Studijski program:	**Građevinarstvo master akademske studije**
Modul:	Građevinska geotehnika / Organizacija, tehnologija i informatika u građevinarstvu
Godina/Semestar:	**I godina / I semestar**
Naziv predmeta (šifra):	**Projektovanje betonskih konstrukcija** (M3O1PB / M3O1PB4)
Nastavnik:	**Milica Vidović / Ivan Milićević**
Naslov vežbi:	Stepenište
Datum :	11. 12. 2025.

Vežbe 2 – Proračun stepeništa:

- *Primeri stepeništa*

  Primeri 1 i 2:

  - *Analiza opterećenja*
  - *Statički sistem*
  - *Granična stanja nosivosti*
  - *Granična stanja upotrebljivosti*
  - *Plan oplate i armature*

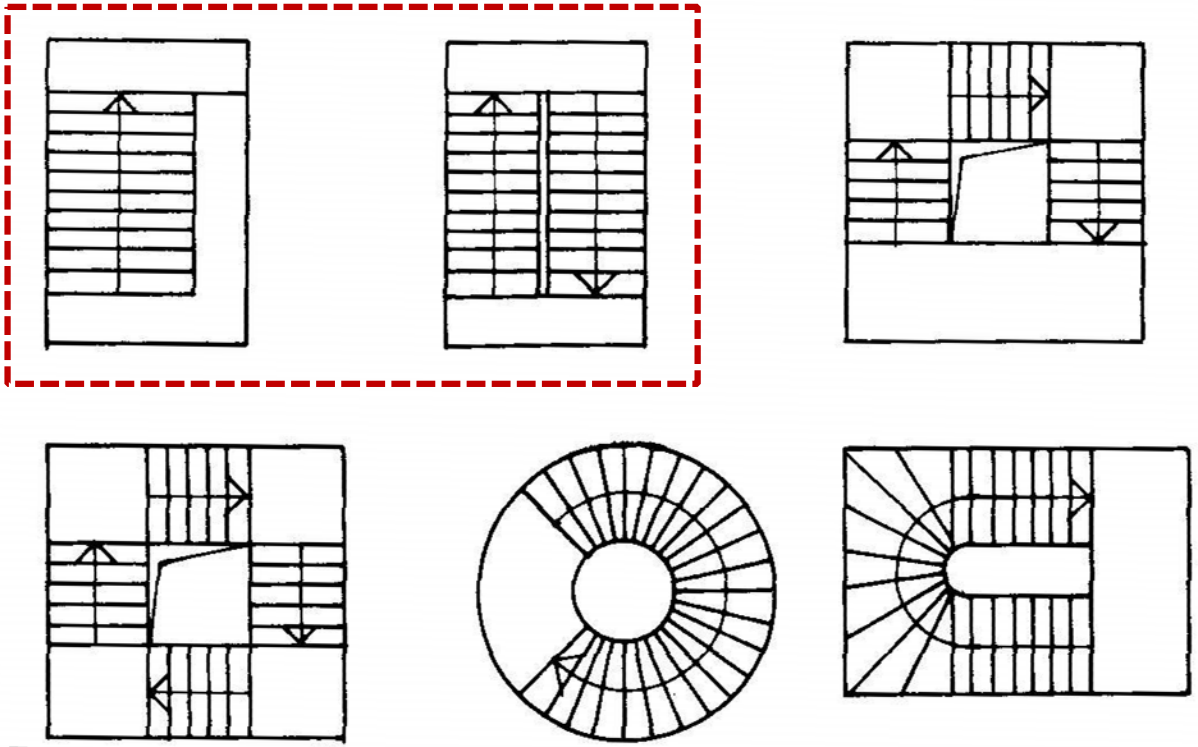
Vežbe 2 – Proračun stepeništa:

- *Literatura:*

  Milosavljević, B. (2019): Armiranobetonske međuspratne konstrukcije - Udžbenik za predmet Projektovanje i građenje betonskih konstrukcija 1, Akademska misao, Beograd.

  Radosavljević, Ž., Bajić, D. (2006): Armirani beton 3, Elementi armiranobetonskih onstrukcija, Građevinska knjiga, Beograd.

Primeri stepeništa



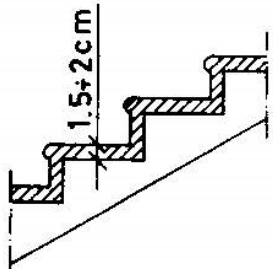
Preporuke:

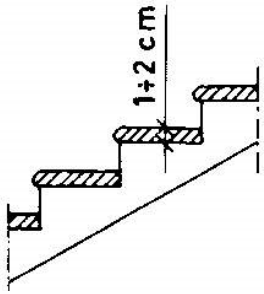
b + 2h = 63 cm (62-65 cm)

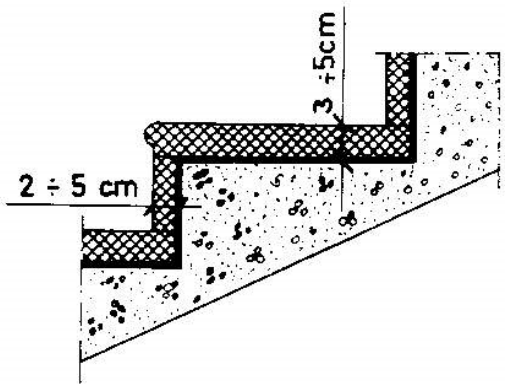
b + h = 46 cm

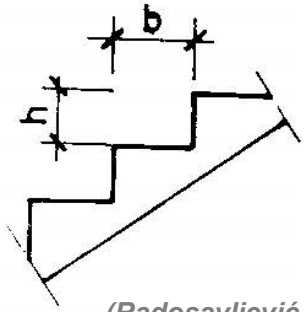
h = 15-17 cm

b/h = 32/15 cm, 30/16 cm,
29/17 cm, 27/18 cm,
26/19 cm, 25/20 cm





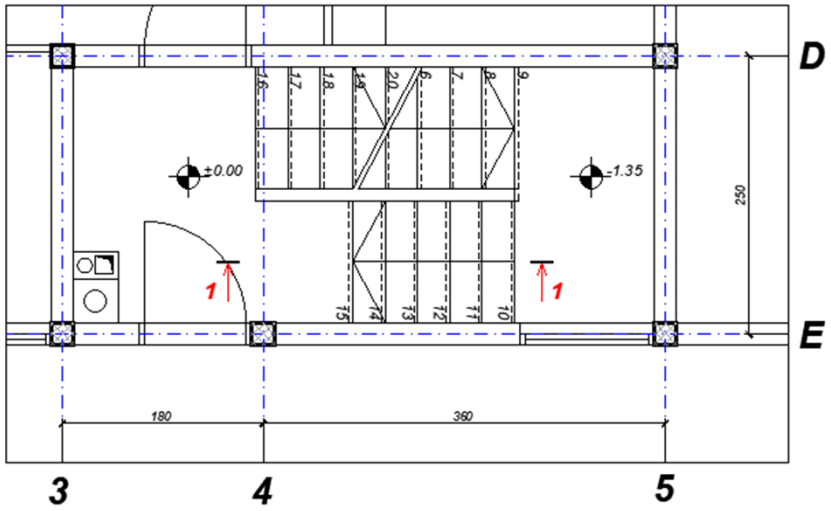




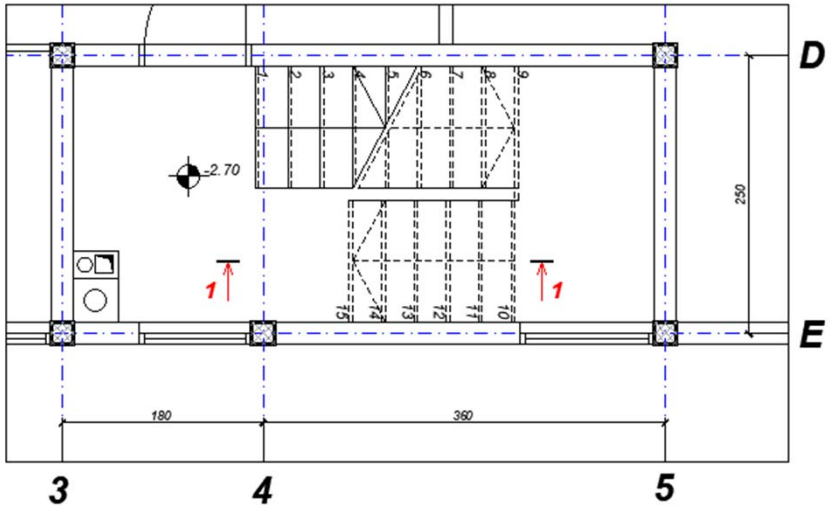
(Radosavljević i Bajić, 2006)

