



Univerzitet u Beogradu – Građevinski fakultet
www.grf.bg.ac.rs

Studijski program:	Građevinarstvo
Godina/Semestar:	III godina / V semestar
Naziv predmeta (šifra):	Teorija betonskih konstrukcija 1 (B3O3B1)
Nastavnik:	Jelena Carević / Stefan Ž. Mitrović / Milica Vidović
Naslov vežbi:	Kombinacije opterećenja. Centrično zatezanje. Mali ekscentricitet- sila zatezanja
Datum :	02.12.2025.

Beograd, 2025.

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Zadatak 15 – KOMBINOVANJE OPTEREĆENJA

Odrediti potrebnu površinu armature za stub poznatih dimenzija, pravougaonog poprečnog preseka, koji je opterećen zadatim uticajima. Podaci za proračun:

$$M_G = 100 \text{ kNm}$$

$$N_G = 500 \text{ kN}$$

$$M_w = \pm 200 \text{ kNm}$$

$$b = 25 \text{ cm}$$

$$h = 65 \text{ cm}$$

C25/30

B500 B



a. zategnuta spoljašnja ivica stuba

$$M_{Ed} = 1.35 \times 100 + 1.5 \times 200 = 435 \text{ kNm}$$

$$N_{Ed} = 1.35 \times 500 = 675 \text{ kN}$$

$$\text{pretp. } d_1 = 7 \text{ cm}$$

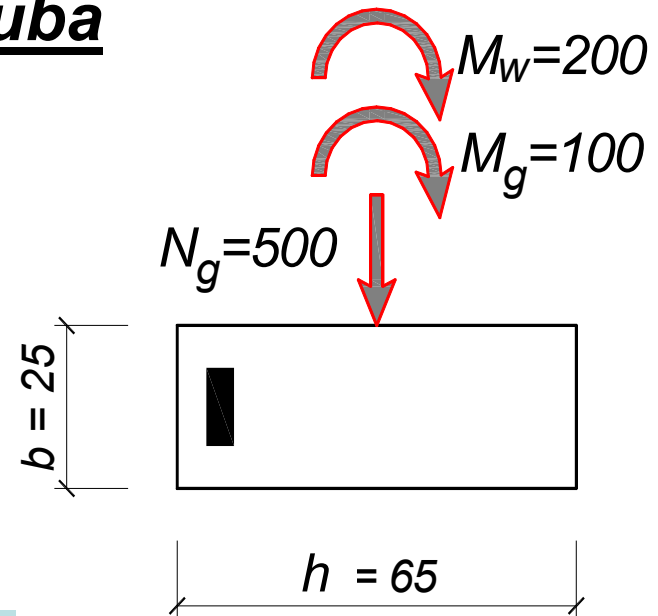
$$d = 65 - 7 = 58 \text{ cm}$$

$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$

$$M_{Eds} = 435 + 675 \times \left(\frac{0.65}{2} - 0.07 \right) = 607.1 \text{ kNm}$$

$$k = \frac{58}{\sqrt{\frac{607.1 \times 10^2}{25 \times 1.42}}} = 1.402 \Rightarrow \varepsilon_{s1} < 2.5\text{‰}$$

Kako je $\varepsilon_{s1} < 2.5\text{‰}$, presek se OBOSTRANO ARMIRA.



usvojeno $\varepsilon_{s1,lim} = 2.5\text{‰} \Rightarrow k_{lim} = 1.672, \omega_{Rd,lim} = 47.222\%$

$$M_{Rd,lim} = \left(\frac{58}{1.672} \right)^2 \times 25 \times 1.42 \times 10^{-2} = 427.2 \text{ kNm}$$

$$\Delta M = 607.1 - 427.2 = 179.9 \text{ kNm}$$

$$\text{pretp. } d_2 = 5 \text{ cm} \Rightarrow A_{s2} = \frac{179.9 \times 10^2}{(58 - 5) \times 43.5} = 7.8 \text{ cm}^2$$

$$A_{s1} = 47.222 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} - \frac{675}{43.5} + 7.8 = 14.63 \text{ cm}^2$$

