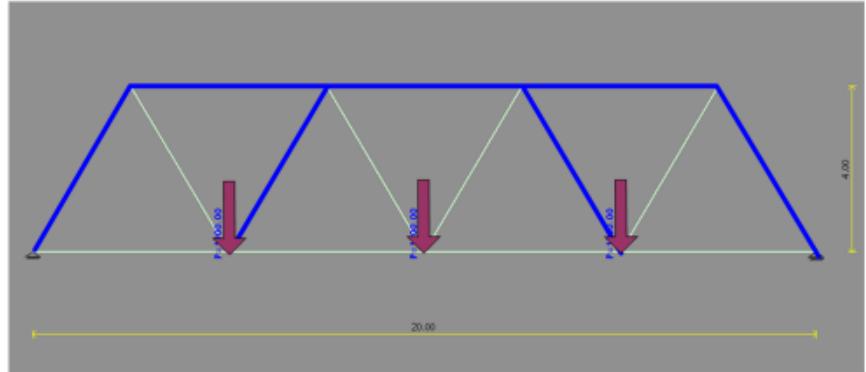
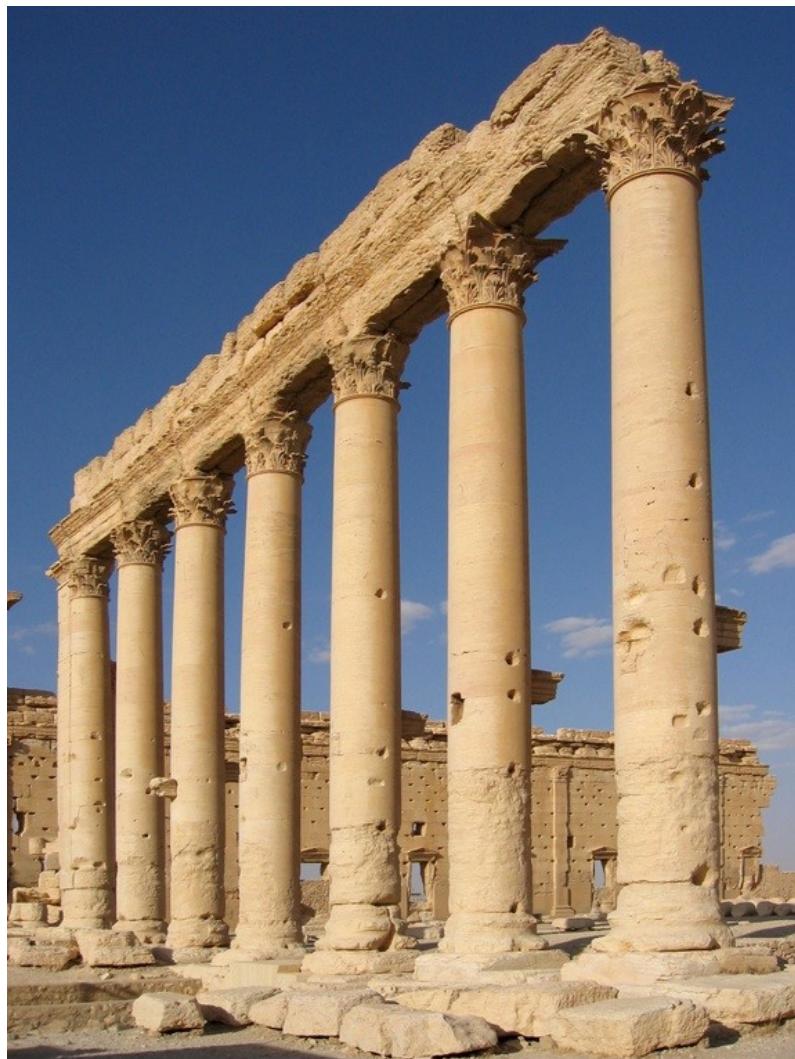
usv.  $d^{II} = 10.5 \text{ cm}$