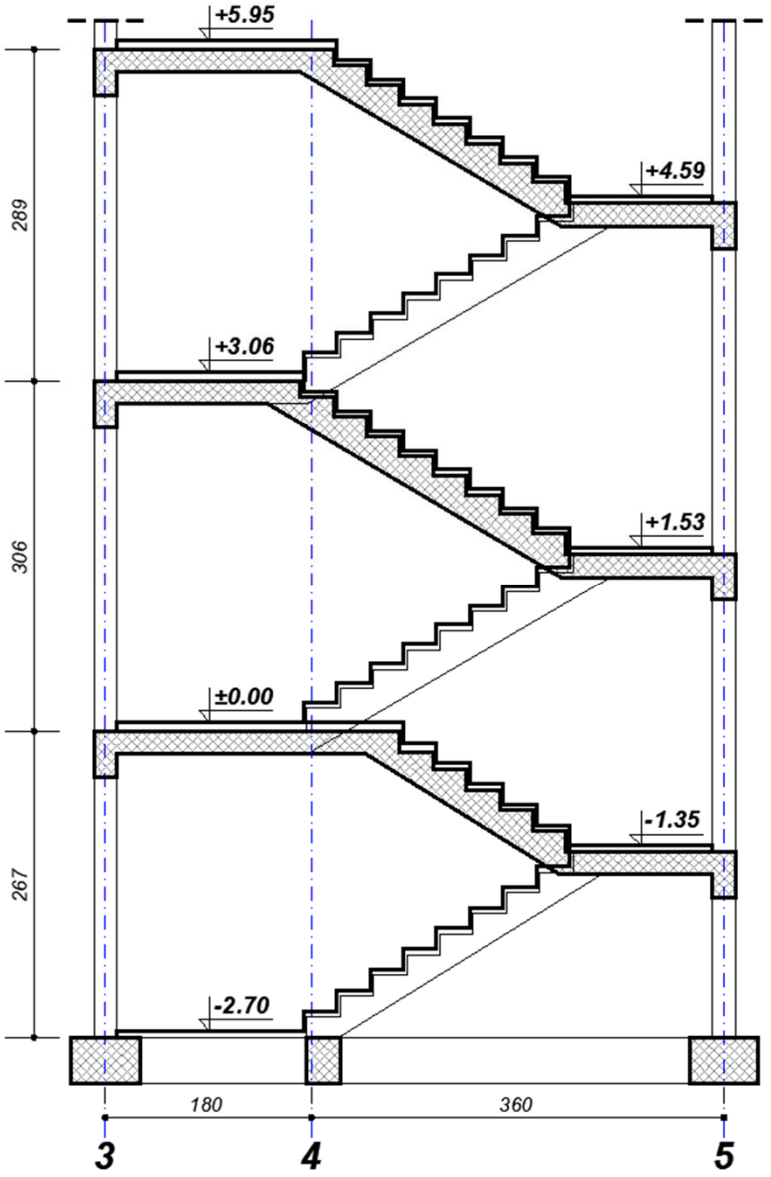
Primeri stepeništa

PRIZEMLJE 1: 50



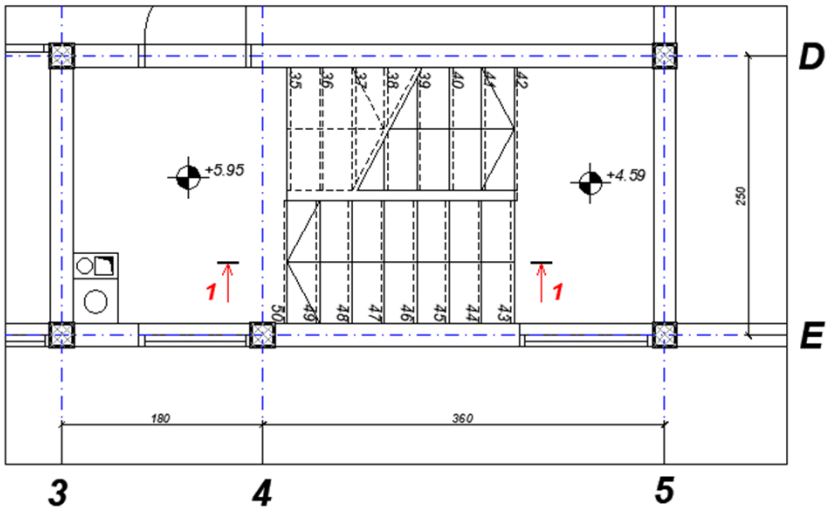
PODRUM 1: 50



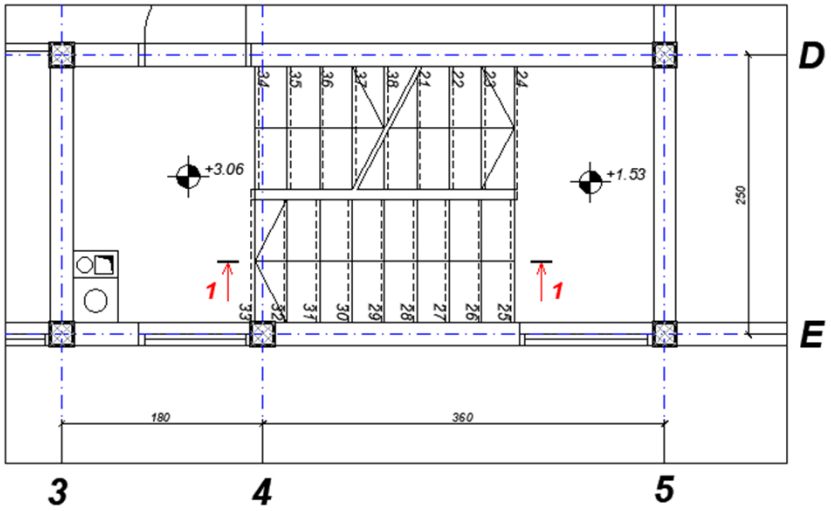


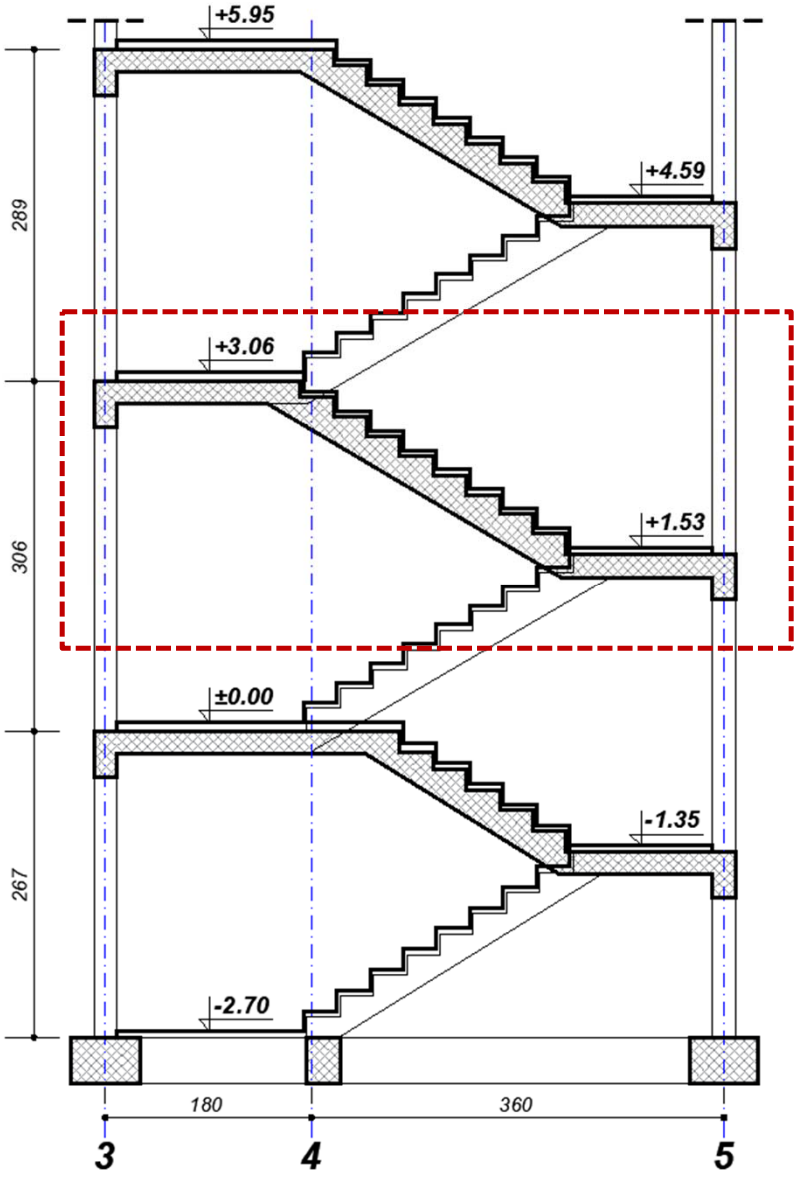
Primeri stepeništa

POTKROVLJE 1: 50

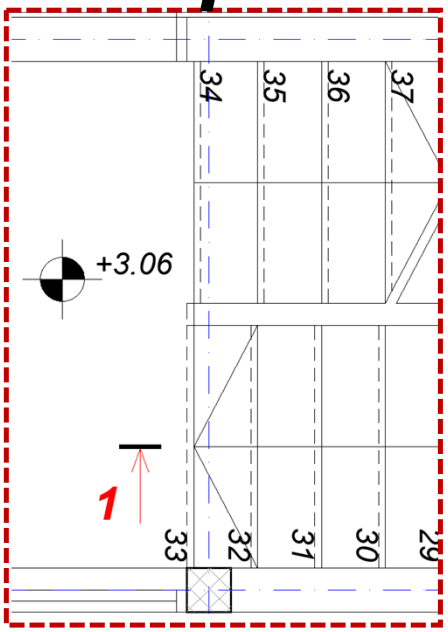


I SPRAT 1: 50





Primeri stepeništa



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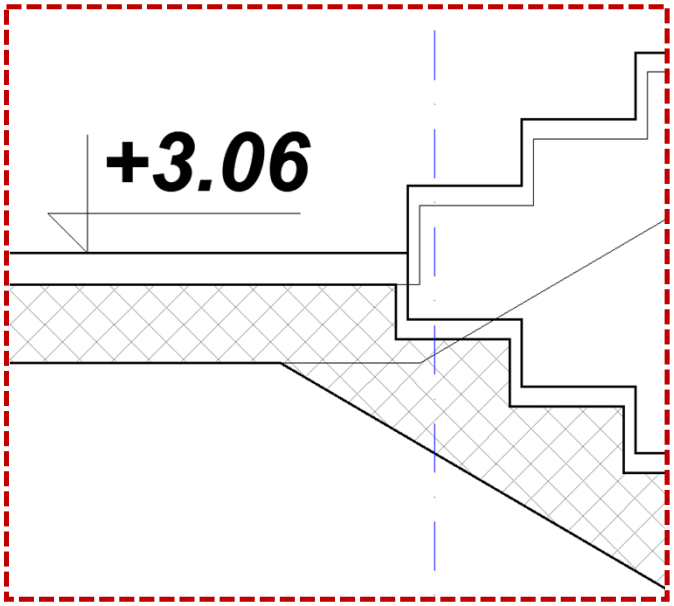
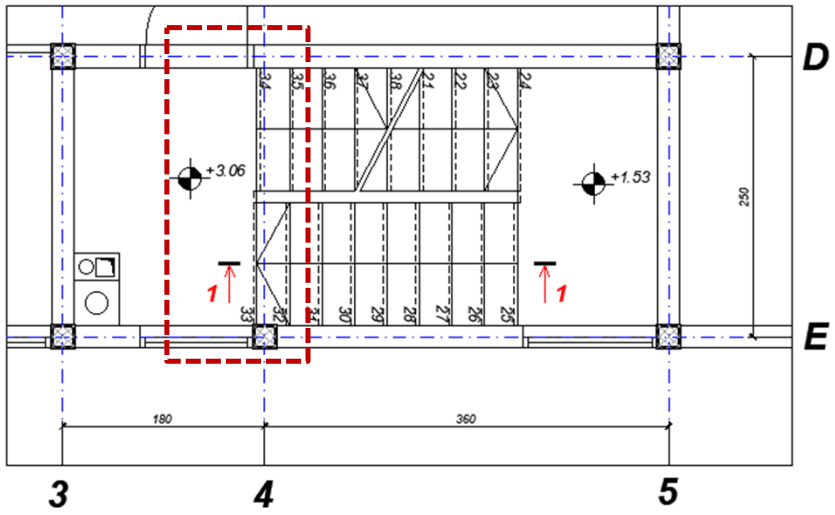
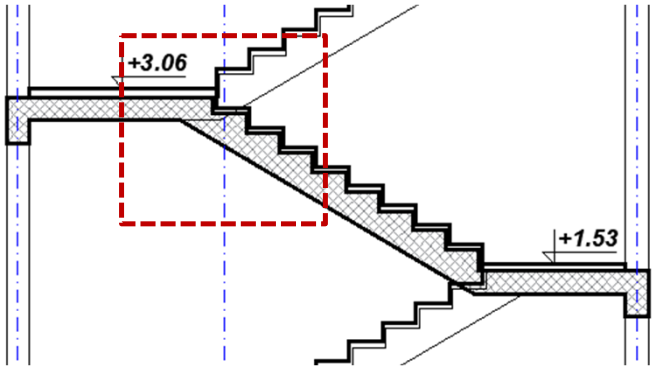
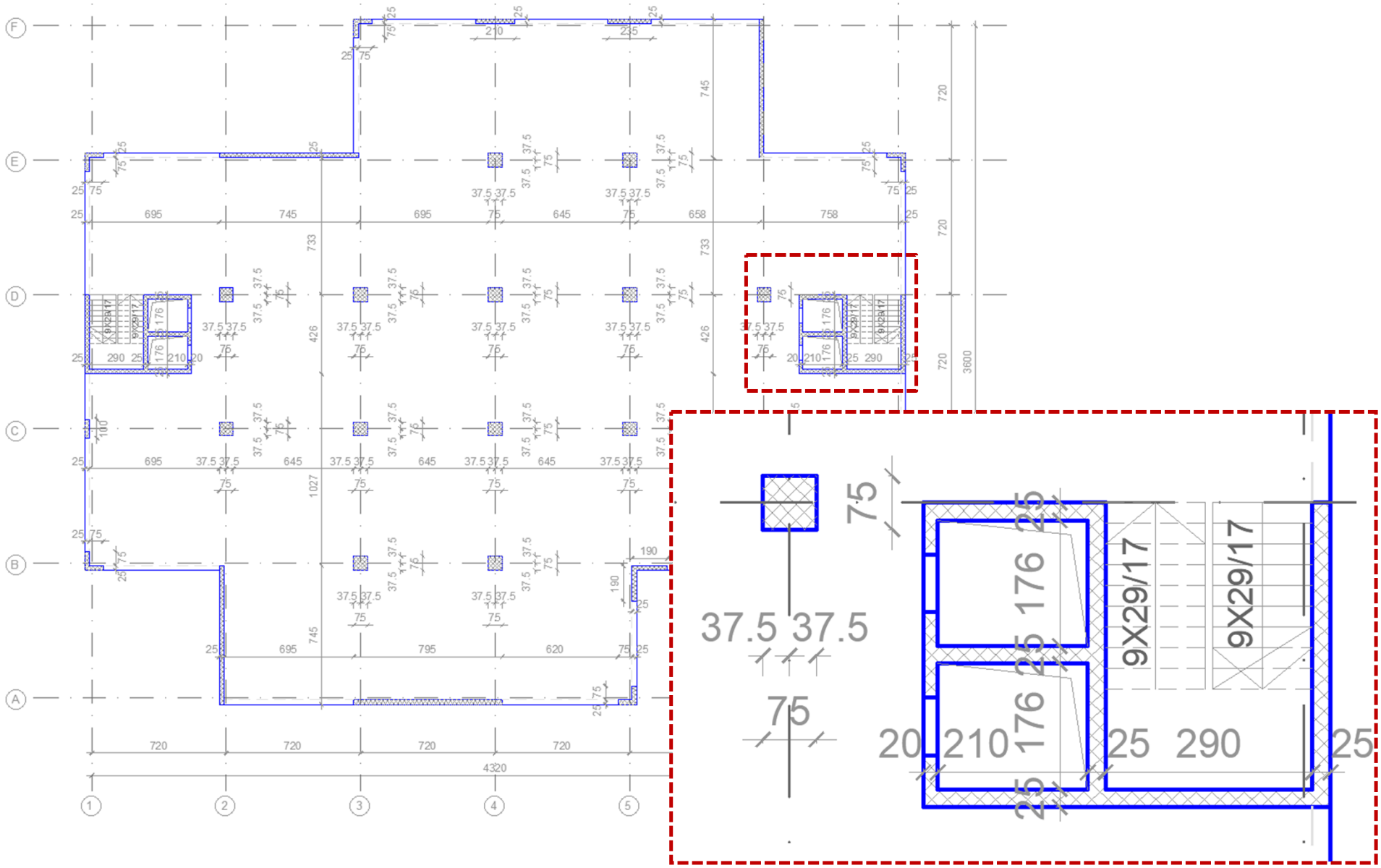


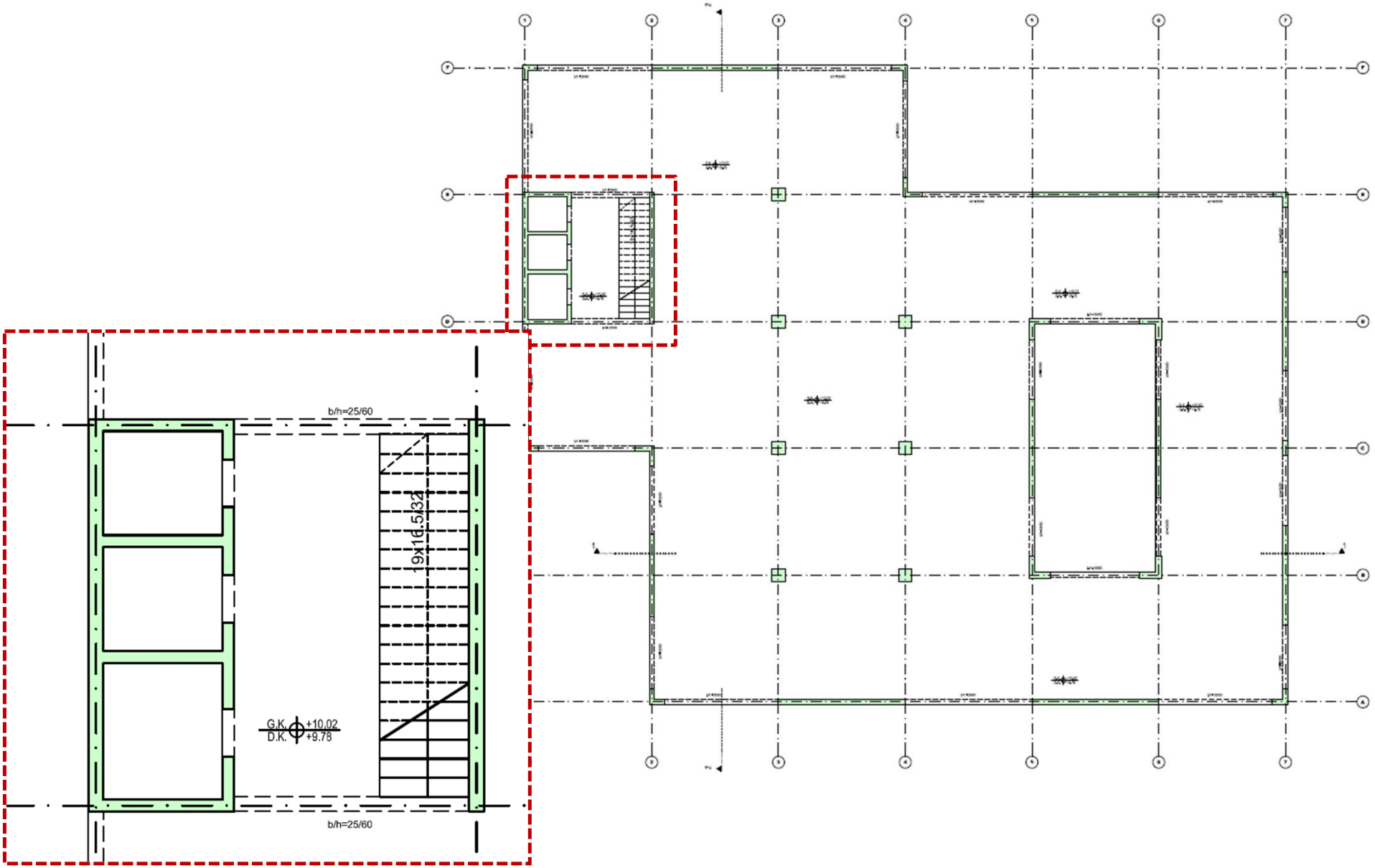




Primeri stepeništa



Primeri stepeništa



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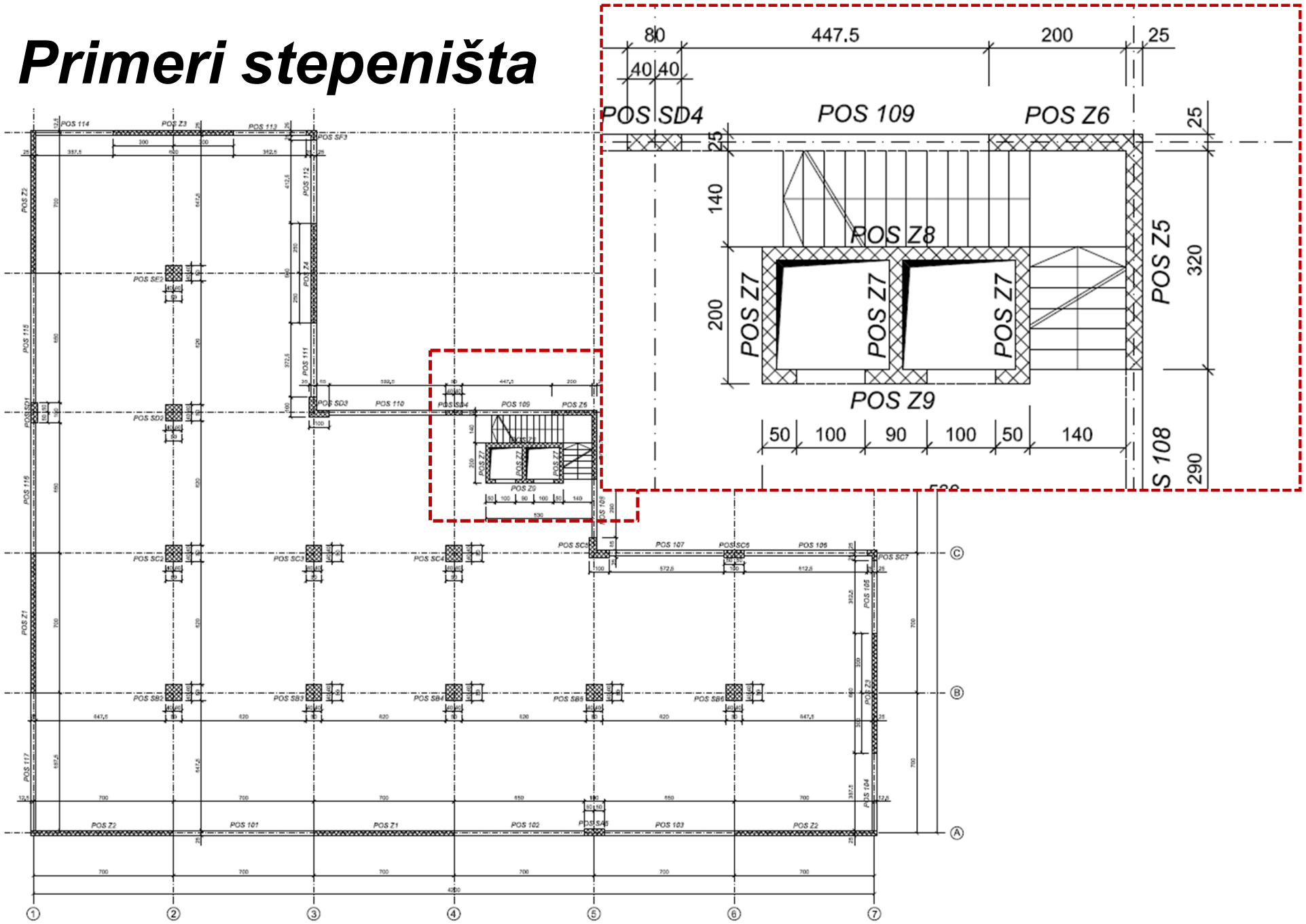