b. zategnuta unutrašnja ivica stuba

$$M_{Ed} = 1.0 \times (-100) + 1.5 \times 200 = 200 \text{ kNm}$$

$$N_{Ed} = 1.0 \times 500 = 500 \text{ kN}$$

$$\text{pretp. } d_1 = 5 \text{ cm}$$

$$d = 65 - 5 = 60 \text{ cm}$$

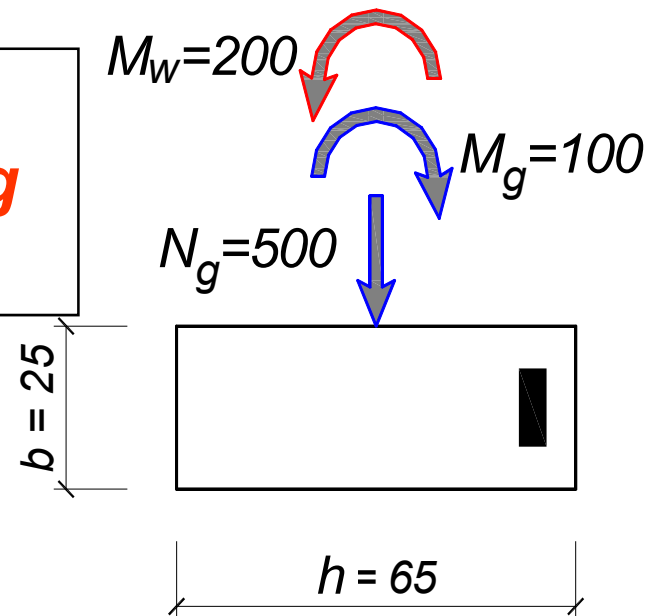
$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$

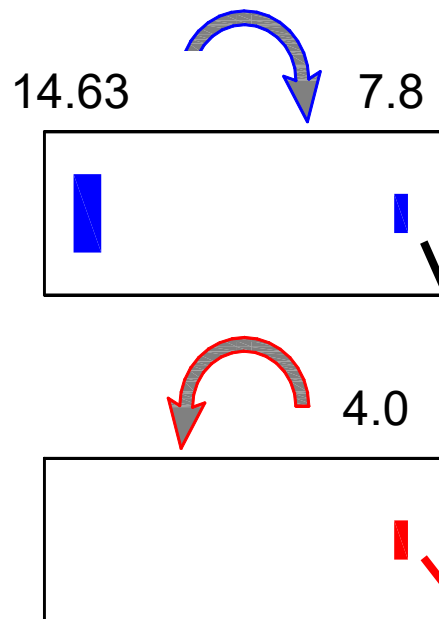
**“povoljno”
dejstvo stalnog
opterećenja**

$$M_{Eds} = 200 + 500 \times \left(\frac{0.65}{2} - 0.05 \right) = 337.5 \text{ kNm}$$

$$k = \frac{60}{\sqrt{\frac{337.5 \times 10^2}{25 \times 1.42}}} = 1.946 \Rightarrow \begin{aligned} \varepsilon_c / \varepsilon_{s1} &= 3.5 / 5.435\text{‰} \\ \omega_1 &= 31.646\% \end{aligned}$$

$$A_{s1} = 31.646 \times \frac{25 \times 60}{100} \times \frac{1.42}{43.5} - \frac{500}{43.5} = 4.0 \text{ cm}^2$$





POTREBNO:



unutra: $A_{s, \text{potr.}} = \max. \left\{ \begin{matrix} 7.8 \\ 4.0 \end{matrix} \right\} = 7.8 \text{ cm}^2$

usvojeno: **3Ø20** (9.42 cm²)

spolja: $A_{s, \text{potr.}} = 14.63 \text{ cm}^2$

usvojeno: **5Ø20** (15.7 cm²)



Zadatak 16 – KOMBINOVANJE OPTEREĆENJA

*Dimenzionisati stub poznatih dimenzija, pravougaonog poprečnog preseka, opterećen zadatim uticajima. Opterećenja **q** i **w** su povremena i NE MORAJU delovati istovremeno. Podaci za proračun:*

