



Studijski program:

Građevinarstvo

Modul:

Konstrukcije

Godina/Semestar:

III godina / V semestar

Naziv predmeta (šifra):

**Teorija betonskih konstrukcija 1
(БЗОЗБ1)**

Nastavnik:

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Naslov vežbi:

Smicanje – 2.deo.

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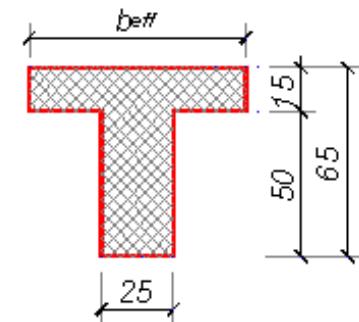
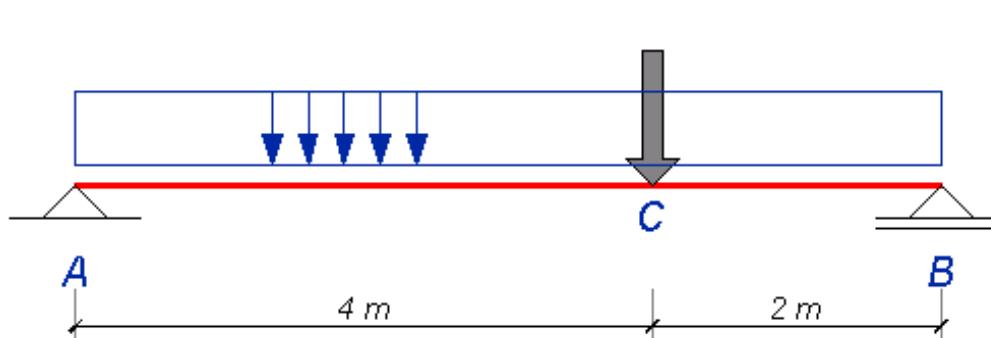


Zadatak 25 – SMICANJE

Za nosač prikazan na skici, čiji su dijagrami proračunskih vrednosti statickih uticaja (M_{Ed} i V_{Ed}) prikazani na sledećem slajdu, izvršiti osiguranje nosača prema glavnim naponima zatezanja.

C25/30 B500B

XC2



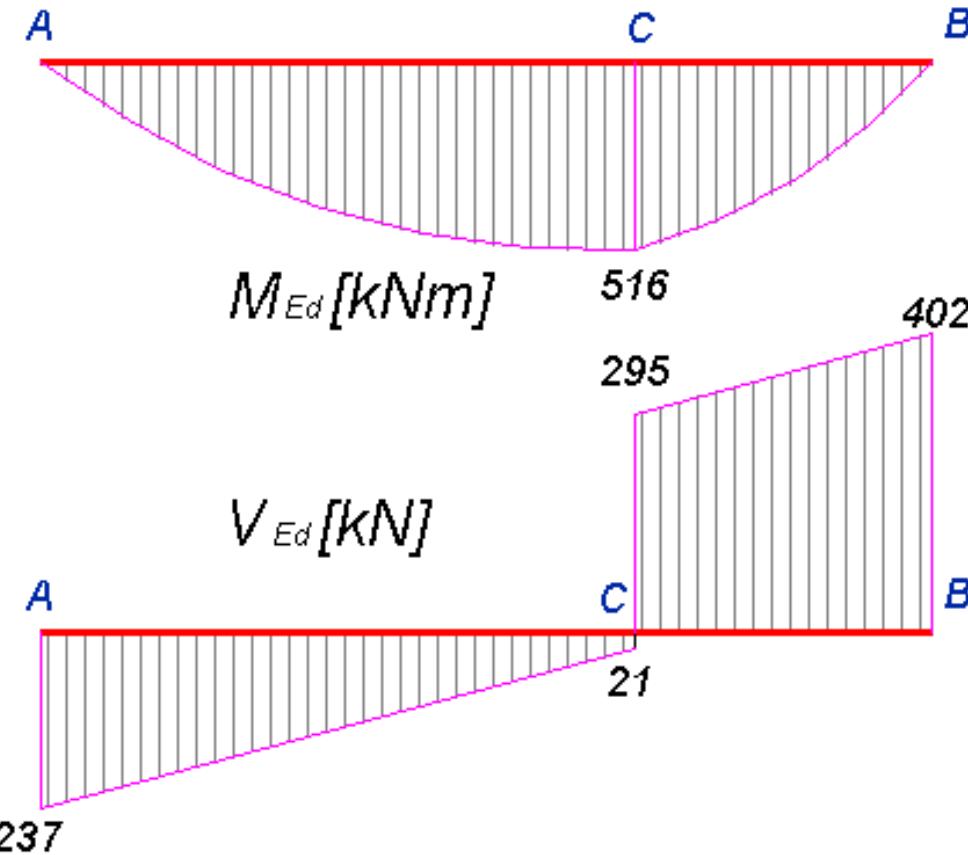
C25/30 $f_{cd} = 0.85 \cdot 25 / 1.5 = 14.2 \text{ MPa} = 1.42 \text{ kN/cm}^2$

B500 B $f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$



Zadatak 25 – SMICANJE

1. Zadati dijagrami statickih uticaja



Zadatak 25 – SMICANJE

2. Dimenzionisanje prema momentima savijanja

1. $M_{Ed}=516 \text{ kNm}$

2. pretp. $d_1 = 7 \text{ cm}$

$$d = h - d_1 = 65 - 7 = 58 \text{ cm}$$

3. Efektivna širina: $b_{eff} = 0.4 \cdot l_0 + b_w = 0.4 \cdot 600 + 25 = 265 \text{ cm}$

4. Računa se:

$$k = \frac{d}{\sqrt{\frac{M_{Ed}}{b_{eff} \cdot f_{cd}}}} = \frac{58}{\sqrt{\frac{516 \cdot 10^2}{265 \cdot 1.42}}} = 4.953$$
$$\varepsilon_c / \varepsilon_{s1} = 3.5 / 65\%$$

$$A_{s1} = \omega_1 b_{eff} d \frac{f_{cd}}{f_{yd}} = 4.086 \cdot \frac{265 \cdot 58}{100} \cdot \frac{1.42}{43.5} = 20.5 \text{ cm}^2$$

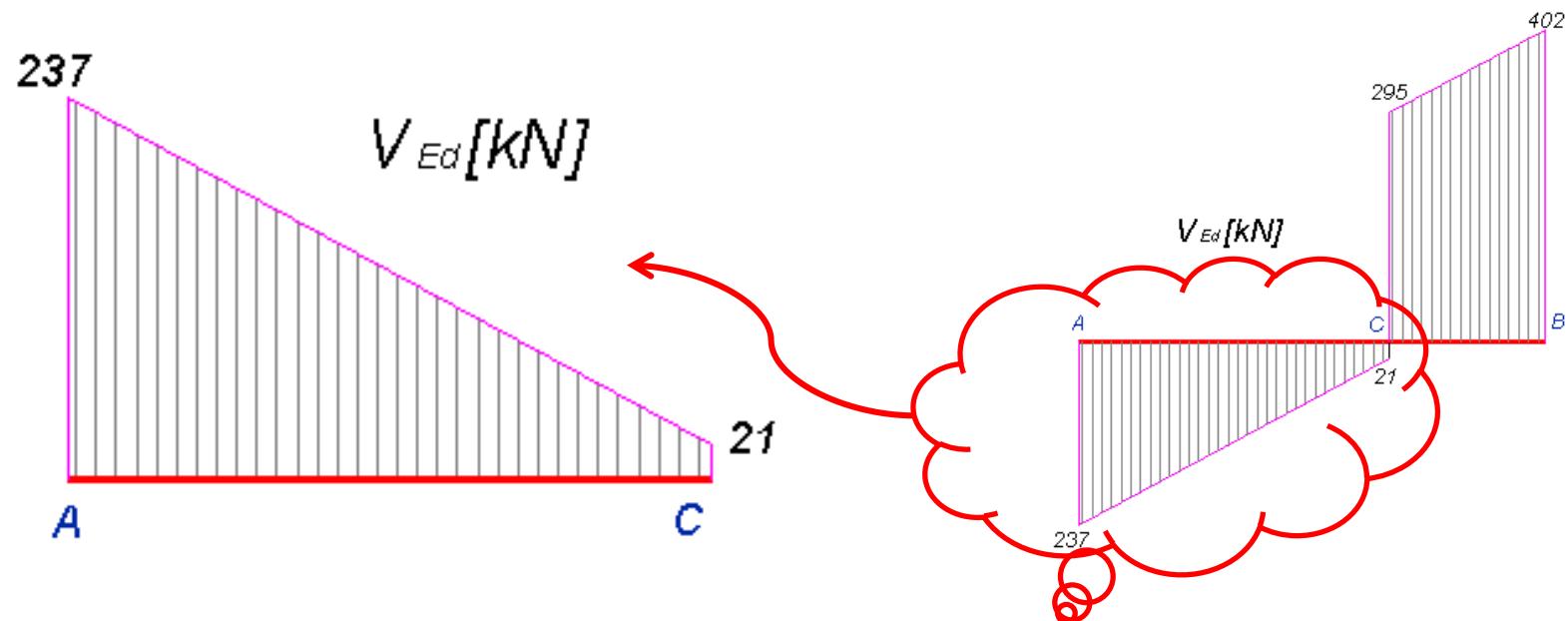
Usvojeno: 5025 (24.55 cm²)