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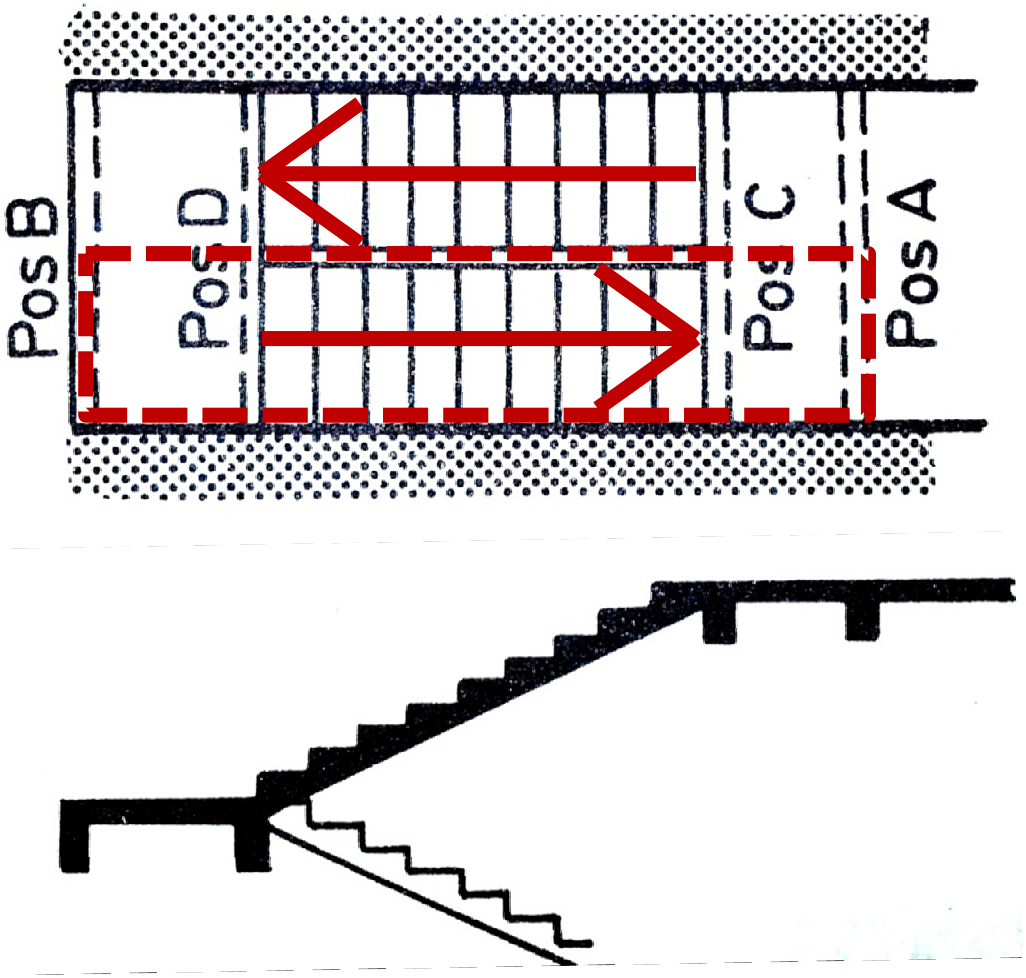
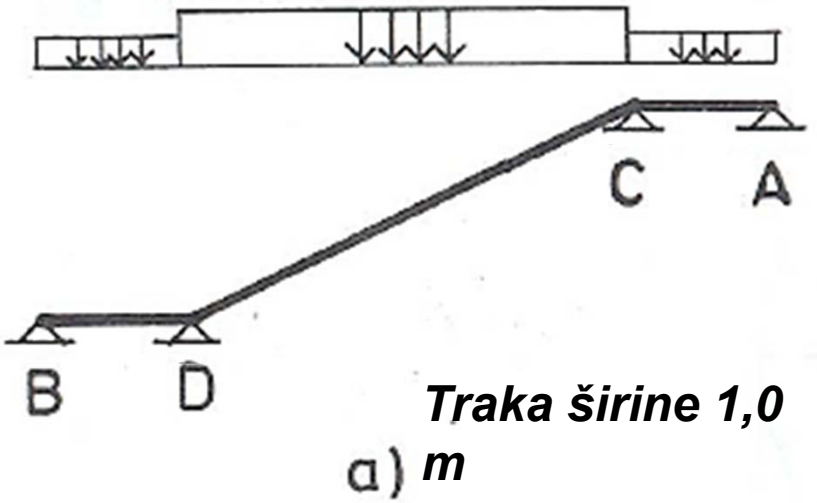
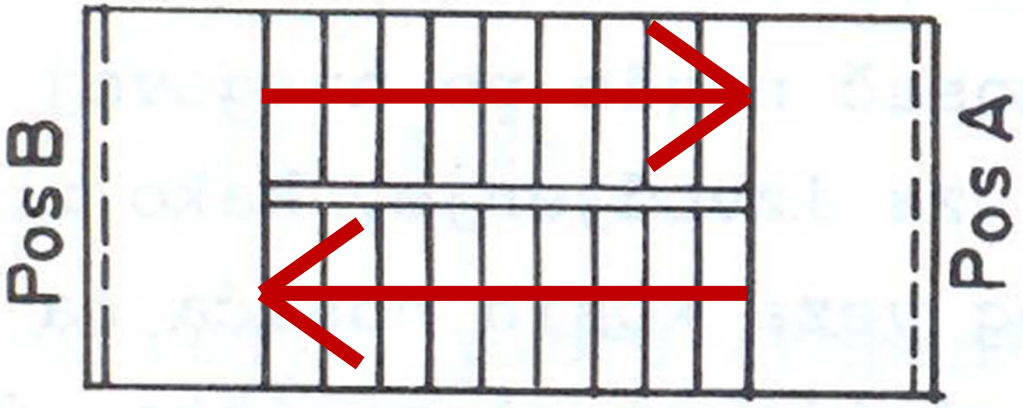


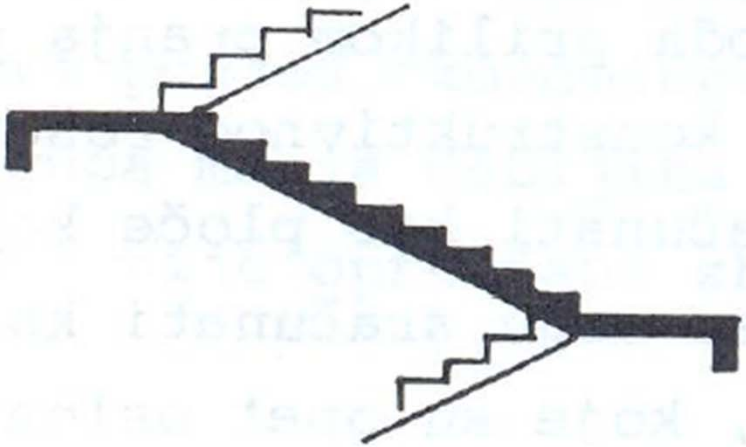


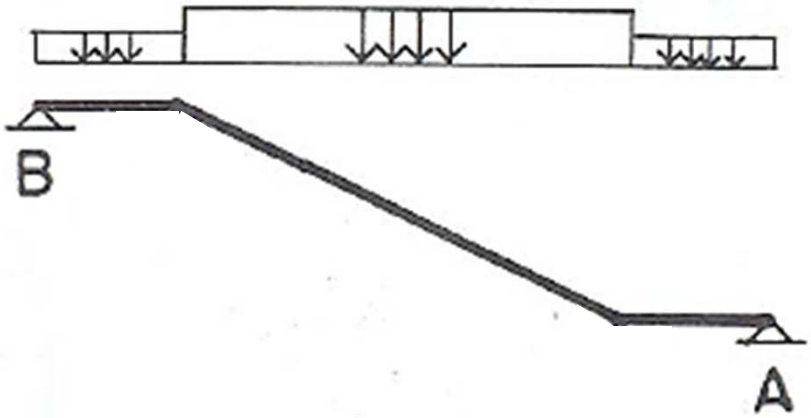
Traka širine 1,0 m

(Radosavljević i Bajić, 2006)

Primeri stepeništa

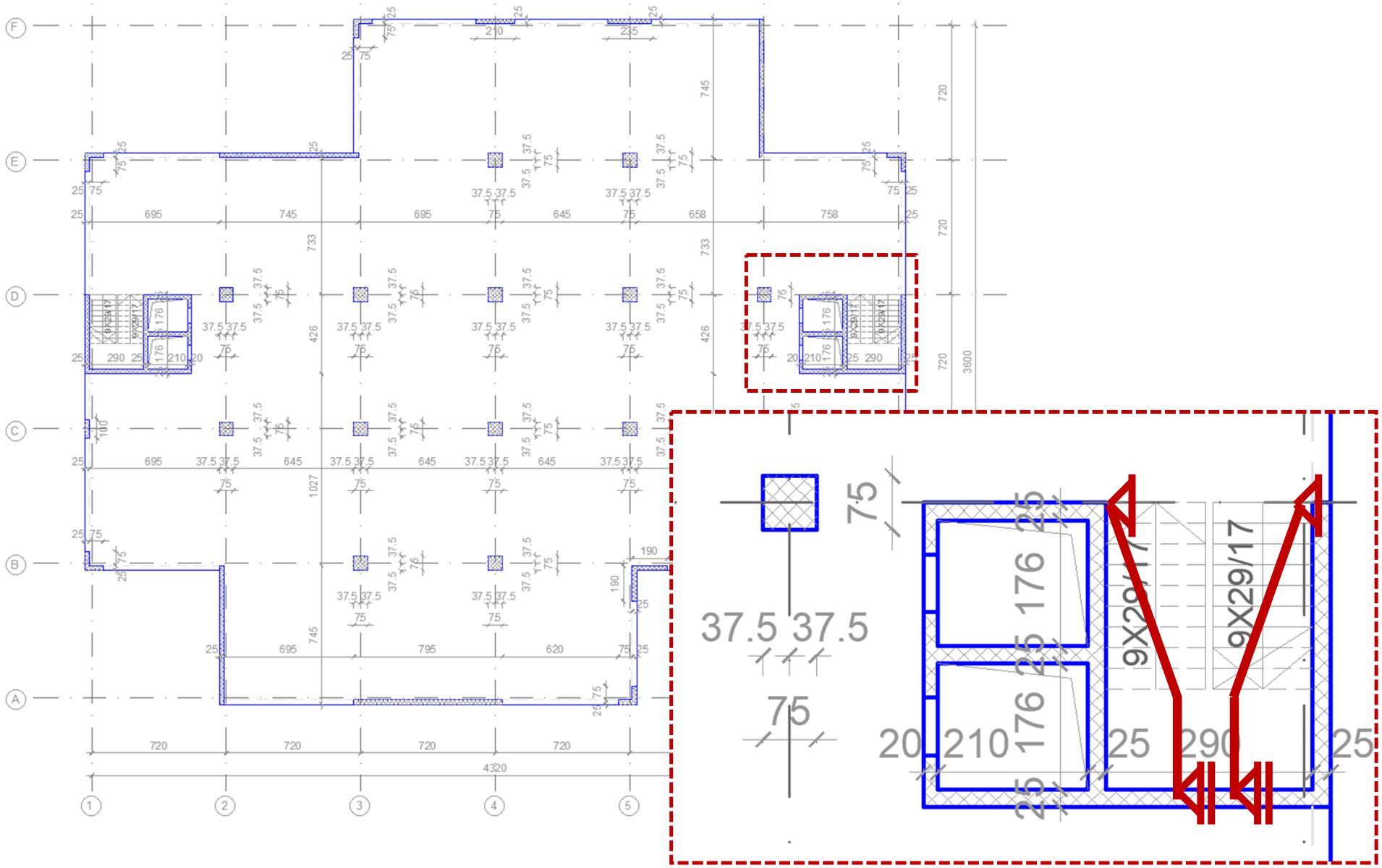






(Radosavljević i Bajić, 2006)

Primeri stepeništa



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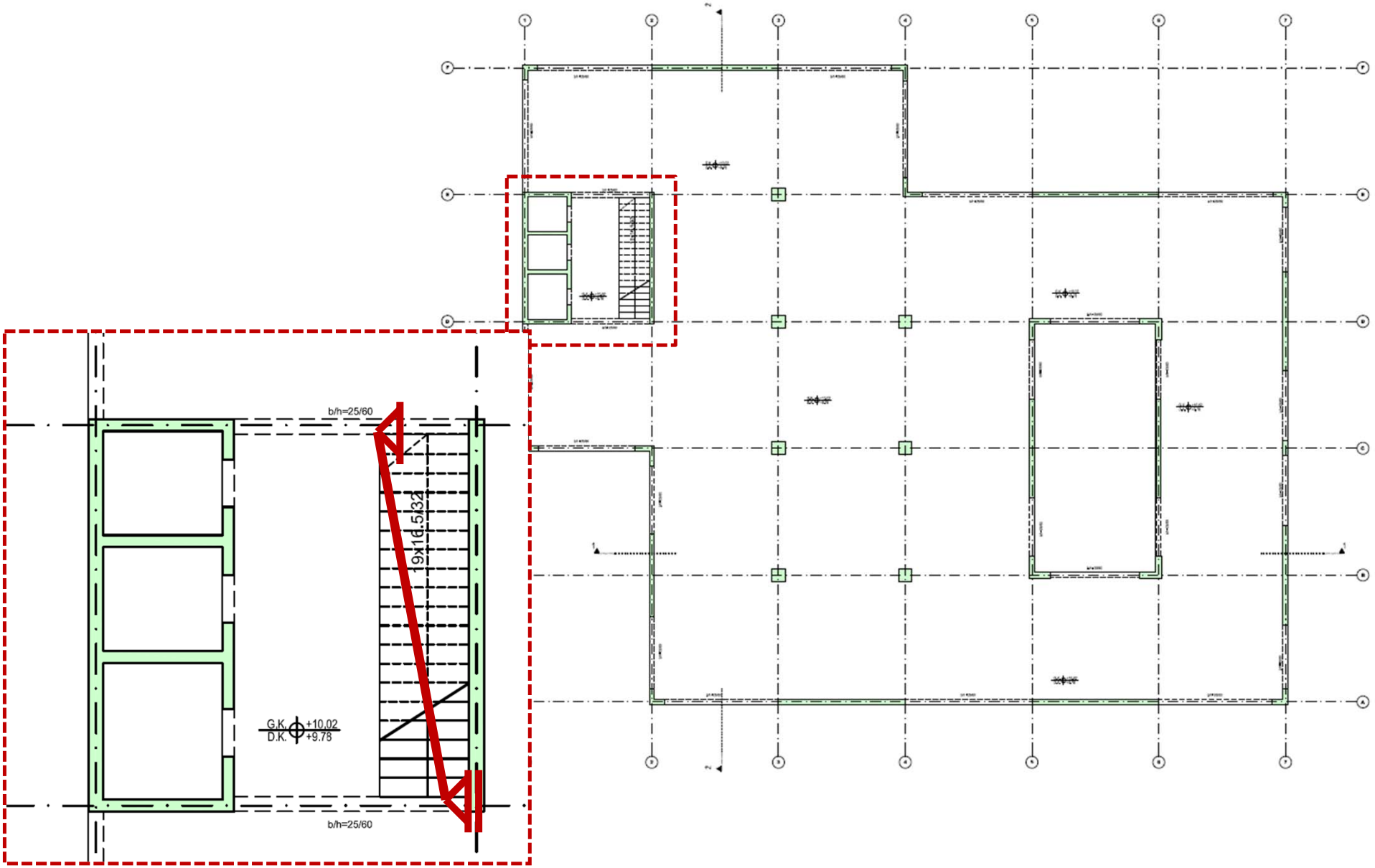
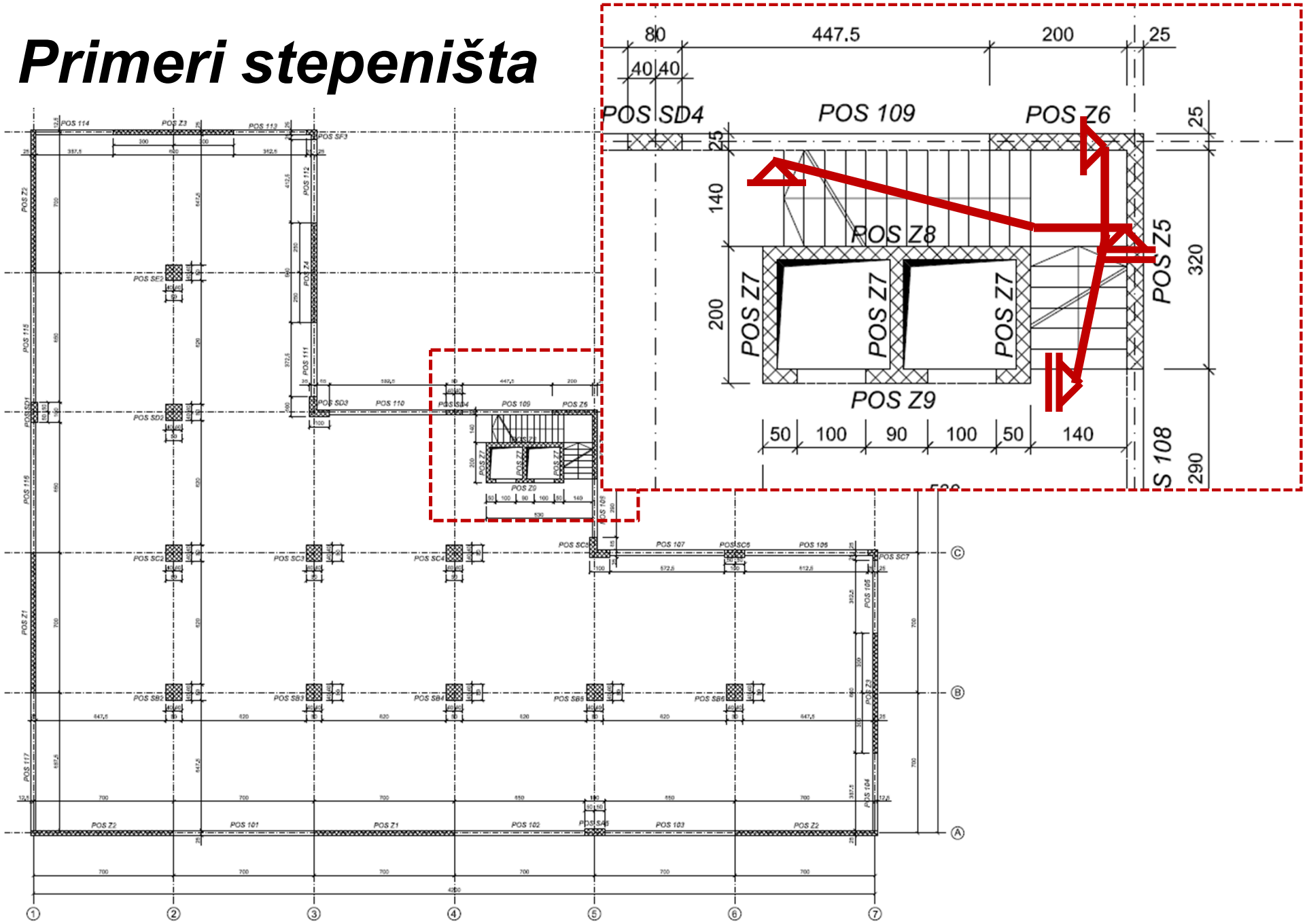
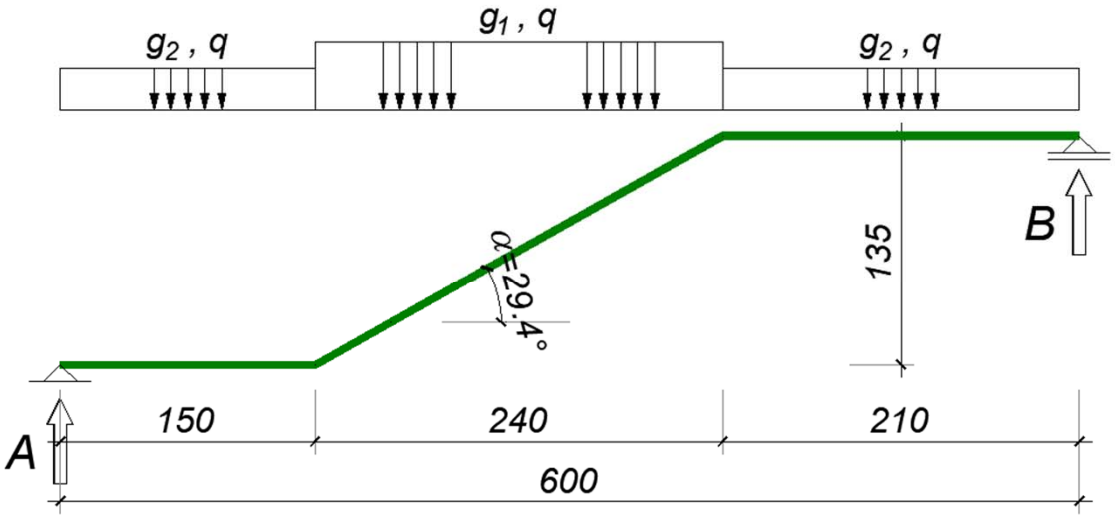