$$a_1 = (5 \times 6 + 5 \times 10.5)/10$$

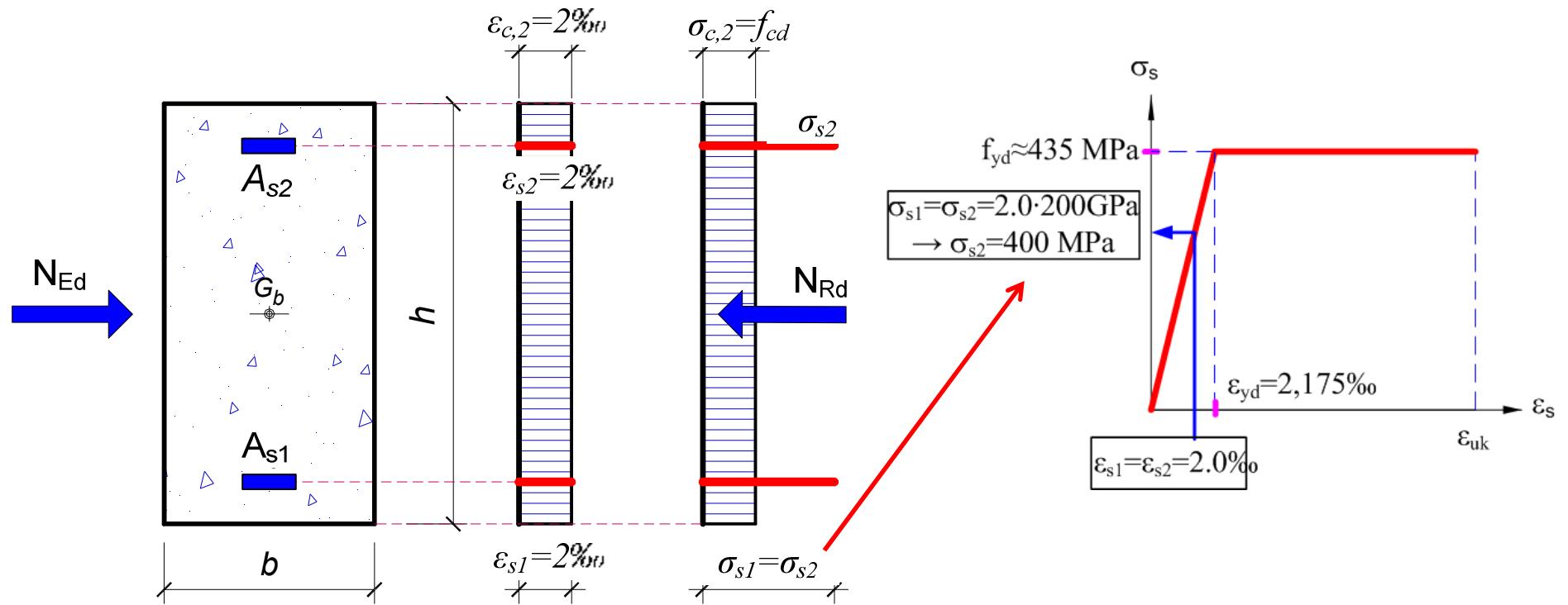
$$d_1 = 8.25 \text{ cm}$$



# Centrični pritisak



# Centrični pritisak



$$\Sigma N = 0 : \quad N_{Rd} = N_{Ed} = A_c \cdot f_{cd} + A_s \cdot \sigma_s$$

## Zadatak 19 – CENTRIČNI PRITISAK

Odrediti potrebnu površinu armature i dimenzije poprečnog preseka, pravougaonog oblika, centrično pritisnutog elementa.  
Podaci za proračun:

$$N_{G,k} = 600 \text{ kN} \quad C25/30 \quad XC3$$

$$N_{Q,k} = 800 \text{ kN} \quad B500 \text{ B} \quad b = 35 \text{ cm}$$

$$C25/30 \rightarrow f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 \text{ MPa} = 1.42 \text{ kN/cm}^2$$

$$B500 \text{ B} \rightarrow f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$$



## Zadatak 19 – CENTRIČNI PRITISAK

Granična sila PRITISKA:

$$N_{Ed} = 1.35 \cdot 600 + 1.5 \cdot 800 = 2010 \text{ kN}$$

Uslov ravnoteže normalnih sila:

$$N_{Rd} = N_{Ed} = A_c \cdot f_{cd} + A_s \cdot \sigma_s$$

$$N_{Ed} = A_c \cdot f_{cd} \left( 1 + \frac{A_s}{A_c} \cdot \frac{\sigma_s}{f_{cd}} \right) = A_c \cdot f_{cd} \left( 1 + \frac{A_s}{A_c} \cdot \frac{\sigma_s}{f_{cd}} \cdot \frac{f_{yd}}{f_{yd}} \right) = A_c \cdot f_{cd} \left( 1 + \omega \cdot \frac{\sigma_s}{f_{yd}} \right)$$

Minimalni geometrijski koeficijent armiranja:

$$\rho_{l,min} = 0.3\% \Rightarrow \omega = 0.3 \cdot 43.5 / 1.42 = 9.19\%$$

$$A_{c,pot} = \frac{N_{Ed}}{f_{cd} \cdot \left( 1 + \omega \frac{\sigma_s}{f_{yd}} \right)} = \frac{2010}{1.42 \cdot \left( 1 + 0.0919 \frac{40}{43.5} \right)} = 1305 \text{ cm}^2$$



## Zadatak 19 – CENTRIČNI PRITISAK

$$h_{\text{pot}} = A_{c,\text{pot}}/b = 1305/35 = 37.3 \text{ cm} \Rightarrow \text{usv. } h = 40 \text{ cm}$$

Sračunavanje potrebne površine armature:

$$A_s = \max \left\{ \begin{array}{l} 0.15 \cdot \frac{N_{Ed}}{f_{yd}} = 0.15 \cdot \frac{2010}{43.5} = 6.93 \text{ cm}^2 \\ 0.003 \cdot A_c = 0.003 \cdot 40 \cdot 35 = 4.2 \text{ cm}^2 \\ 4\varnothing 12 = 4 \cdot 1.13 = 4.48 \text{ cm}^2 \end{array} \right\} = 6.93 \text{ cm}^2$$

**8 Ø12 (8.96 cm<sup>2</sup>)**

Maksimalno rastojanje poprečne armature (EC2):

$$s_{cl,t \max} = \min \left\{ \begin{array}{l} 20\varnothing_{\min} \\ \min(b, h) \\ 40 \text{ cm} \end{array} \right\} = \min \left\{ \begin{array}{l} 20 \cdot 1.2 = 24 \text{ cm} \\ \min(b, h) = 35 \text{ cm} \\ 40 \text{ cm} \end{array} \right\} = 24 \text{ cm}$$



## Zadatak 19 – CENTRIČNI PRITISAK

Maksimalno rastojanje poprečne armature (**EN1992-1-1/NA**):

$$s_{cl,t \max} = \min \left\{ \begin{array}{l} 12\varnothing_{\min} \\ \min(b, h) \\ 30cm \end{array} \right\} = \min \left\{ \begin{array}{l} 12 \cdot 1.2 = 14.4cm \\ \min(b, h) = 35cm \\ 30cm \end{array} \right\} = 14.4cm$$



## Zadatak 19 – CENTRIČNI PRITISAK

Usvojena podužna armatura:  $8 \varnothing 12$  ( $8.96 \text{ cm}^2$ )

Usvojena poprečna armatura:  $\varnothing 6/12.5$