Zadatak 25 – SMICANJE

3. Dimenzionisanje prema smičućim silama: **DEO A-C**

1. Proračunske vrednosti uticaja na posmatranom segmentu:



*Dimenziioniše se prvo presek sa najvećim uticajima!
Samo za potrebe ovog zadatka uradjeno je drugačije.*



Zadatak 25 – SMICANJE

2. Proračun nosivosti betona na smicanje:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_i f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{580}} = 1.587$$

→ Plan armature!

$$\rho_i = \frac{A_{sl}}{b_w \cdot d} = \frac{2 \cdot 4.91}{25 \cdot 58} = 0.0068 < 0.02$$

$$V_{Rd,c} = \left[0.12 \cdot 1.587 \cdot (100 \cdot 0.0068 \cdot 25)^{1/3} \right] \cdot \frac{25 \cdot 58}{10} = 70.9 kN$$



Zadatak 25 – SMICANJE

3. Minimalna nosivost betona na smicanje:

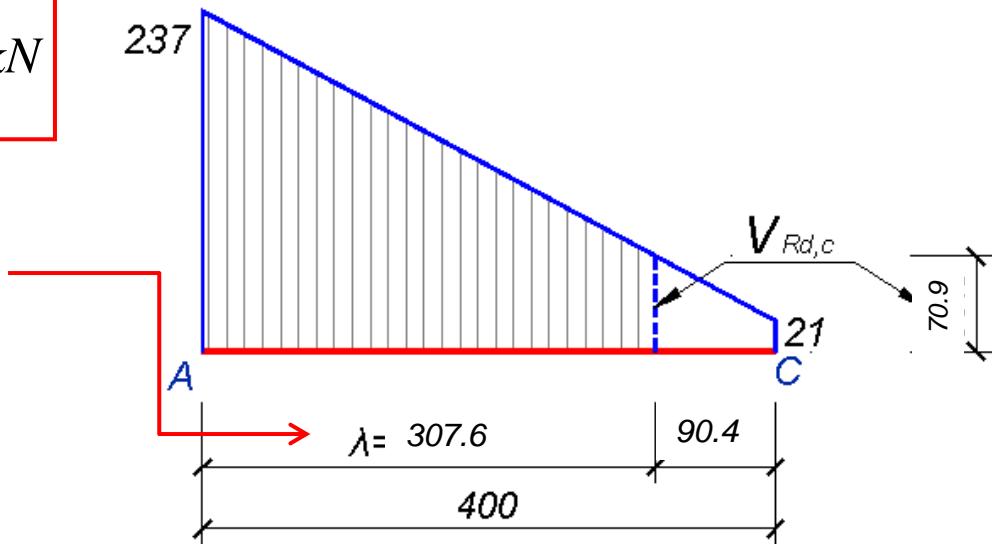
$$V_{\min} = [v_{\min} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d = [0.0035 \cdot k^{3/2} \cdot f_{ck}^{1/2}] \cdot b_w \cdot d$$

$$V_{\min} = [0.0035 \cdot 1.587^{3/2} \cdot 25^{1/2}] \cdot 25 \cdot 58 = 50.7 \text{ kN}$$

4. Nosivost betona na smicanje:

$$V_{Rd,c} = \max \left\{ \begin{array}{l} 70.9 \text{ kN} \\ 50.7 \text{ kN} \end{array} \right\} = 70.9 \text{ kN}$$

Na delu nosača – **dužini osiguranja (λ)** potrebno osiguranje poprečnom armaturom!



Zadatak 25 – SMICANJE

5. Određivanje potrebne armature za smicanje

Na dužini osiguranja:

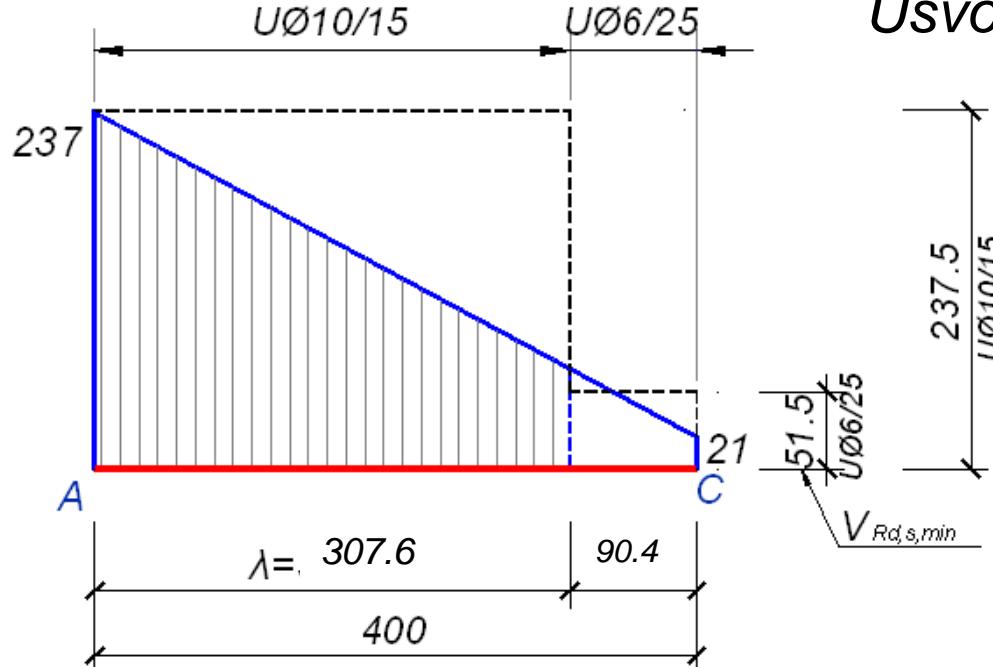
$$V_{Ed} = V_{Rd,s}$$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = V_{Ed}$$

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$

$$s_{rac} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{237} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 15.04 \text{ cm}$$