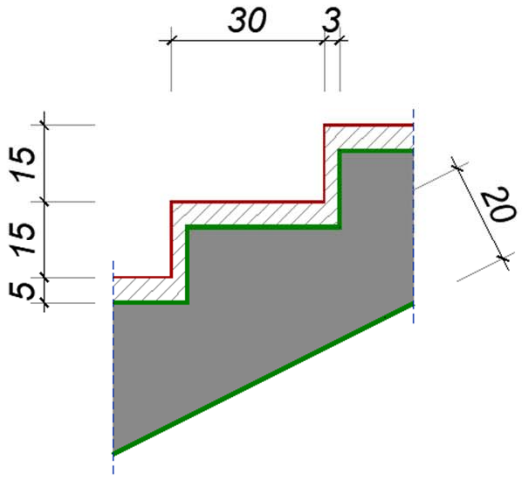
Primeri stepeništa



Primer 1 – Proračun konstrukcije stepeništa ST1

Stepenište je statičkog sistema kolenaste ploče, debljine h_p = 20 cm. Savladava visinsku razliku od 135 cm (9 visina po 15 cm). Stepenici su dimenzija b/h = 30/15 cm. Debljina vertikalne obloge stepenika je 3 cm, a horizontalne 5 cm. Debljina podne obloge na međupodestu je 6 cm, a na podestu 8 cm (usvojiti za težinu obe obloge 1.5 kN/m²). Širina stepenišnog kraka je 125 cm.





$$tg\alpha = 135/240 = 0.5625 \quad \Rightarrow \quad \alpha = 29.4^{o}$$

Primer 1 – Proračun konstrukcije stepeništa ST1

1. Analiza opterećenja

Kategorija	Karakteristično opterećenje q_k [kN/m^2]
A (Stambeni prostor):	
Tavanice	1.5 – **2.0**
Stepeništa	**2.0** – 4.0
Balkoni	**2.5** – 4.0
B (Kancelarijski prostor)	2.0 – **3.0**
C (Prostorije gde se okuplja veći broj ljudi):	
C1 (Površine sa stolovima)	2.0 – **3.0**
C2 (Površine sa fiksiranim sedištima)	3.0 – **4.0**
C3 (Površine pez prepreka za kretanje ljudi)	3.0 – **5.0**
C4 (Površine na kojima su moguće fizičke aktivnosti)	4.5 – **5.0**
C5 (Površine na kojima je moguča ljudska navala)	**5.0** – 7.5
D:	
D1 (Površine u maloprodajnim objektima)	**4.0** – 5.0
D2 (Površine u robnim kućama)	4.0 – **5.0**

Kada je neophodno, opterećenja q_k treba da se uvećaju u proračunu (na primer za stepeništa i balkone, u zavisnosti od korišćenja i dimenzija)

Primer 1 – Proračun konstrukcije stepeništa ST1

1. Analiza opterećenja

Opterećenje na kosom delu:

- **stalno opterećenje**

težina ploče	$0.20 \cdot 25.0/\cos 29.4°$	$= 5.74$ kN/m^2
težina stepenika	$0.5 \cdot 0.15 \cdot 24.0$	$= 1.80$ kN/m^2
horizontalna obloga	$0.05 \cdot 24$	$= 1.20$ kN/m^2
vertikalna obloga	$15/30 \cdot 0.03 \cdot 24$	$= 0.36$ kN/m^2
ukupno:		$g_1 = 9.10$ kN/m^2

- **povremeno opterećenje** $q = 3.00$ kN/m^2

Opterećenje na horizontalnim delovima:

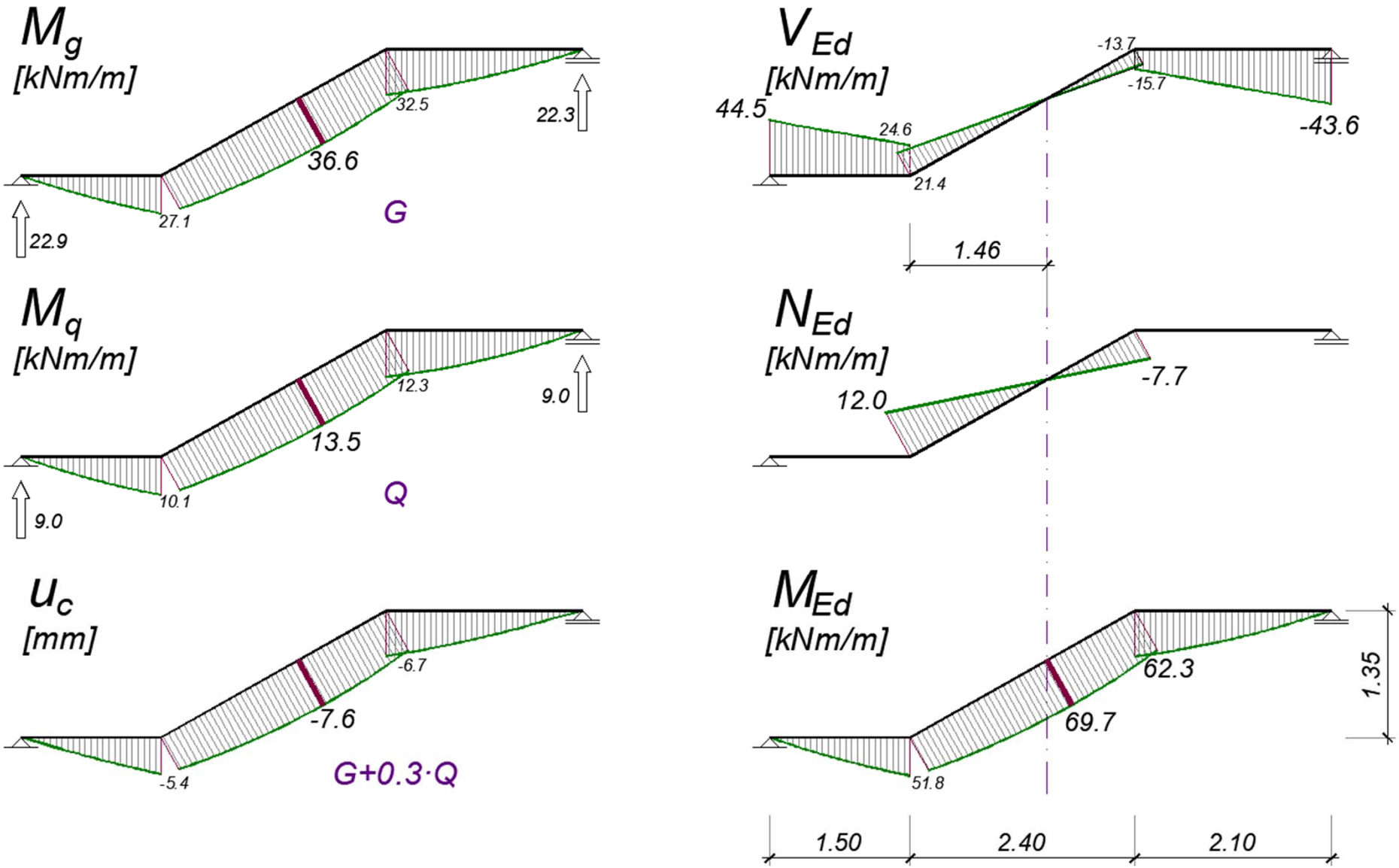
- **stalno opterećenje**

težina ploče	$0.20 \cdot 25.0$	$= 5.00$ kN/m^2
pod, plafon		$= 1.50$ kN/m^2
ukupno:		$g_2 = 6.50$ kN/m^2

- **povremeno opterećenje** $q = 3.00$ kN/m^2

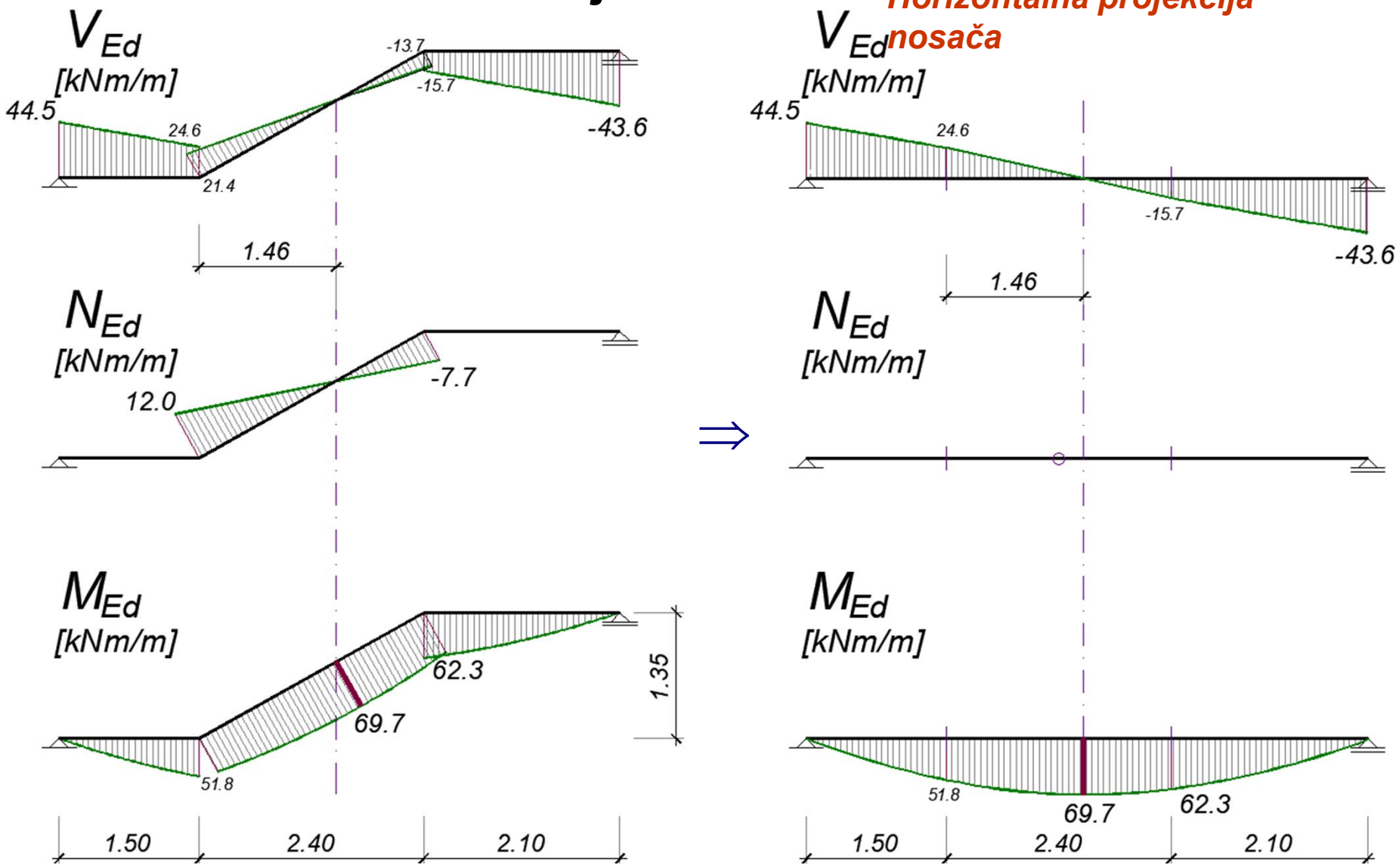
Primer 1 – Proračun konstrukcije stepeništa ST1

2. Proračun statičkih uticaja



Primer 1 – Proračun konstrukcije stepeništa ST1

2. Proračun statičkih uticaja



Primer 1 – Proračun konstrukcije stepeništa ST1

3. Dimenzionisanje

3.1) Dimenzionisanje preseka u sredini raspona prema M_{Ed}

$$M_{Ed} = 69.7\ kNm/m$$

pretp. d_1 = 4 cm $\Rightarrow$ d = 20 – 4 = 16 cm

$$k = \frac{16.0}{\sqrt{\frac{67.9}{1.70}}} = 2.499 \quad \Rightarrow \ \varepsilon_{c,2} / \varepsilon_{s1} = 3.5 / -12.6‰$$

$$\Rightarrow \ \omega_1 = 0.176$$

$$A_{s1} = 0{,}176 \cdot 100 \cdot 16.0 \cdot \frac{1{,}7}{43{,}5} = 11{,}01 cm^2/m$$

$$A_{s1,min} = \begin{cases} 0.013 \cdot 100 \cdot 16 = 2.08\ cm^2/m \\ 0.26 \frac{2.9}{500} \cdot 100 \cdot 16 = 2.41\ cm^2/m \end{cases}$$

Usvojeno:
Ø12/10 (11.3 cm²/m)

Primer 1 – Proračun konstrukcije stepeništa ST1

3. Dimenzionisanje

3.1) Dimenzionisanje preseka u sredini raspona prema M_{Ed}

Podeona armatura:

$A_{s,p} = 0.2 \cdot 11.01\ cm^2/m = 2.20\ cm^2/m$

Usvojeno:
Ø8/20 (2.51 cm^2/m)

3.2) Dimenzionisanje merodavnog preseka prema V_{Ed}

$$d = 16.0\ cm,\ k = 1 + \sqrt{200/160} = 2.12 \le 2.0$$
$$\Rightarrow v_{min} = 0.542\ MPa$$
$$\Rightarrow V_{Rd,c,min} = v_{min} \cdot b \cdot d = 84.3\ kN/m$$

$$V_{Ed} = 44.5\ kN/m$$

$$\Rightarrow V_{Rd,c} > V_{Ed}$$

Primer 1 – Proračun konstrukcije stepeništa ST1

3. Dimenzionisanje

3.3) Dimenzionisanje preseka prema M_{Ed} na kosom delu

$$M_{Ed} = 62.3\ kNm/m \quad Z_{Ed} = 7.7\ kN/m \Rightarrow M_{Ed,s} = 62.2\ kNm/m$$

pretp. d_1 = 4 cm $\Rightarrow$ d = 20 – 4 = 16 cm

$$k = \frac{16.0}{\sqrt{\frac{62.2}{1.70}}} = 2.645 \Rightarrow \varepsilon_{c,2} / \varepsilon_{s1} = 3.5 / -14.7‰$$

$$\Rightarrow \omega_1 = 0.155 + 0.003 = 0.158$$

≈ 0

$$A_{s1} = 9.90\,cm^2/m$$

Usvojeno:
glavna arm. Ø12/10 (11.3 cm²/m)
podeona arm. Ø8/20 (2.51 cm²/m)

Primer 1 – Proračun konstrukcije stepeništa ST1

3. Dimenzionisanje

3.4) Kontrola prslina bez direktnog proračuna

$$M_{Ed,qp} = 40.7\ kNm/m$$

Usvojena površina zategnute armature: $A_{s1,prov} = 11.3\,cm^2/m, d = 16.0cm$

Minimalna površina armature iz uslova prslina:

$$A_{s1,min} = \frac{k_c \cdot k \cdot f_{ct,eff} \cdot 0.5 \cdot 100 \cdot 16}{500} = \frac{0.4 \cdot 1.0 \cdot 2.9 \cdot 0.5 \cdot 100 \cdot 16}{500} = 1.86\,cm^2/m$$

Napon zatezanja u armaturi (približan postupak):

$$\sigma_{s1,qp} = \frac{M_{Ed,qp}}{z \cdot A_{s,prov}} = \frac{40.7 \cdot 100}{0.9 \cdot 16 \cdot 11.3} = 250.1\,MPa$$

Primer 1 – Proračun konstrukcije stepeništa ST1

3. Dimenzionisanje

3.4) Kontrola prslina bez direktnog proračuna

$$\sigma_{s1,qp} = \frac{M_{Ed,qp}}{z \cdot A_{s,prov}} = \frac{40.7 \cdot 100}{0.9 \cdot 16 \cdot 11.3} = 250.1\,MPa$$

w_k = 0.3 mm $\Rightarrow$

max $Ø_s$*= 15.0 mm

max $e_Ø$ = 187 mm

Kontrola maksimalnog prečnika armature

$Ø_s$*

$$Ø_{s,max} = Ø_s^* \cdot \frac{f_{ct,eff}}{2.9} \cdot \frac{k_c h_{cr}}{2(h-d)} = 15.0 \cdot \frac{2.9}{2.9} \cdot \frac{0.4 \cdot 0.5 \cdot 200}{2(200-160)} = 7.5\ mm$$

$$Ø_{s,max} = 7.5\ mm < 12\ mm\ !$$

Nije zadovoljeno

Kontrola maksimalnog rastojanja šipki $e_Ø$:

max $e_Ø$ = 187 mm > $e_Ø$ = 100 mm

Zadovoljeno

Primer 1 – Proračun konstrukcije stepeništa ST1

3. Dimenzionisanje

3.5) Kontrola ugiba bez direktnog proračuna

Potrebna površina zategnute armature: $A_{s1,req} = 11.01\,cm^2/m$

Usvojena površina zategnute armature: $A_{s1,prov} = 11.3\,cm^2/m, d = 16.0\,cm$

Potrebna površina pritisnute armature: $A_{s2,req} = 0.00\,cm^2$

$$\rho_0 = \sqrt{f_{ck}}\,10^{-3} = 0.0055$$

$$\rho_1 = A_{s1,req}/b{\cdot}d = 0.0069 > \rho_0 \Rightarrow Jedn.\ 7.16b\,(EC2)$$

$$\rho' = 0.000$$

$$K = 1.0\,(prosta\ greda)$$

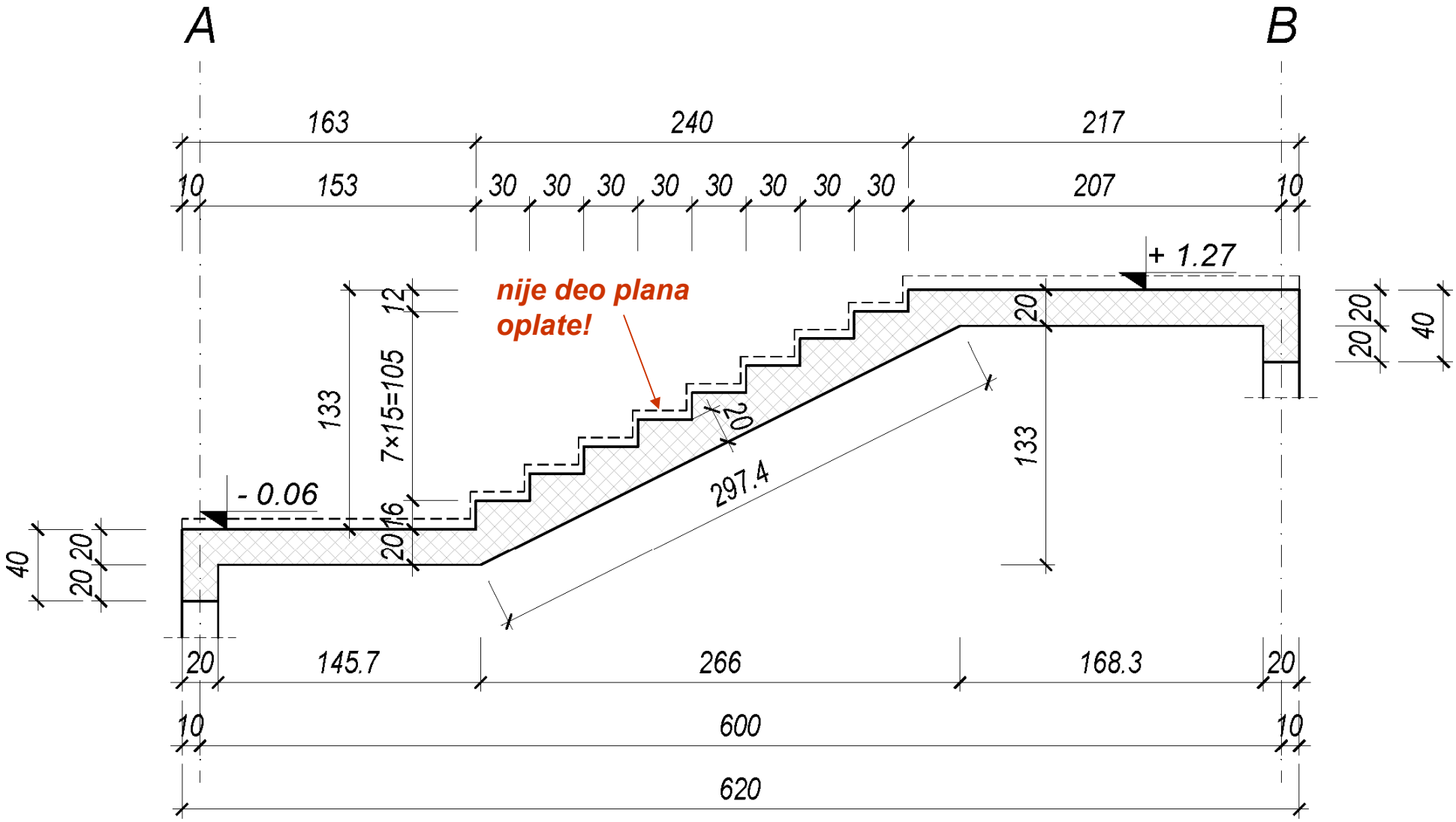
$$\left(\frac{L}{d}\right) = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}\right] = 17.53$$

Nije zadovoljeno!

$$\left(\frac{L}{d}\right)_{lim} = \frac{500{\cdot}11.30}{500{\cdot}11.01}\cdot\left(\frac{L}{d}\right) = 17.99 > \left(\frac{L}{d}\right)_{stv} = 600/16.0 = 37.5\ \ !$$