Usvojeno: $\text{UØ}10/15$

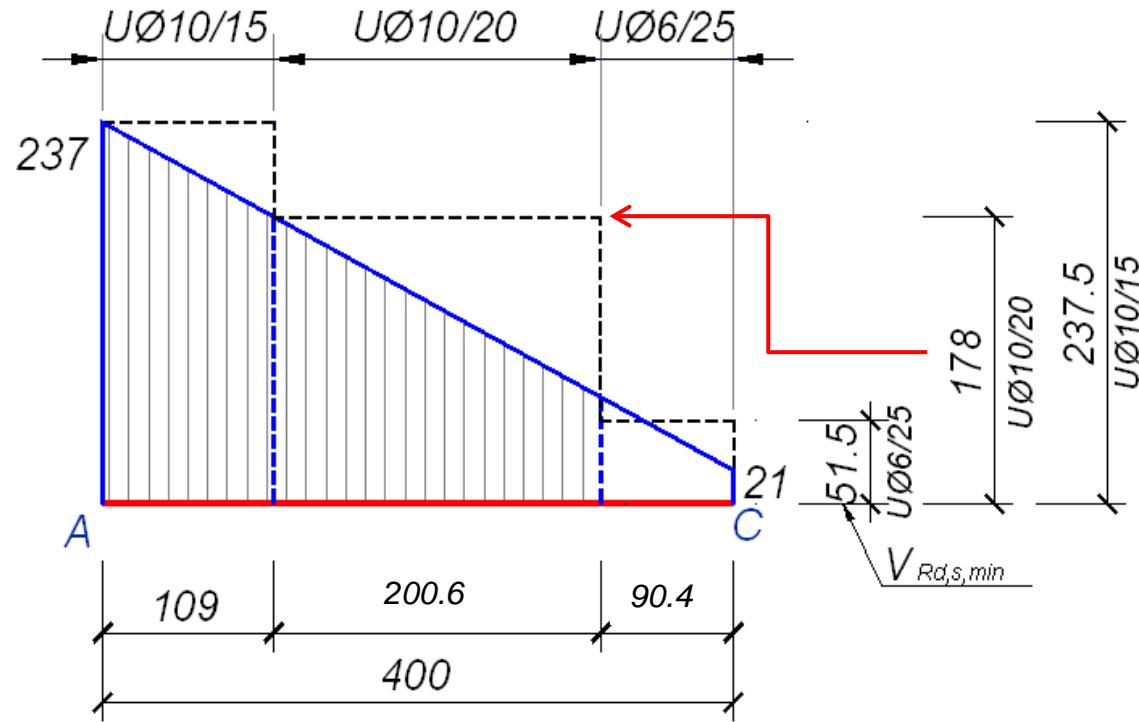


Zadatak 25 – SMICANJE

5a. Varijantno rešenje – optimizacija količine uzengija:

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}10$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$ $s=20 \text{ cm}$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{20} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 178.2 \text{ kN}$$



Zadatak 25 – SMICANJE

6. Kontrola minimalne armature za smicanje

$$\rho_{w,\min} = 0.08 \cdot \frac{\sqrt{f_{ck}}}{f_{yk}} = 0.08 \cdot \frac{\sqrt{25}}{500} = 0.0008 = 0.08\%$$

$$\rho_w = \frac{m \cdot a_{sw}^{(1)}}{s \cdot b_w} \quad \xrightarrow{\text{red arrow}} \quad s_{\rho,\min} = \frac{m \cdot a_{sw}^{(1)}}{\rho_{w,\min} \cdot b_w} = \frac{2 \cdot 0.785}{0.0008 \cdot 25} = 78.5 \text{ cm}$$

Usvojeno: UØ6/25

$$s_{\rho,\min} = \frac{m \cdot a_{sw}^{(1)}}{\rho_{w,\min} \cdot b_w} = \frac{2 \cdot 0.283}{0.0008 \cdot 25} = 28.3 \text{ cm}$$

7. Kontrola maksimalne smičuće nosivosti:

$$V_{Rd,\max} = \frac{\alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd}}{ctg\theta + tg\theta} = \frac{1.0 \cdot 25 \cdot 0.9 \cdot 58 \cdot 0.54 \cdot 1.42}{1+1} = 500.3 \text{ kN}$$



Zadatak 25 – SMICANJE

8. Maksimalno podužno rastojanje armature s_{max}

Табела 1 – Највеће подужно растојање између елемената арматуре за смицање $s_{l,max}$

	Прорачунска вредност силе смицања V_{Ed}^*	Класе чврстоће бетона	
		$\leq C 50/60$	$> C 50/60$
1.	$V_{Ed} \leq 0,3V_{Rd,max}$	$0,75 d^{**}) \leq 300 \text{ mm}$	$0,75 d \leq 200 \text{ mm}$
2.	$0,3V_{Rd,max} \leq V_{Ed} \leq 0,6V_{Rd,max}$	$0,55 d \leq 300 \text{ mm}$	$0,55 d \leq 200 \text{ mm}$
3.	$V_{Ed} > 0,6V_{Rd,max}$		$0,3 d \leq 200 \text{ mm}$

* $V_{Rd,max}$ може да се одреди поједностављено са $\theta = 40^\circ$ ($\cot \theta = 1,2$).

** За греде код којих је $h < 200 \text{ mm}$ и $V_{Ed} \leq V_{Rd,s}$ растојање не мора да буде мање од 150 mm.

$$0.3 \cdot V_{Rd,max} = 0.3 \cdot 500.3 = 150.1 kN < V_{Ed} < 0.6 \cdot V_{Rd,max} = 0.6 \cdot 500.3 = 300.2 kN$$

$$s_{max} = \min \{0.55d; 30cm\} = \min \{0.55 \cdot 58 = 31.9; 30cm\} = 30cm$$



Zadatak 25 – SMICANJE

9. Usvajanje armature za smicanje

Usvojeni prečnik: $\text{Ø}10$

Usvojeno rastojanje:

$$s = \min \left\{ s_{\rho,\min}, s_{\max} \right\} = \begin{cases} 15.04 \\ 78.5 \\ 30.0 \end{cases} = 15.04 \text{ cm}$$

Usvojeno: $U\text{Ø}10/15$

10. Nosivost usvojene armature za smicanje:

$$V_{Rd,s} = \frac{2 \cdot 0.785}{15} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 237.7 \text{ kN}$$

$$V_{Rd,s} = 237.7 \text{ kN} \begin{cases} > 237 \text{ kN} = V_{Ed} \\ < 500.3 \text{ kN} = V_{Rd,max} \end{cases} \rightarrow \begin{array}{l} \text{Nosivost uzengija veća od proračunske sile smicanja} \\ \text{Osigurano od loma pritisnute betonske dijagonale} \end{array}$$



Zadatak 25– SMICANJE

11. Dužina osiguranja: **λ=307.6 cm**

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \begin{cases} V_{Ed}, & V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot ctg\theta, & V_{Ed} > V_{Rd,c} \end{cases} = 0.5 \cdot 237 \cdot 1.0 = 118.5 kN$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{118.8}{43.5} = 2.72 cm^2 < 2 \cdot 4.91 = 9.82 cm^2$$