Primer 1 – Proračun konstrukcije stepeništa ST1

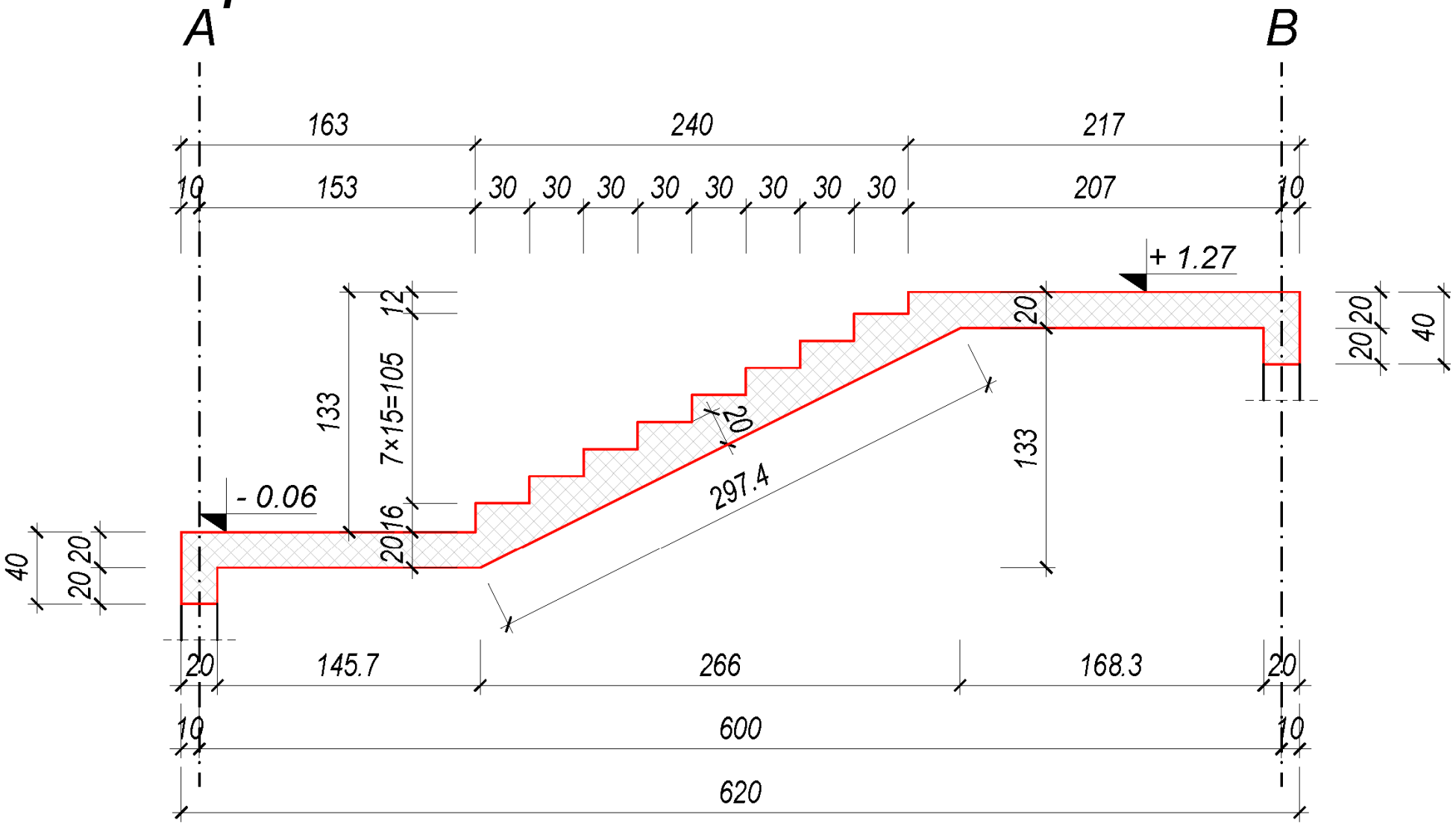
4. Plan oplate



C30/37 R 1:50

Primer 1 – Proračun konstrukcije stepeništa ST1

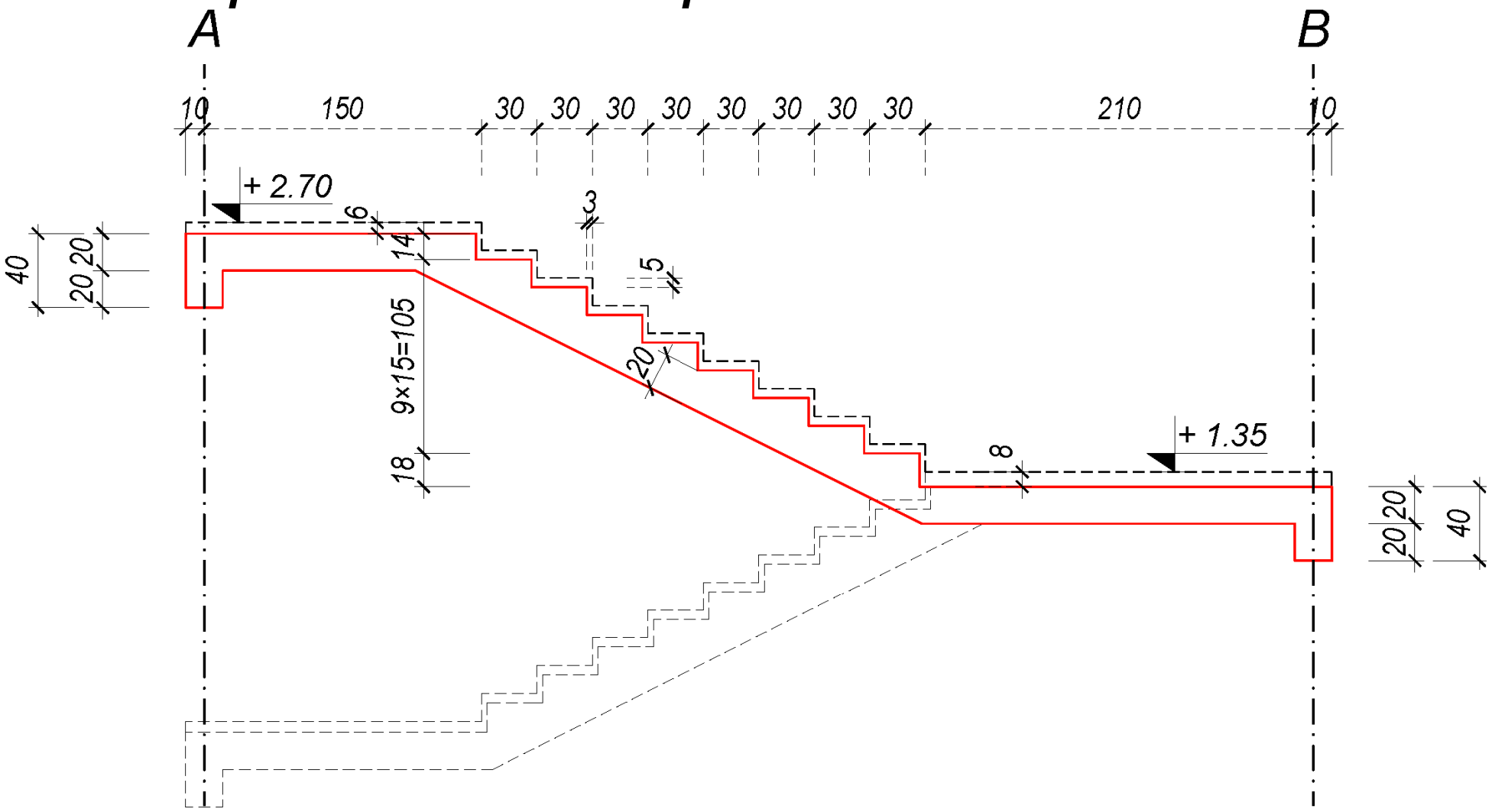
4. Plan oplate



C30/37 R 1:50

Primer 1 – Proračun konstrukcije stepeništa ST1

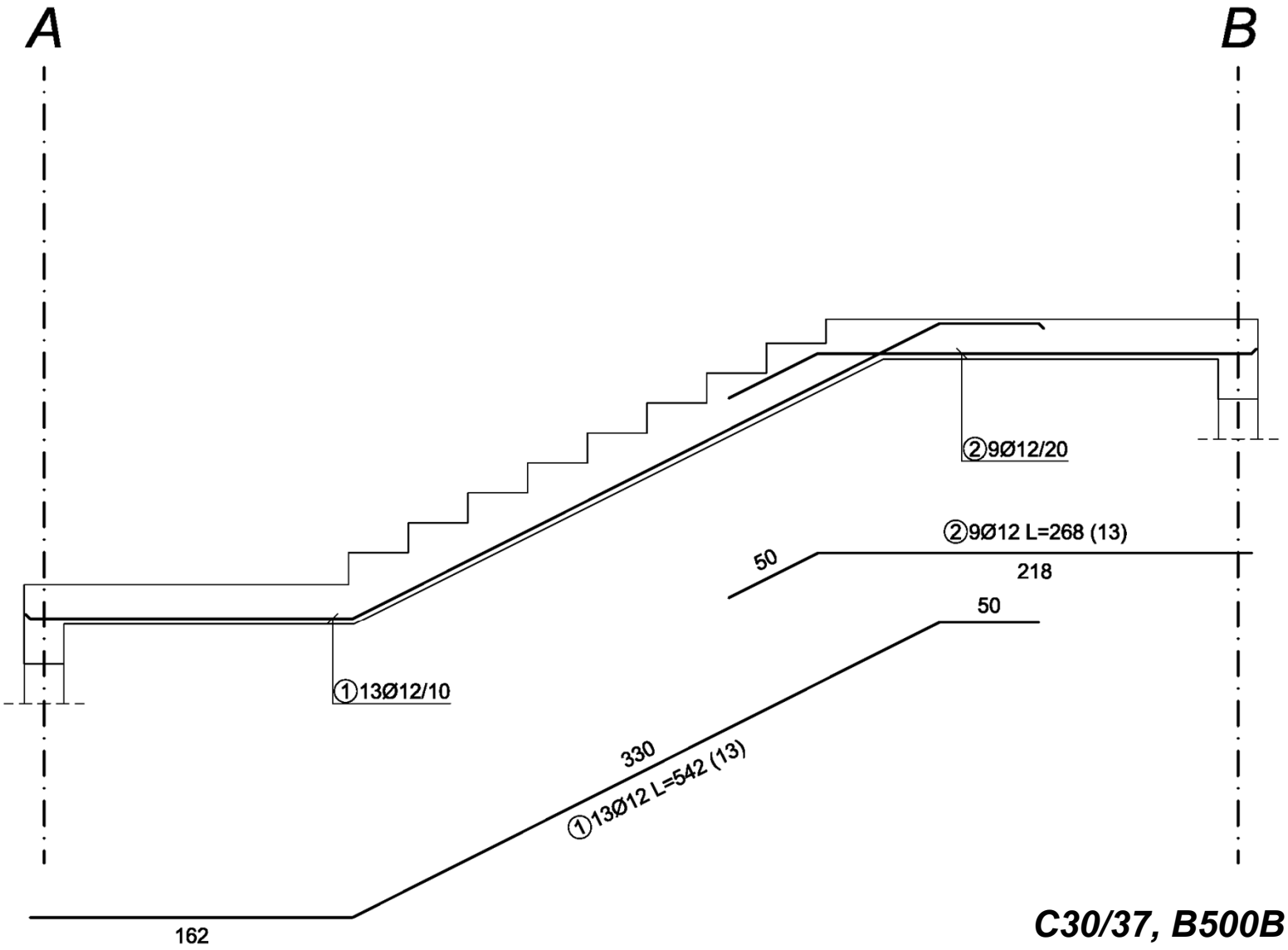
4. Plan oplate – Dvokrako stepenište



C30/37 R 1:50

Primer 1 – Proračun konstrukcije stepeništa ST1

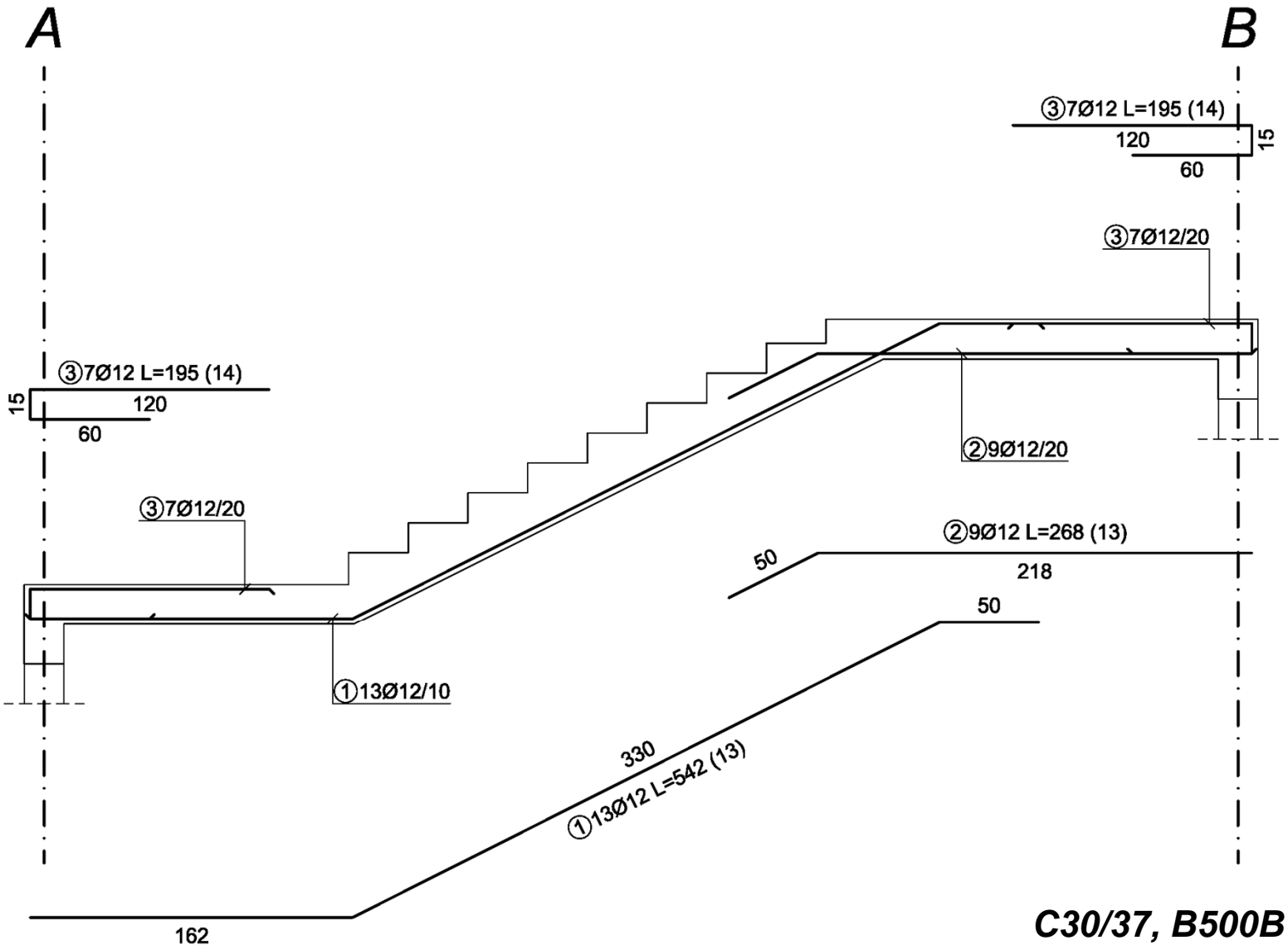
5. Plan armature



C30/37, B500B R 1:50

Primer 1 – Proračun konstrukcije stepeništa ST1

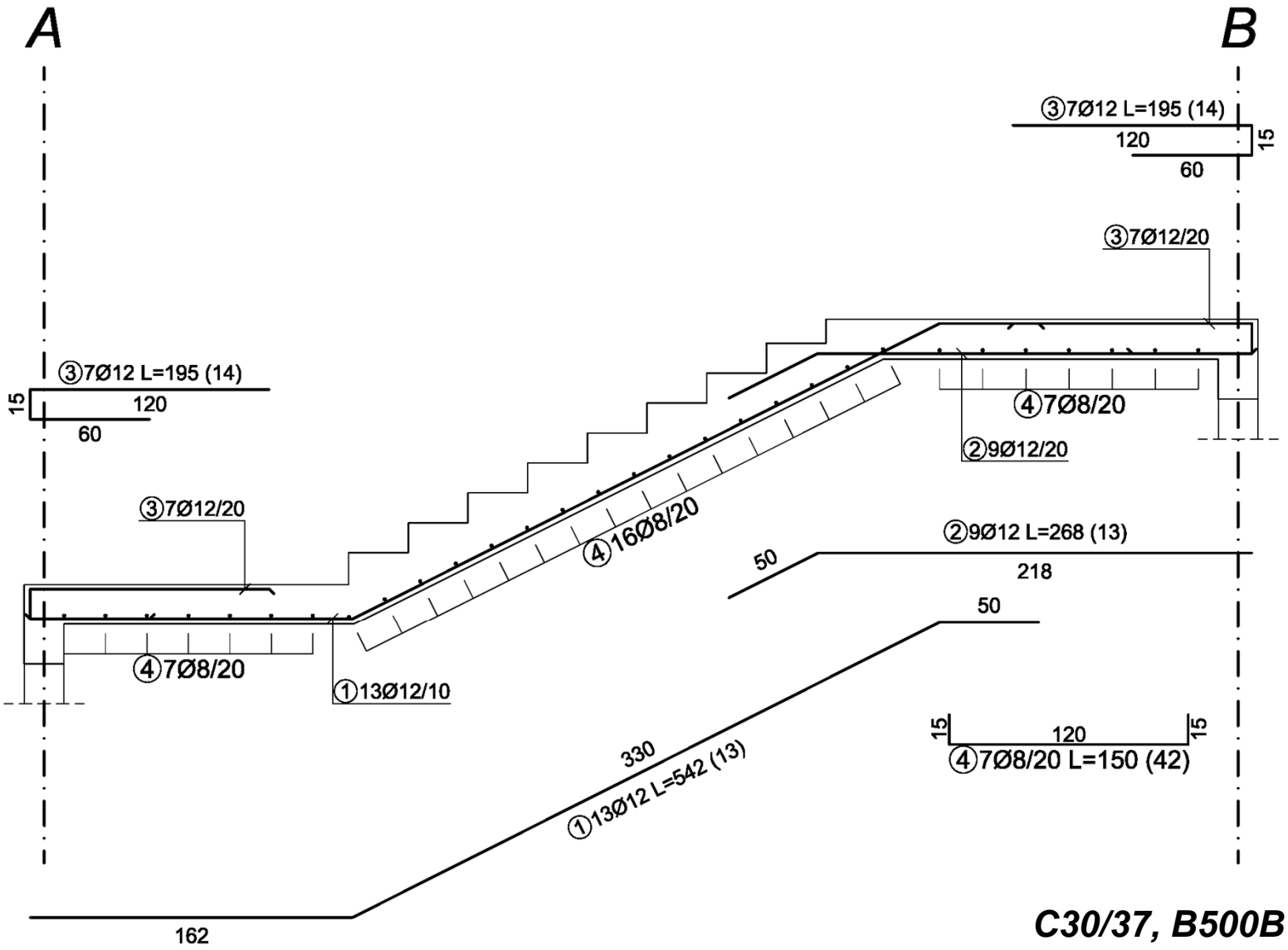
5. Plan armature



C30/37, B500B R 1:50

Primer 1 – Proračun konstrukcije stepeništa ST1

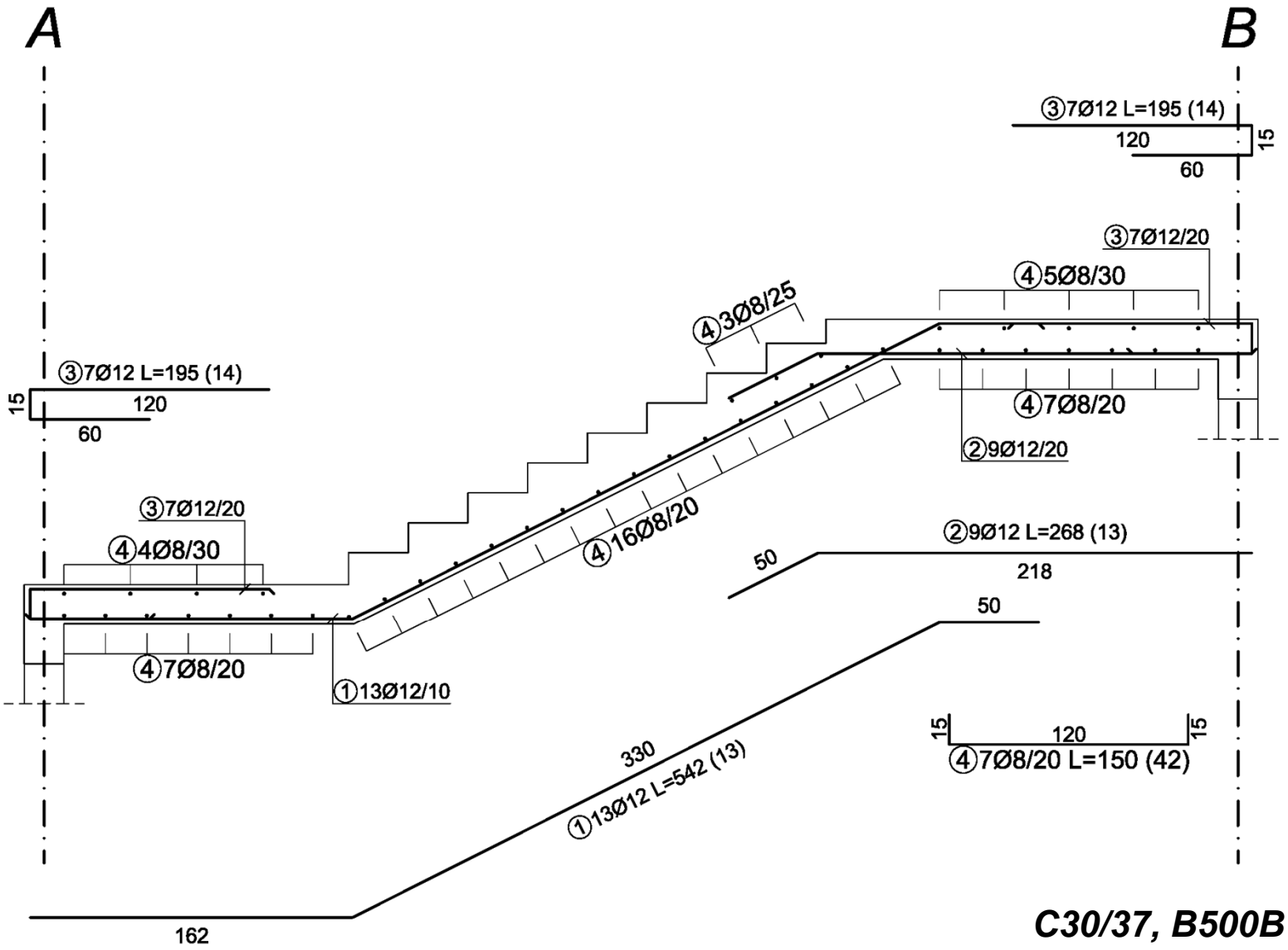
5. Plan armature



C30/37, B500B R 1:50

Primer 1 – Proračun konstrukcije stepeništa ST1

5. Plan armature

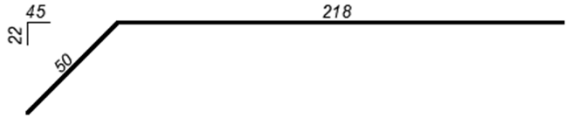
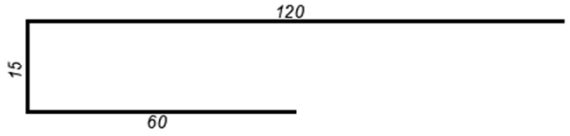
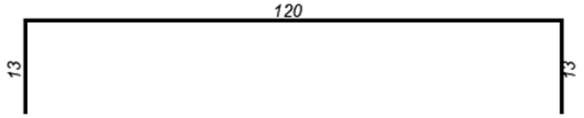


C30/37, B500B R 1:50

Primer 1 – Proračun konstrukcije stepeništa ST1

5. Plan armature

Šipke - specifikacija

ozn.	oblik i mere [cm]	ozn.	Ø	lg [m]	n [kom]	lgn [m]
POS ST1 (1 kom)						
1		B500B	12	5.42	13	70.46
2		B500B	12	2.68	13	34.84
3		B500B	12	1.95	14	27.30
4		B500B	8	1.46	42	61.32

Šipke - rekapitulacija

Ø [mm]	lgn [m]	Jedinična težina [kg/m']	Težina [kg]
B500B			
8	61.32	0.395	24.22
12	132.60	0.888	117.75
Ukupno (B500B)			141.97