Armatura koju treba
obezbediti u donjoj zoni na
mestu slobodnog oslonca A

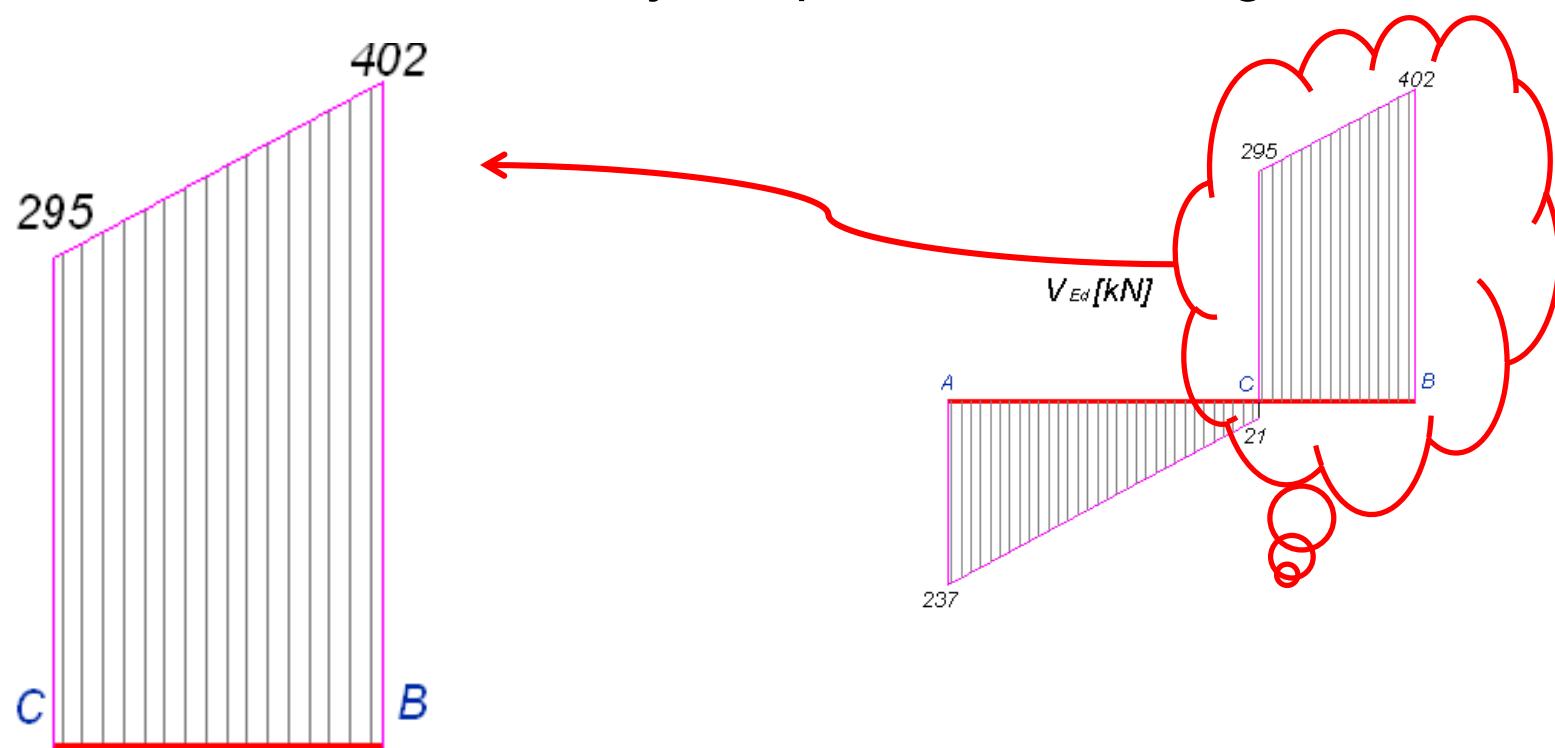
Armatura iz dimenzionisanja prema
momentima savijanja – plan armature



Zadatak 25 – SMICANJE

3. Dimenzionisanje prema smičućim silama: **DEO B-C**

1. Proračunske vrednosti uticaja na posmatranom segmentu:



Zadatak 25 – SMICANJE

2. Proračun nosivosti betona na smicanje:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_i f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{580}} = 1.587$$

Plan armature!

$$\rho_i = \frac{A_{sl}}{b_w \cdot d} = \frac{2 \cdot 4.91}{25 \cdot 58} = 0.0068 < 0.02$$

$$V_{Rd,c} = \left[0.12 \cdot 1.587 \cdot (100 \cdot 0.0068 \cdot 25)^{1/3} \right] \cdot \frac{25 \cdot 58}{10} = 70.9 kN$$



Zadatak 25 – SMICANJE

3. Minimalna nosivost betona na smicanje:

$$V_{\min} = [v_{\min} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d = [0.0035 \cdot k^{3/2} \cdot f_{ck}^{1/2}] \cdot b_w \cdot d$$

$$V_{\min} = [0.0035 \cdot 1.587^{3/2} \cdot 25^{1/2}] \cdot 25 \cdot 58 = 50.7 \text{ kN}$$

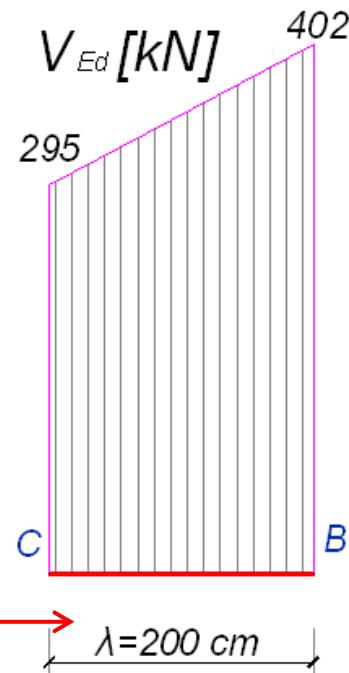
4. Nosivost betona na smicanje:

$$V_{Rd,c} = \max \begin{cases} 70.9 \text{ kN} \\ 50.7 \text{ kN} \end{cases} \quad 70.9 \text{ kN} < 295 \text{ kN}$$

Ceo segment nosača B-C
potrebno je osigurati
poprečnom armaturom.



**Dužina osiguranja (λ)
jednaka je dužini
segmenta B-C**



Zadatak 25 – SMICANJE

5. Određivanje potrebne armature za smicanje

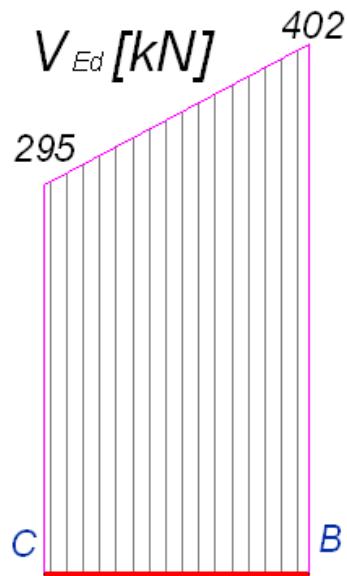
Na dužini osiguranja:

$$V_{Ed} = V_{Rd,s}$$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta = V_{Ed}$$

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\textcolor{red}{\emptyset 10}$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$

$$s_{rac} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 0.785}{402} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 8.9 \text{ cm}$$



Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\textcolor{red}{\emptyset 12}$, $a_{sw}^{(1)}=1.13 \text{ cm}^2$

$$s_{rac} \leq \frac{m \cdot a_{sw}^{(1)}}{V_{Ed}} \cdot z \cdot f_{ywd} \cdot ctg\theta = \frac{2 \cdot 1.13}{402} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 12.7 \text{ cm}$$

Usvojeno: **U∅12/12.5**



Zadatak 25 – SMICANJE

5. Nosivost usvojenih uzengija:

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, $\text{Ø}12$, $a_{sw}^{(1)}=0.785 \text{ cm}^2$ $s=12.5 \text{ cm}$

$$V_{Rd,s} = \frac{m \cdot a_{sw}^{(1)}}{s} \cdot z \cdot f_{ywd} \cdot ctg\theta$$

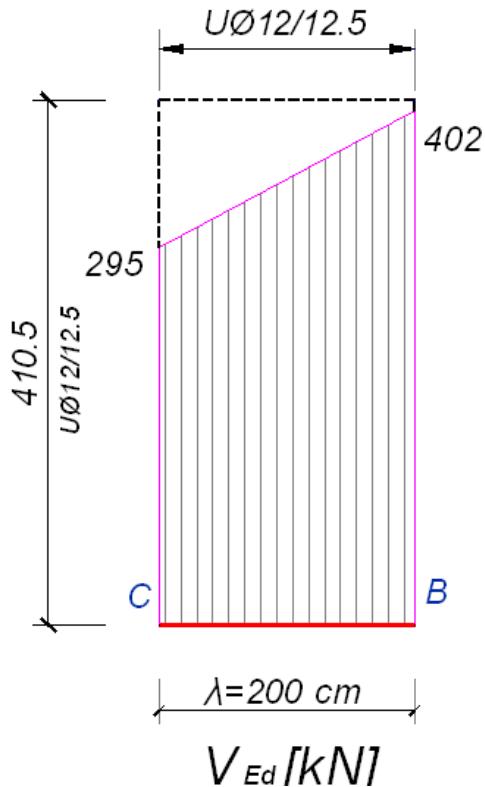
$$V_{Rd,s} = \frac{2 \cdot 1.13}{12.5} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 410.5 \text{ kN} \left\{ \begin{array}{l} > 51.4 = V_{Rd,s,\min} \\ < 500.3 = V_{Rd,s,\max} \end{array} \right.$$