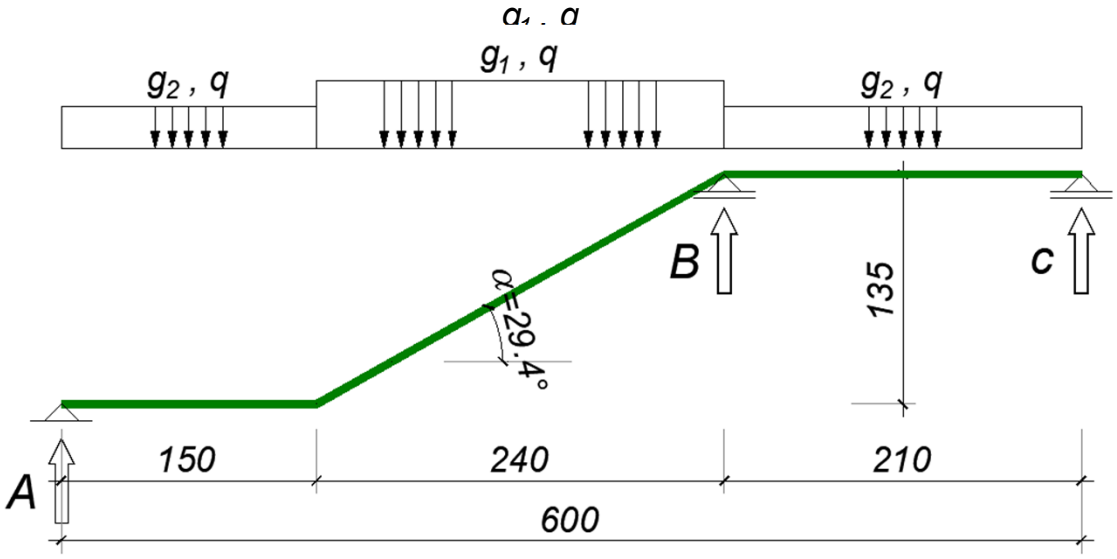
$V_b = 1.97\ m^3$

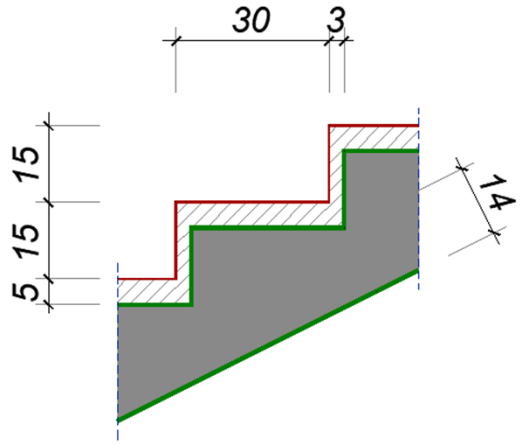
$m_a/V_b = 141.97/1.97$

$m_a/V_b = 72.07\ kg/m^3$

Primer 2 – Proračun konstrukcije stepeništa ST2

Stepenište je statičkog sistema kontinualne kolenaste ploče, debljine h_p = 14 cm. Savladava visinsku razliku od 135 cm (9 visina po 15 cm). Stepenici su dimenzija b/h = 30/15 cm. Debljina vertikalne obloge stepenika je 3 cm, a horizontalne 5 cm. Debljina podne obloge na međupodestu je 6 cm, a na podestu 8 cm (usvojiti za težinu obe obloge 1.5 kN/m^2). Širina stepenišnog kraka je 125 cm.





$$tg\alpha = 135/240 = 0.5625 \Rightarrow \alpha = 29.4^{o}$$

Primer 2 – Proračun konstrukcije stepeništa ST2

1. Analiza opterećenja

Opterećenje na kosom delu:

- **stalno opterećenje**

težina ploče	$0.14 \cdot 25.0/\cos 29.4°$	$= 4.02$ kN/m^2
težina stepenika	$0.5 \cdot 0.15 \cdot 24.0$	$= 1.80$ kN/m^2
horizontalna obloga	$0.05 \cdot 24$	$= 1.20$ kN/m^2
vertikalna obloga	$15/30 \cdot 0.03 \cdot 24$	$= 0.36$ kN/m^2
ukupno:		$g_1 = 7.38$ kN/m^2

- **povremeno opterećenje** $q = 3.00$ kN/m^2

Opterećenje na horizontalnim delovima:

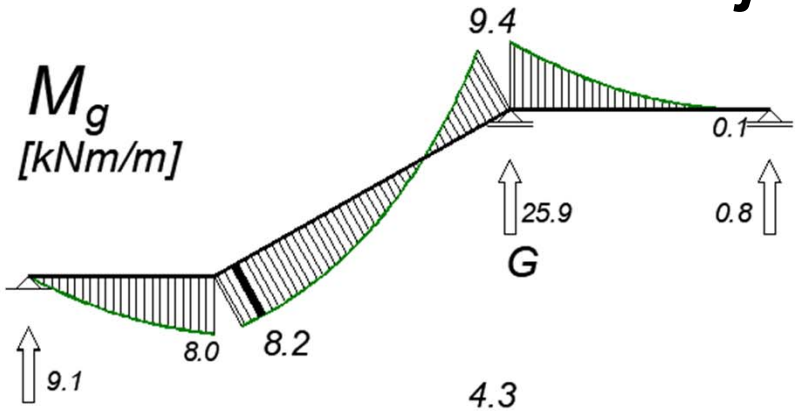
- **stalno opterećenje**

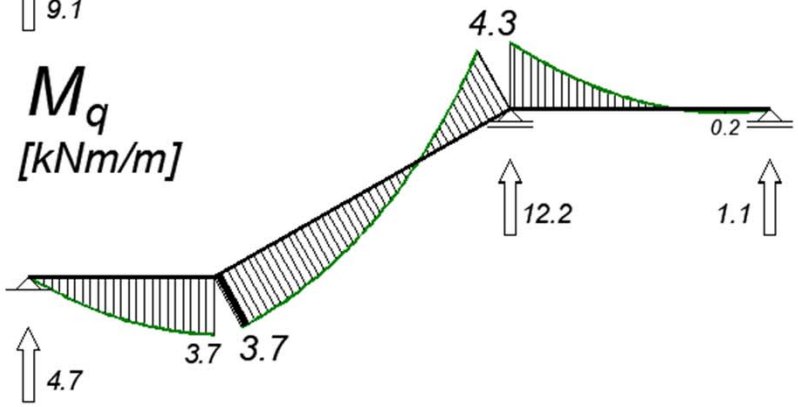
težina ploče	$0.14 \cdot 25.0$	$= 3.50$ kN/m^2
pod, plafon		$= 1.50$ kN/m^2
ukupno:		$g_2 = 5.00$ kN/m^2

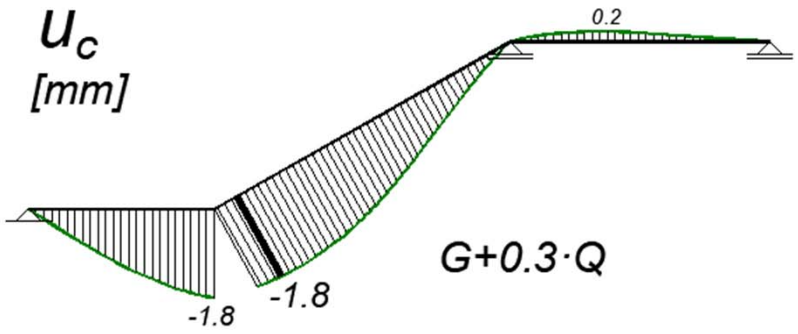
- **povremeno opterećenje** $q = 3.00$ kN/m^2

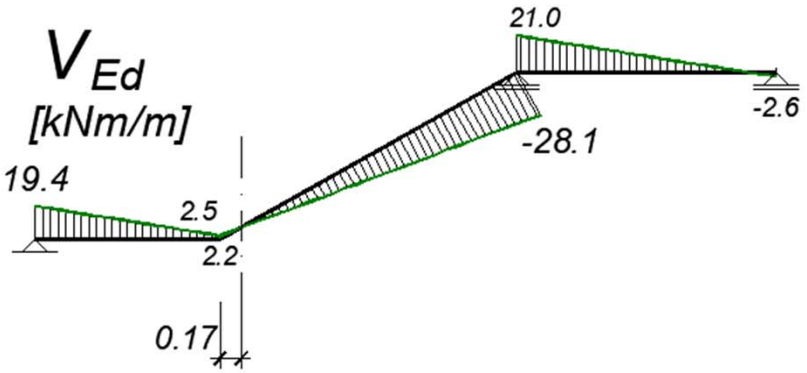
Primer 2 – Proračun konstrukcije stepeništa ST2

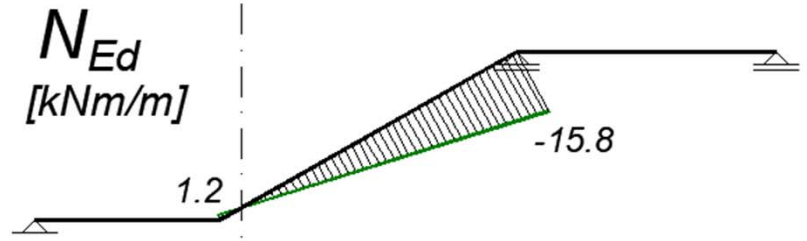
2. Proračun statičkih uticaja

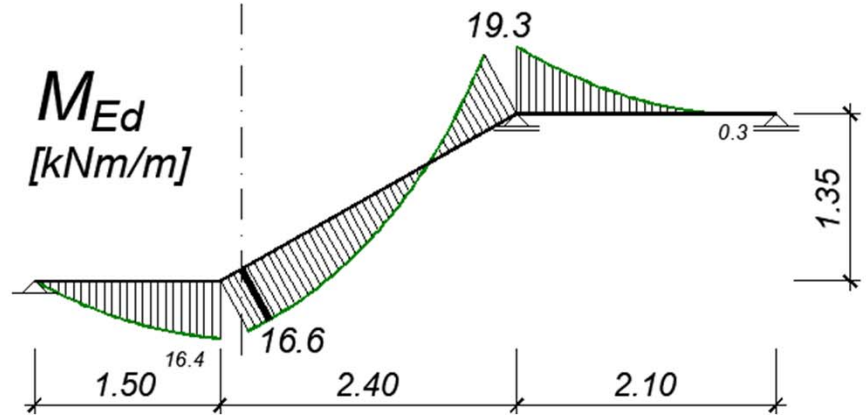












Primer 2 – Proračun konstrukcije stepeništa ST2

3. Dimenzionisanje

3.1) Dimenzionisanje preseka prema M_{Ed} na kosom delu

$$M_{Ed} = 19.3\ kNm/m \quad Z_{Ed} = 15.8\ kN/m \Rightarrow M_{Ed,s} = 19.1\ kNm/m$$

pretp. d_1 = 4 cm $\Rightarrow$ d = 20 – 4 = 16 cm

$$k = \frac{10.0}{\sqrt{\frac{19.1}{1.70}}} = 2.983 \Rightarrow \varepsilon_{c,2} / \varepsilon_{s1} = 3.5 / -20.2‰$$

$$\Rightarrow \omega_1 = 0.120 + 0.009 = 0.129$$

≈ 0

$$A_{s1} = 5.05\,cm^2/m$$

$$A_{s,p} = 1.01\,cm^2/m$$

$$A_{s,min} = 1.51\,cm^2/m$$

Usvojeno:
glavna arm. Ø12/20 (5.65 cm²/m)
podeona arm. Ø8/30 (1.68 cm²/m)

Primer 2 – Proračun konstrukcije stepeništa ST2

3. Dimenzionisanje

3.2) Dimenzionisanje merodavnog preseka prema V_{Ed}

$$d = 10.0\,cm,\ k = 1 + \sqrt{200/100} = 2.41 \le 2.0$$

$$\Rightarrow v_{min} = 0.542\,MPa$$

$$\Rightarrow V_{Rd,min} = v_{min} \cdot b \cdot d = 54.2\,kN/m$$

$$V_{Ed} = 28.1\,kN/m$$

$$(v_{Ed} = 0.281\,MPa)$$

$$\Rightarrow V_{Rd,c} > V_{Ed}$$

3.3) Kontrola prslina bez direktnog proračuna (kod srednjeg oslonca)

Kontrola maksimalnog prečnika armature $Ø_s$*

max $Ø_s$*= 22.4 mm

max $Ø_s$ = 7.9 mm < Ø = 12 mm — **Nije zadovoljeno**

Kontrola maksimalnog rastojanja šipki $e_Ø$:

max $e_Ø$ = 236 mm > $e_Ø$ = 200 mm — **Zadovoljeno**

Primer 2 – Proračun konstrukcije stepeništa ST2

3. Dimenzionisanje

3.4) Kontrola ugiba bez direktnog proračuna (u polju)

Potrebna površina zategnute armature: $A_{s1,req} = 4.01\,cm^2/m$

Usvojena površina zategnute armature: $A_{s1,prov} = 5.65\,cm^2/m, d = 10.0\,cm$

Potrebna površina pritisnute armature: $A_{s2,req} = 0.00\,cm^2$

$$\rho_0 = \sqrt{f_{ck}}\,10^{-3} = 0.0055$$

$$\rho_1 = A_{s1,req} / b{\cdot}d = 0.0040 < \rho_0 \Rightarrow Jedn.\ 7.16a\,(EC2)$$

$$\rho' = 0.000$$

$$K = 1.3\,(kontinualni\ nosač)$$

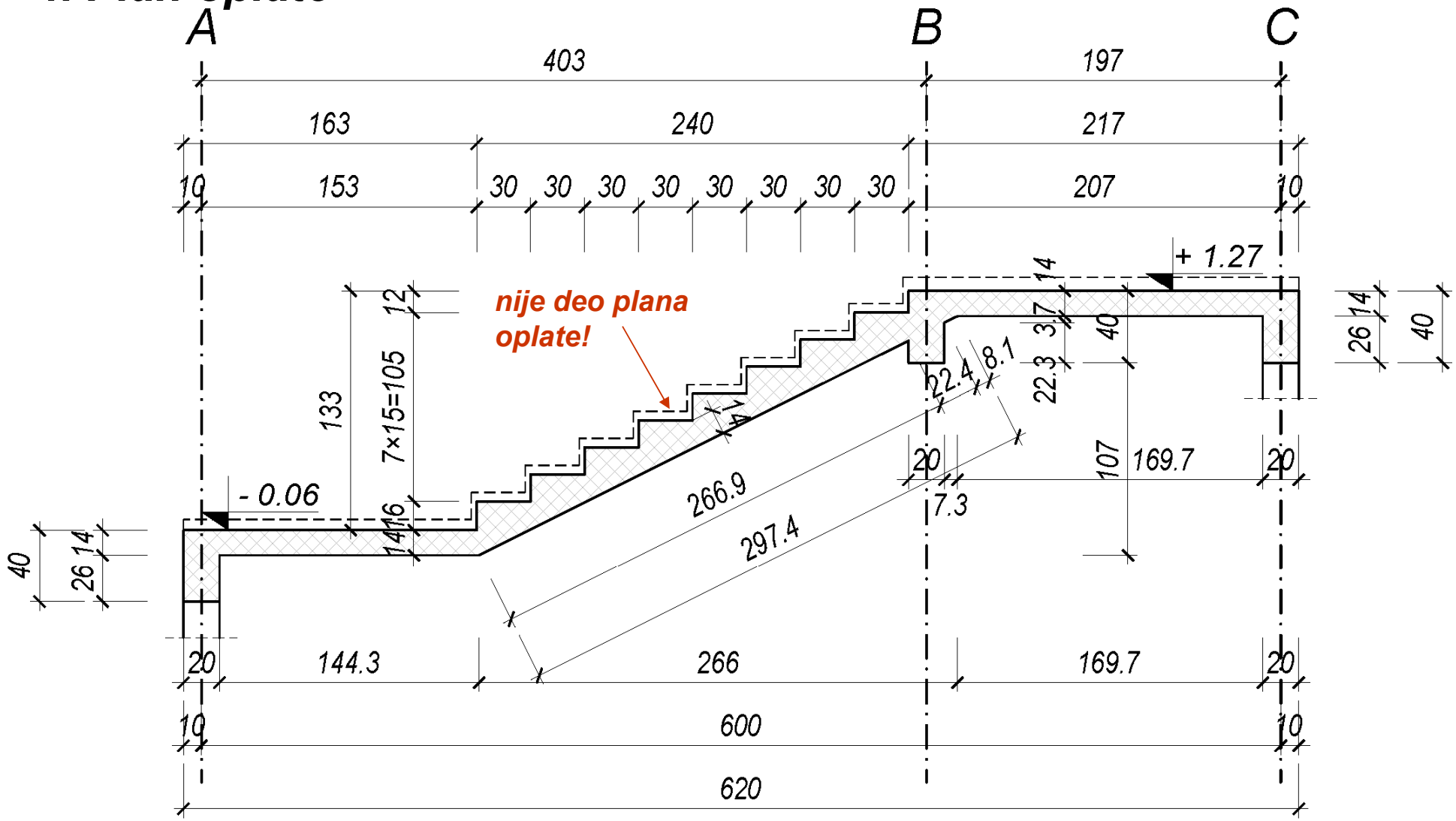
$$\left(\frac{L}{d}\right) = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right] = 33.93$$

Zadovoljeno!

$$\left(\frac{L}{d}\right)_{lim} = \frac{500{\cdot}5.65}{500{\cdot}4.01}\cdot\left(\frac{L}{d}\right) = 47.8 \quad > \quad \left(\frac{L}{d}\right)_{stv} = 390/10.0 = 39.0$$