6. Nosivost minimalne armature za smicanje

$$V_{Rd,s,\min} = \frac{2 \cdot 0.283}{25} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 51.4 \text{ kN}$$

7. Kontrola maksimalne smičuće nosivosti:

$$V_{Rd,max} = \frac{\alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd}}{ctg\theta + tg\theta} = \frac{1.0 \cdot 25 \cdot 0.9 \cdot 58 \cdot 0.54 \cdot 1.42}{1+1} = 500.3 \text{ kN}$$



Zadatak 25– SMICANJE

11. Dužina osiguranja: dužina **$\lambda=200 \text{ cm}$**

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \begin{cases} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot \operatorname{ctg} \theta, V_{Ed} > V_{Rd,c} \end{cases} = 0.5 \cdot 402 \cdot 1.0 = 201 \text{ kN}$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{201}{43.5} = 4.62 \text{ cm}^2 < 2 \cdot 4.91 = 9.82 \text{ cm}^2$$



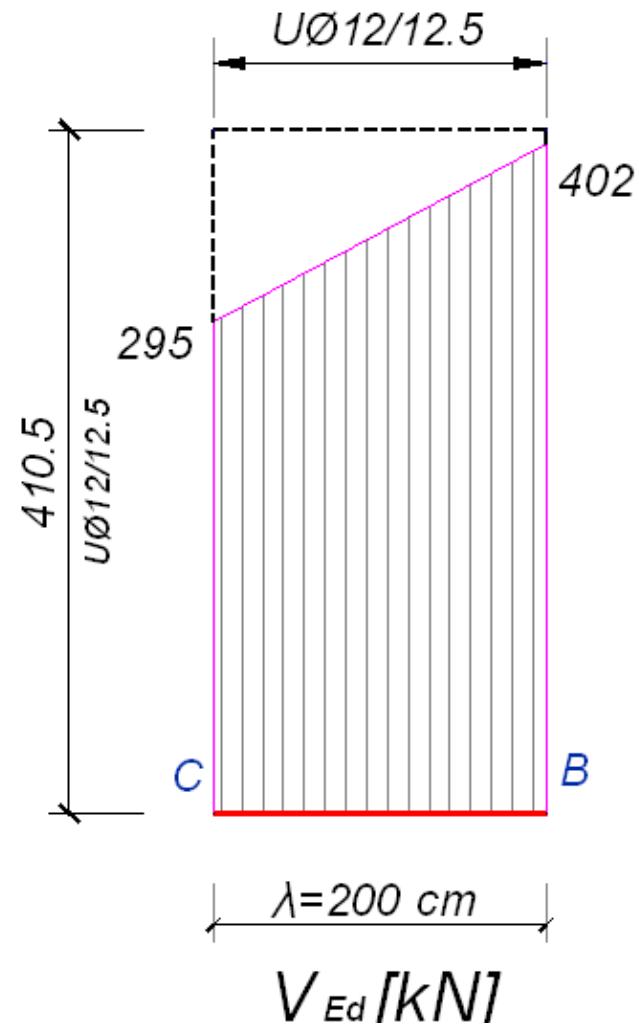
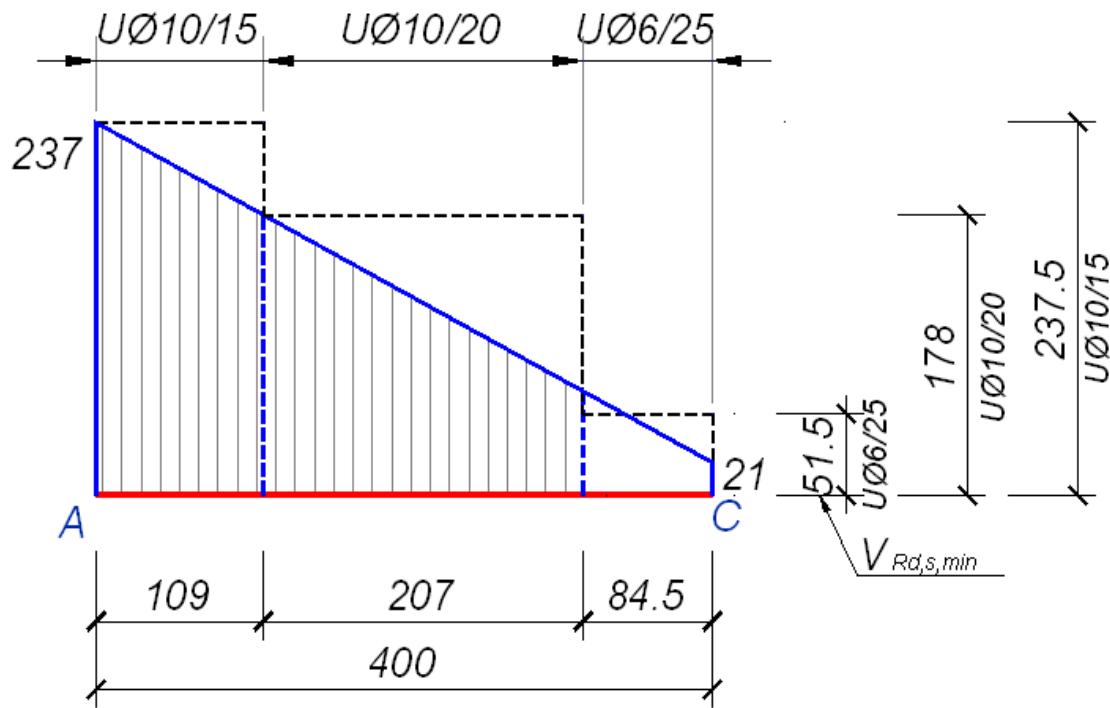
Armatura koju treba
obezbediti u donjoj zoni na
mestu slobodnog oslonca B

Armatura iz dimenzionisanja prema
momentima savijanja – plan armature



Zadatak 25 – SMICANJE

Konačno rešenje:



Zadatak 25 – SMICANJE

5a. Varijantno rešenje: **uzengije+kosa armatura**

Usvojeno: $\theta=45^\circ$, $\operatorname{ctg}\theta=1$, $m=2$, **$\text{Ø}10$** , $a_{sw}^{(1)}=0.785 \text{ cm}^2$ **$s=20\text{cm}$**

Deo smicanja koji se poverava uzengijama:

$$V_{Rd,s} = \frac{2 \cdot 0.785}{20} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 178.2 \text{ kN}$$

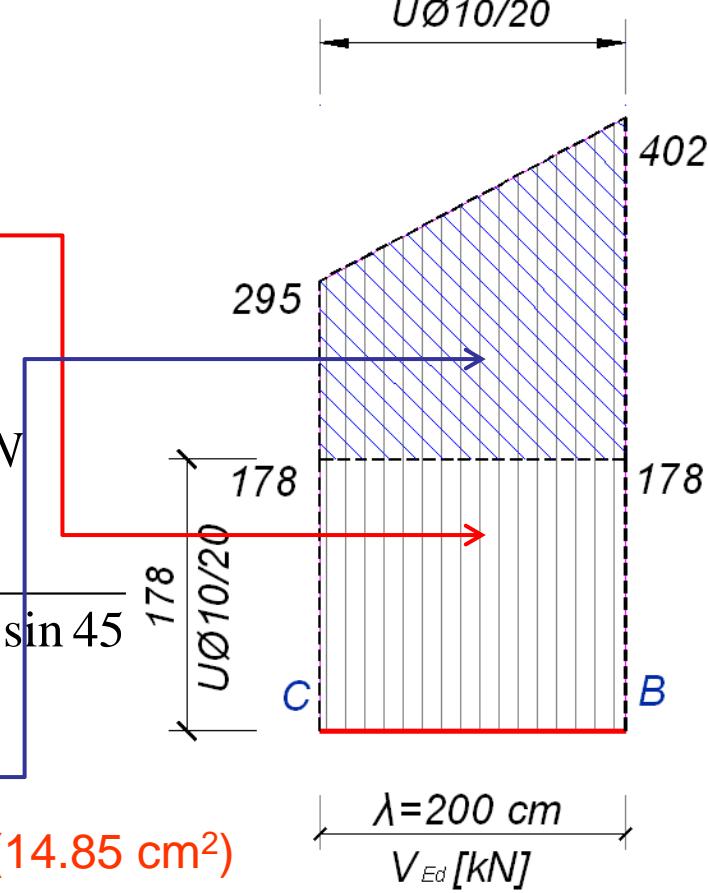
Deo smicanja koji se poverava kosoj armaturi:

$$V_{Rd,s,k} = \frac{(402 - 178) + (295 - 178)}{2} \cdot 200 \cdot \frac{1}{0.9 \cdot 58} = 653.2 \text{ kN}$$

$$A_{sw,k} = \frac{V_{Rd,s,k}}{f_{ykw} \cdot (\operatorname{ctg}\theta + \operatorname{ctg}\alpha) \cdot \sin \alpha} = \frac{653.2 \text{ kN}}{43.5 \cdot (\operatorname{ctg}45 + \operatorname{ctg}45) \cdot \sin 45}$$

$$A_{sw,k} = \frac{653.2 \text{ kN}}{43.5 \cdot (1+1) \cdot \frac{\sqrt{2}}{2}} = \frac{653.2 \text{ kN}}{43.5 \cdot \sqrt{2}} = 10.6 \text{ cm}^2$$

Usvojeno: **$3\text{Ø}25$ (14.85 cm^2)**



Zadatak 25 – SMICANJE

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \begin{cases} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot (\operatorname{ctg}\theta - \operatorname{ctg}\alpha), V_{Ed} > V_{Rd,c} \end{cases} = 0.5 \cdot (V_{Rd,s} + V_{Rd,s,k}) \cdot (\operatorname{ctg}\theta - \operatorname{ctg}\alpha) = 0.5 \cdot V_{Rd,s} \cdot \operatorname{ctg}\theta$$

$$\Delta F_{td} = 0.5 \cdot 178.2 \cdot 1.0 = 89.1 kN$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{89.1}{43.5} = 2.05 cm^2 < \underline{2 \cdot 4.91 = 9.82 cm^2}$$


Armatura koju treba obezbediti u donjoj zoni na mestu slobodnog oslonca B

Armatura iz dimenzionisanja prema momentima savijanja – plan armature



Zadatak 25 – SMICANJE

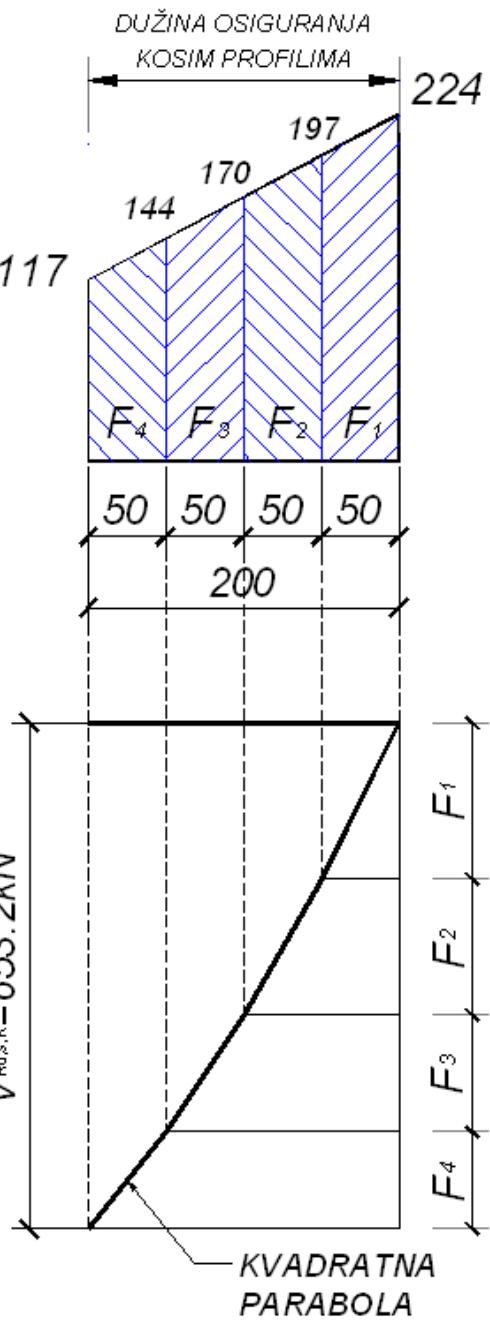
13. Određivanje mesta povijanja kosih profila

$$F_1 = \frac{224+197}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 201.6 \text{ kN}$$

$$F_2 = \frac{197+170}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 175.8 \text{ kN}$$

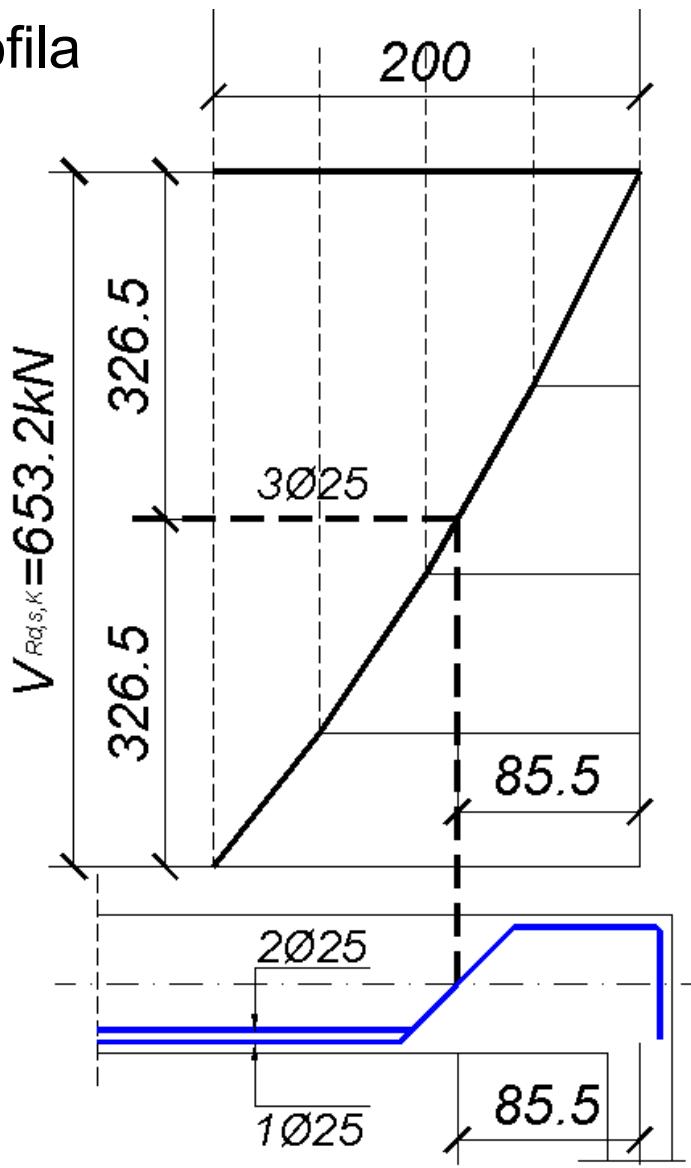
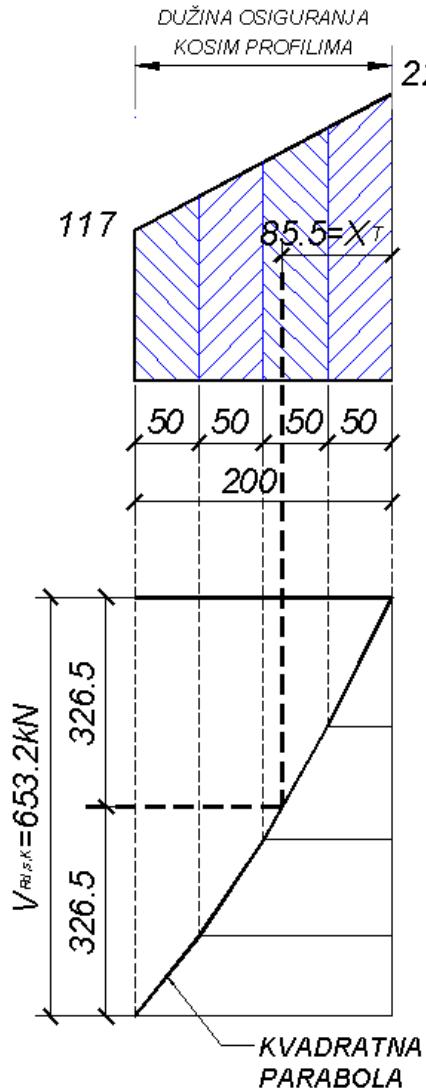
$$F_3 = \frac{170+144}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 150.4 \text{ kN}$$