Primer 2 – Proračun konstrukcije stepeništa ST2

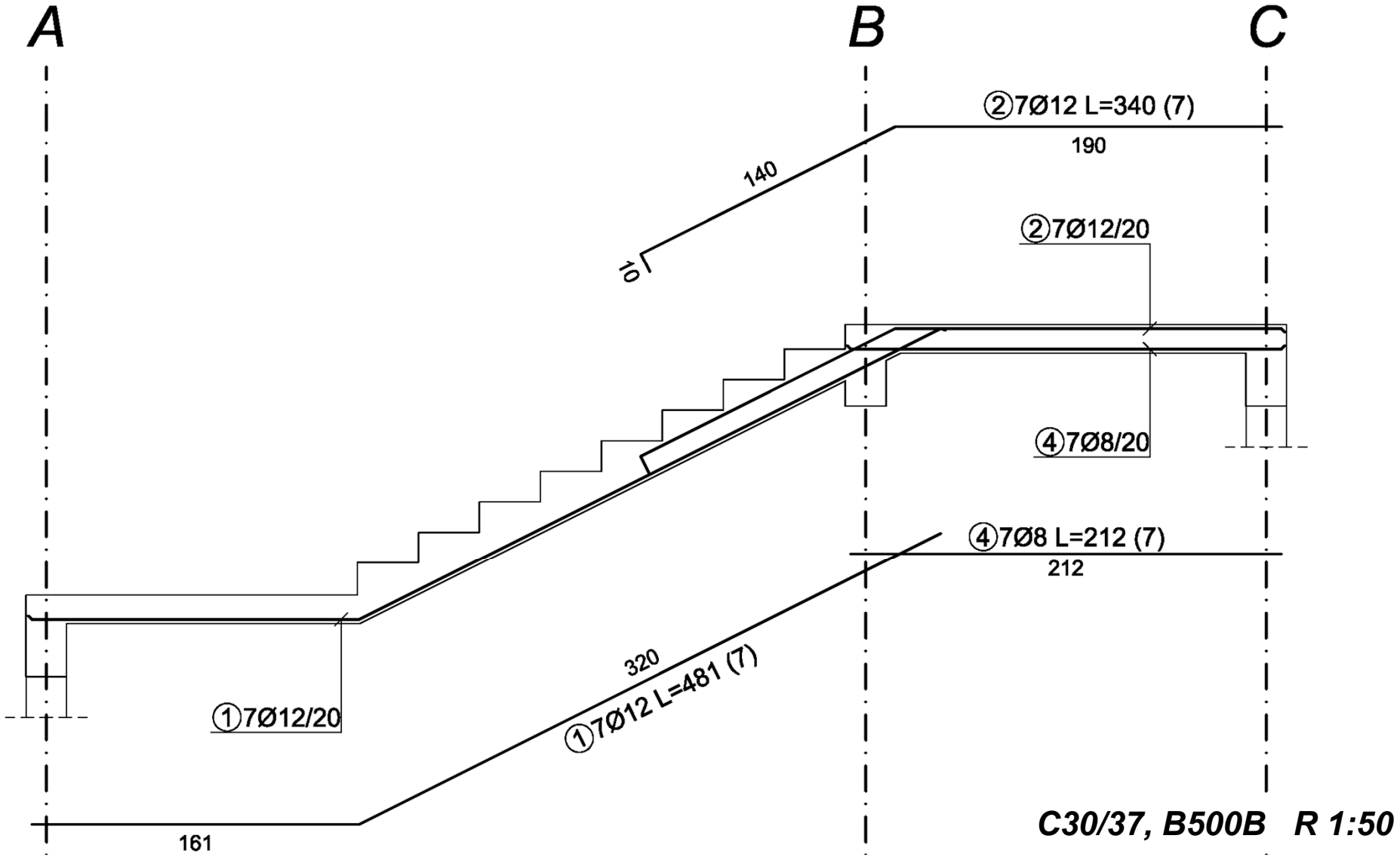
4. Plan oplate



C30/37 R 1:50

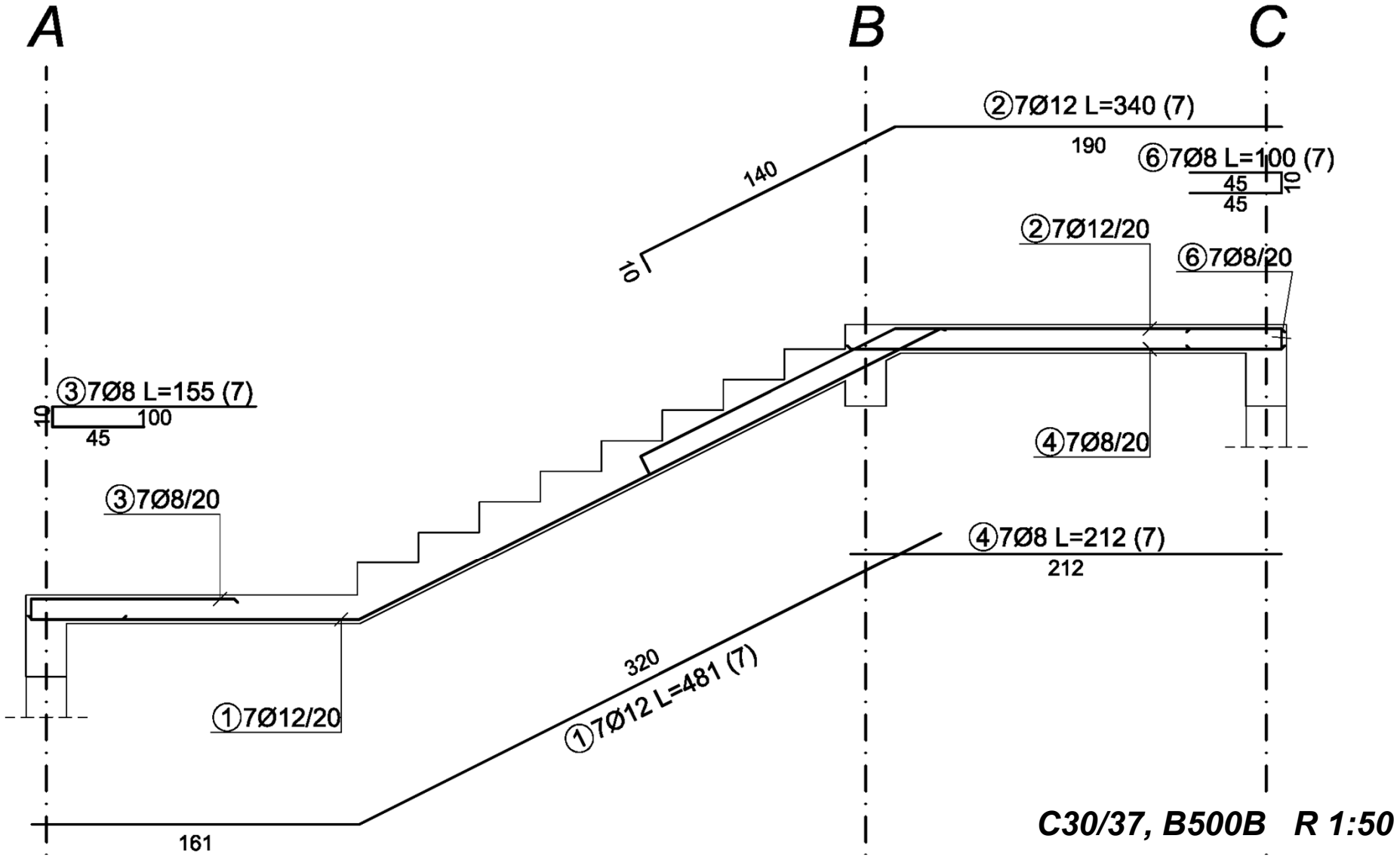
Primer 2 – Proračun konstrukcije stepeništa ST2

5. Plan armature



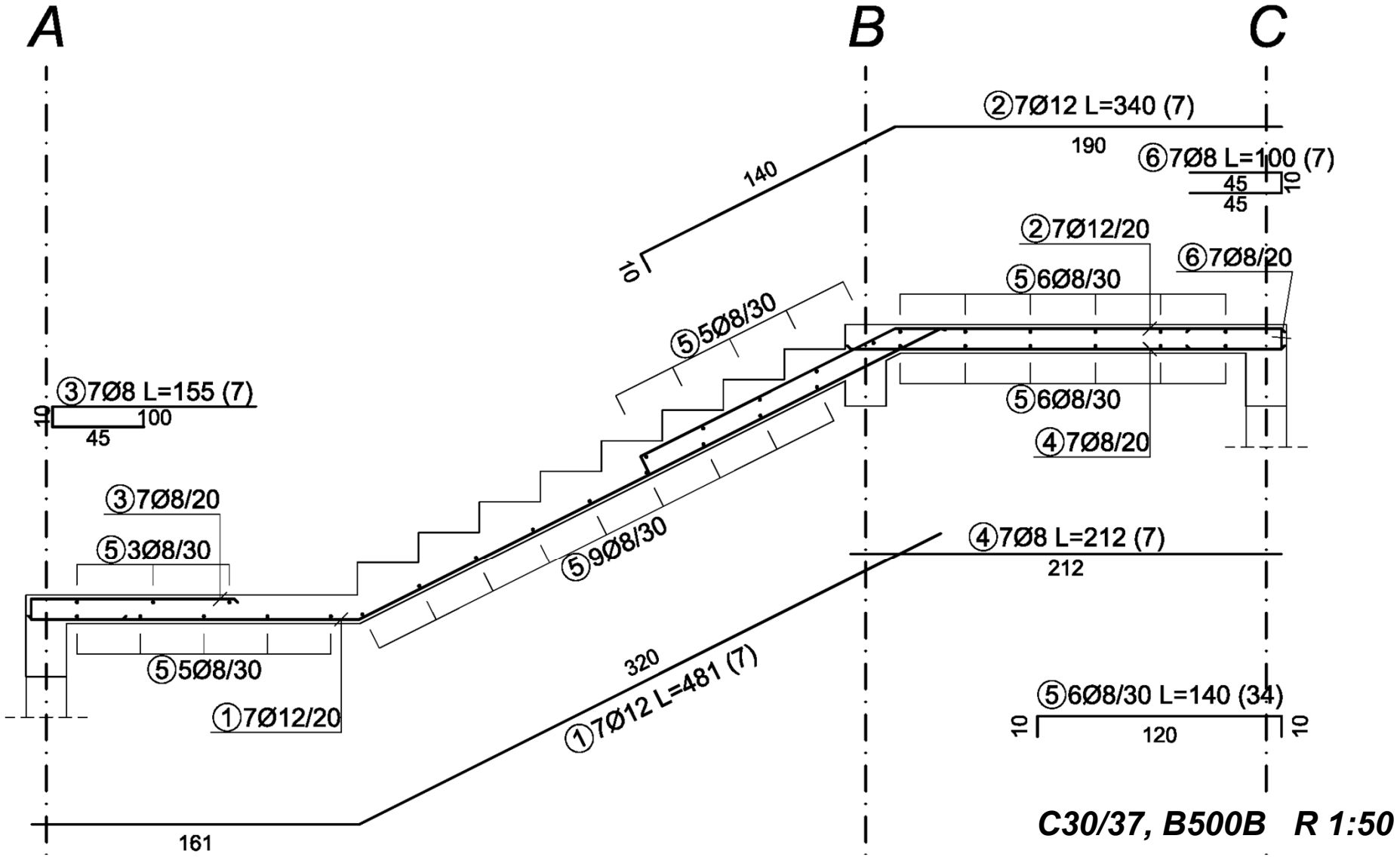
Primer 2 – Proračun konstrukcije stepeništa ST2

5. Plan armature



Primer 2 – Proračun konstrukcije stepeništa ST2

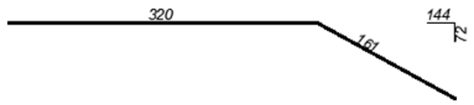
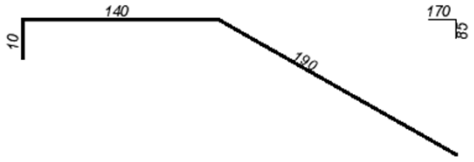
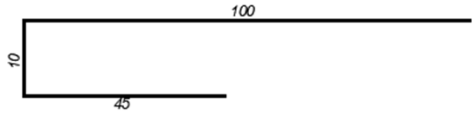
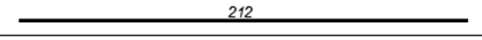
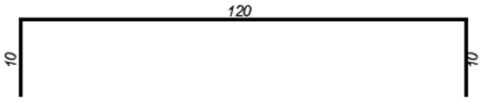
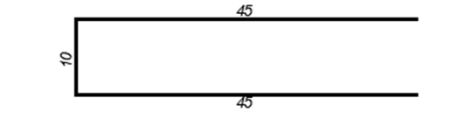
5. Plan armature



Primer 2 – Proračun konstrukcije stepeništa ST2

5. Plan armature

Šipke - specifikacija

ozn.	oblik i mere [cm]	ozn.	Ø	lg [m]	n [kom]	lgn [m]
POS ST2 (1 kom)						
1		B500B	12	4.81	7	33.67
2		B500B	12	3.40	7	23.80
3		B500B	8	1.55	7	10.85
4		B500B	8	2.12	7	14.84
5		B500B	8	1.40	34	47.60
6		B500B	8	1.00	7	7.00

Šipke - rekapitulacija

Ø [mm]	lgn [m]	Jedinična težina [kg/m']	Težina [kg]
B500B			
8	80.29	0.395	31.71
12	57.47	0.888	51.03
Ukupno (B500B)			82.75

$V_b = 1.97\ m^3$

$m_a/V_b = 82.75/1.97$

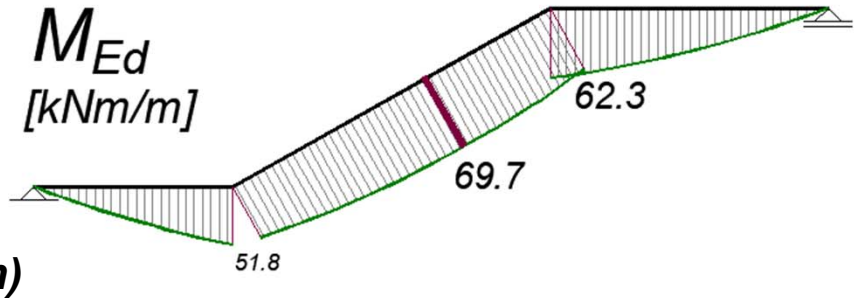
$m_a/V_b = 42.00\ kg/m^3$

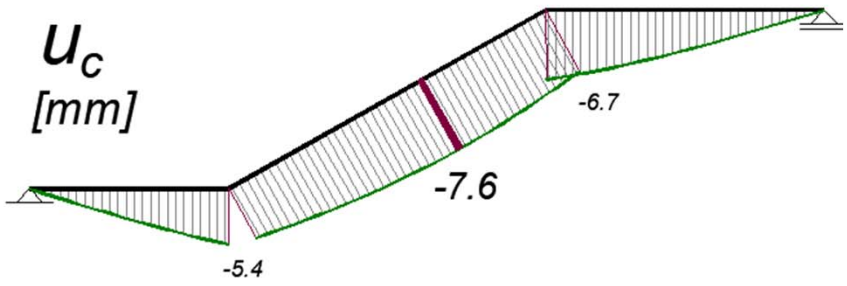
C30/37, B500B R 1:50

Primeri 1, 2 i 3 – Proračun konstrukcije stepeništa

ST1

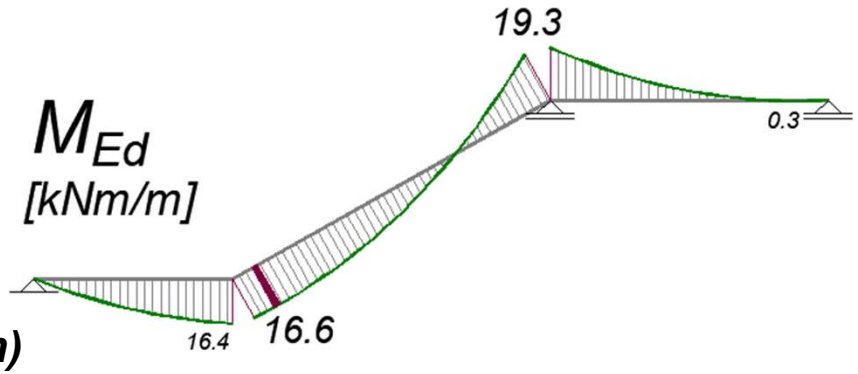
(h_p = 20 cm)

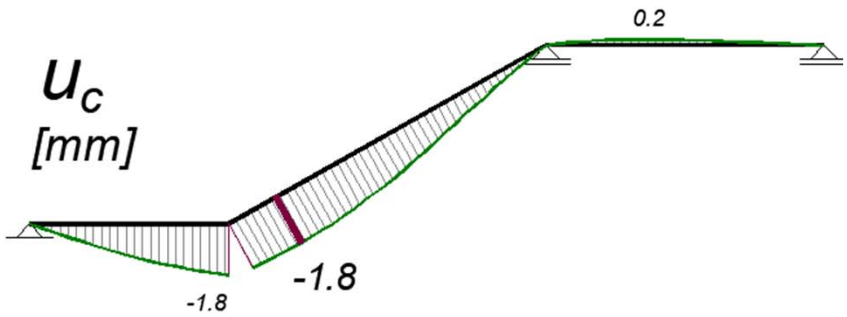




ST2

(h_p = 14 cm)





ST3

(h_p = 14 cm)