$$F_4 = \frac{144+117}{2} \cdot 50 \cdot \frac{1}{0.9 \cdot 58} = 125 \text{ kN}$$



Zadatak 25 – SMICANJE

13. Određivanje mesta povijanja kosih profila



Zadatak 25 – SMICANJE

5b. Varijantno rešenje: **uzengije+kosa armatura**

Usvojeno: $\theta=45^0$, $ctg\theta=1$, $m=2$, **$\Ø 10$** , $a_{sw}^{(1)}=0.785 \text{ cm}^2$ **$s=10\text{cm}$**

Deo smicanja koji se poverava uzengijama:

$$V_{Rd,s} = \frac{2 \cdot 0.785}{10} \cdot 0.9 \cdot 58 \cdot 43.5 \cdot 1.0 = 356.4 \text{ kN}$$

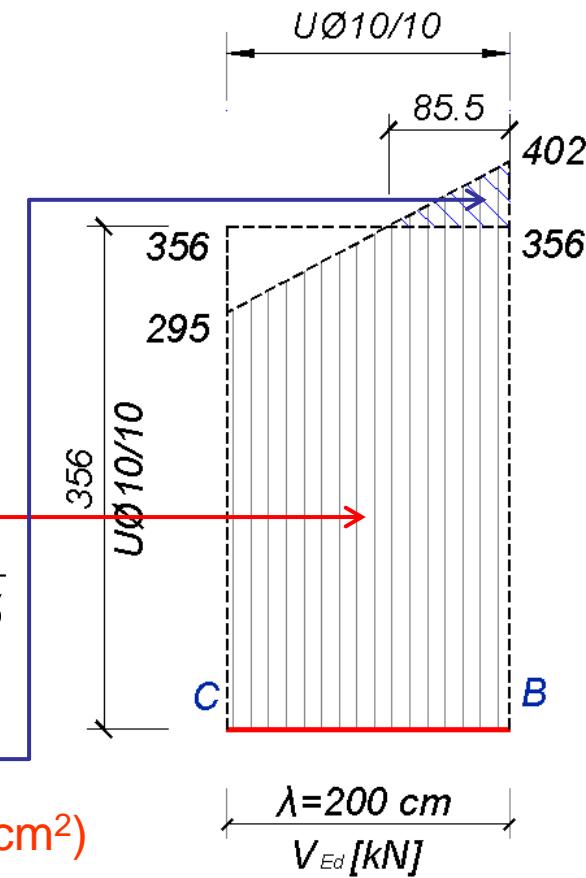
Deo smicanja koji se poverava kosoj armaturi:

$$V_{Rd,s,k} = \frac{(402 - 356)}{2} \cdot 85.5 \cdot \frac{1}{0.9 \cdot 58} = 37.7 \text{ kN}$$

$$A_{sw,k} = \frac{V_{Rd,s}}{f_{ykw} \cdot (ctg\theta + ctg\alpha) \cdot \sin \alpha} = \frac{37.7 \text{ kN}}{43.5 \cdot (ctg 45 + ctg 45) \cdot \sin 45}$$

$$A_{sw,k} = \frac{37.7 \text{ kN}}{43.5 \cdot (1+1) \cdot \frac{\sqrt{2}}{2}} = \frac{37.7 \text{ kN}}{43.5 \cdot \sqrt{2}} = 0.6 \text{ cm}^2$$

Usvojeno: **$Ø 25$ (4.91 cm^2)**



Zadatak 25 – SMICANJE

12. Dodatna zategnuta armatura

$$\Delta F_{td} = \begin{cases} V_{Ed}, V_{Ed} < V_{Rd,c} \\ 0.5 \cdot V_{Ed} \cdot (\operatorname{ctg}\theta - \operatorname{ctg}\alpha), V_{Ed} > V_{Rd,c} \end{cases} = 0.5 \cdot (V_{Rd,s} + V_{Rd,s,K}) \cdot (\operatorname{ctg}\theta - \operatorname{ctg}\alpha) = 0.5 \cdot V_{Rd,s} \cdot \operatorname{ctg}\theta$$

$$\Delta F_{td} = 0.5 \cdot 356 \cdot 1.0 = 178 kN$$

$$\Delta A_{s1} = \frac{\Delta F_{td}}{f_{yd}} = \frac{178}{43.5} = 4.1 cm^2 < \underline{2 \cdot 4.91 = 9.82 cm^2}$$



Armatura koju treba
obezbediti u donjoj zoni na
mestu slobodnog oslonca B

Armatura iz dimenzionisanja prema
momentima savijanja – plan armature



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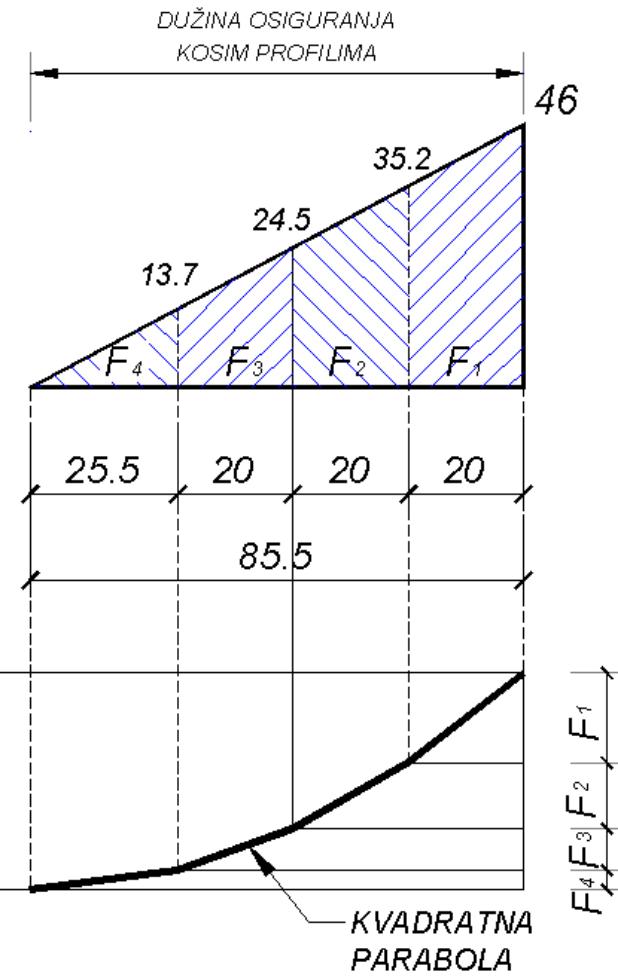
13. Određivanje mesta povijanja kosih profila

$$F_1 = \frac{46 + 35.2}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 15.6 \text{ kN}$$

$$F_2 = \frac{35.2 + 24.5}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 11.4 \text{ kN}$$

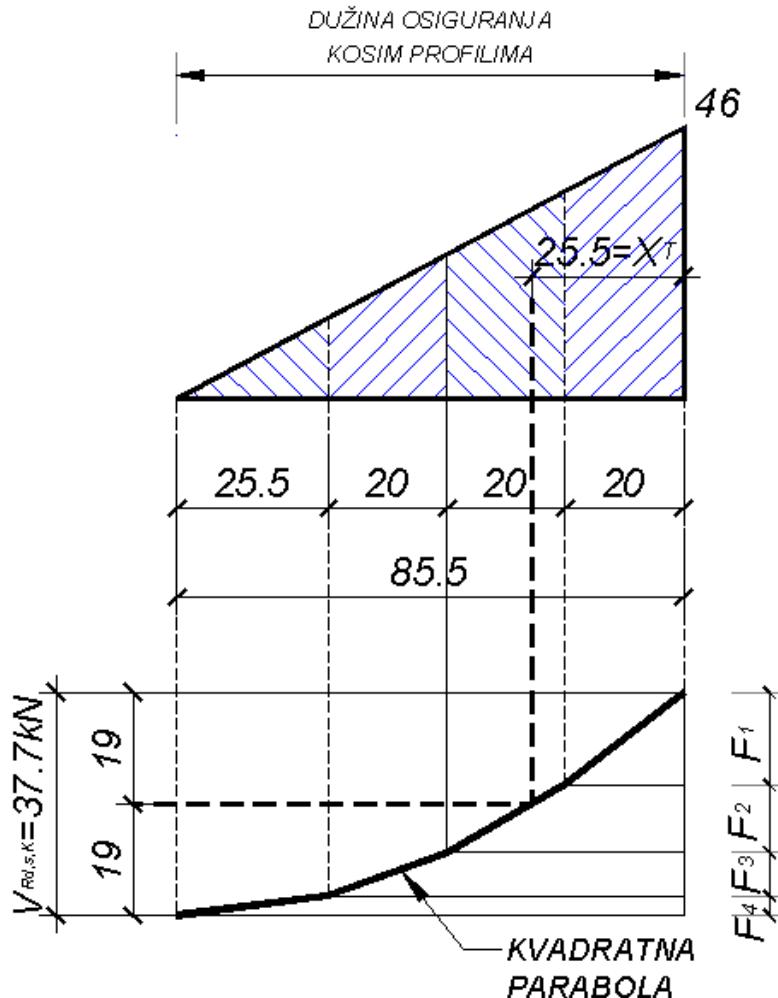
$$F_3 = \frac{24.5 + 13.7}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 7.3 \text{ kN}$$

$$F_4 = \frac{13.7 + 25.5}{2} \cdot 20 \cdot \frac{1}{0.9 \cdot 58} = 3.3 \text{ kN}$$



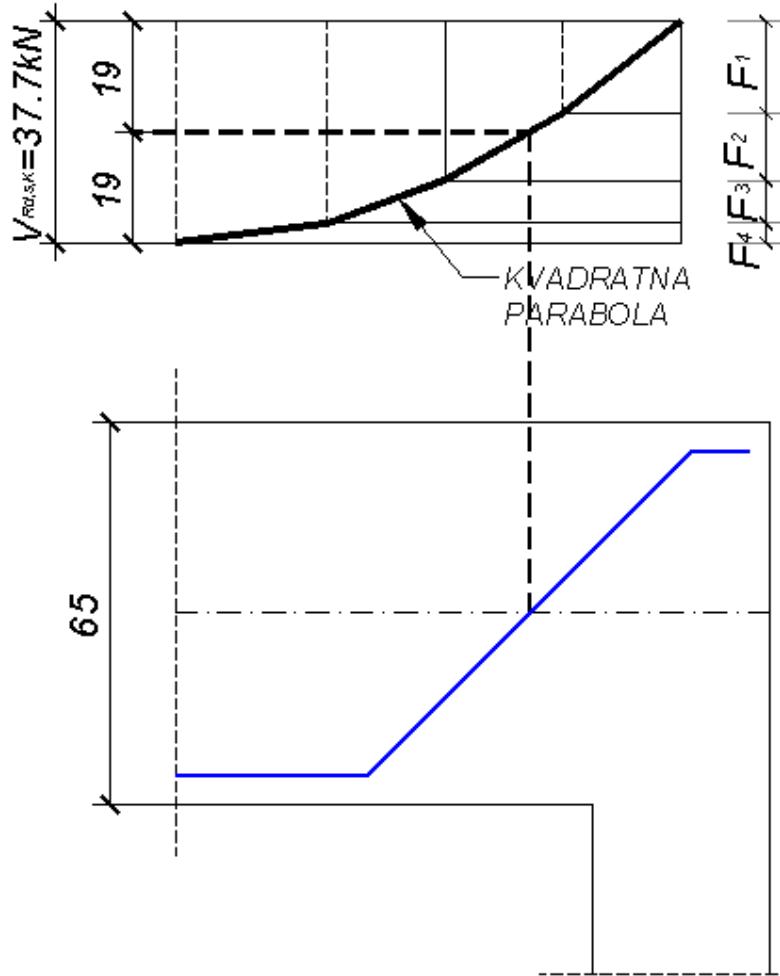
Zadatak 25 – SMICANJE

13. Određivanje mesta povijanja kosih profila

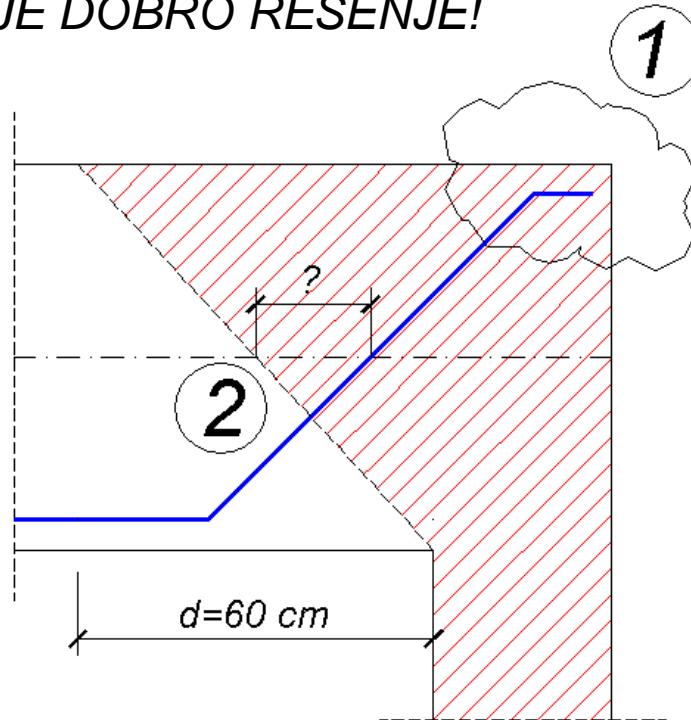


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13. Određivanje mesta povijanja kosih profila



NIJE DOBRO REŠENJE!



Zadatak 25 – SMICANJE

PLAN ARMATURE