$$M_G = 100 \text{ kNm}$$

$$N_Q = 500 \text{ kN}$$

$$M_w = \pm 200 \text{ kNm}$$

$$b = 25 \text{ cm}$$

$$h = 65 \text{ cm}$$

C25/30

B500B



a. zategnuta spoljašnja ivica stuba

a.1 Maksimalni momenat - MINIMALNA sila pritiska

$$M_{Ed} = 1.35 \times 100 + 1.5 \times 200 = 435 \text{ kNm}$$

$$N_{Ed} = 0$$

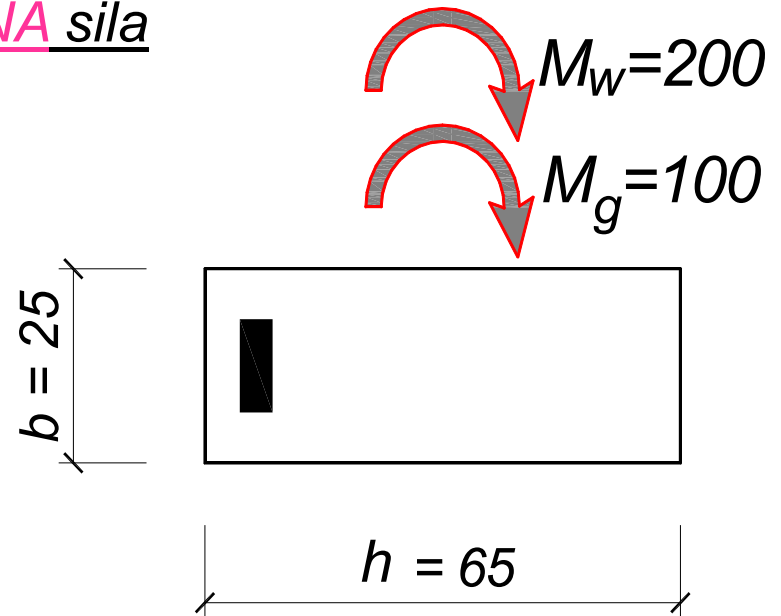
$$\text{pretp. } d_1 = 7 \text{ cm} \Rightarrow d = 65 - 7 = 58 \text{ cm}$$

$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$

$$k = \frac{58}{\sqrt{\frac{435 \times 10^2}{25 \times 1.42}}} = 1.657$$

$$\varepsilon_c / \varepsilon_{s1} = 3.5 / 2.321\text{‰} \Rightarrow \omega_1 = 48.673\%$$

$$A_{s1} = 48.673 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} = 23.04 \text{ cm}^2$$



a. zategnuta spoljašnja ivica stuba

a.2 Maksimalni momenat - **MAKSIMALNA** sila pritiska

a.2.1 dominantno promenljivo – **VETAR**

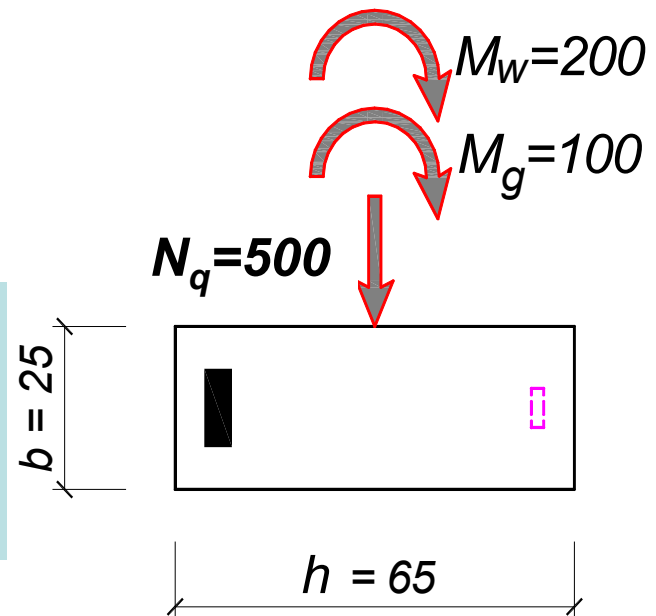
$$1,35 \cdot G_k + 1,5 \cdot (F_{k,w} + 0,5 \cdot Q_{k,n} + 0,7 \cdot Q_{k,es}) = 1,35 \cdot G_k + 1,5 \cdot F_{k,w} + 0,75 \cdot Q_{k,n} + 1,05 \cdot Q_{k,es}$$

$$M_{Ed} = 1,35 \times 100 + 1,5 \times 200 = 435 \text{ kNm}$$

$$N_{Ed} = 1,5 \times 0,7 \times 500 = 525 \text{ kN}$$

$$M_{Eds} = 435 + 525 \times \left(\frac{0,65}{2} - 0,07 \right) = 568,9 \text{ kNm}$$

$$k = \frac{58}{\sqrt{\frac{568,9 \times 10^2}{25 \times 1,42}}} = 1,449 \Rightarrow \varepsilon_{s1} < 2,5\text{‰}$$



Kako je $\varepsilon_{s1} < 2,5\text{‰}$, presek se OBOSTRANO ARMIRA.

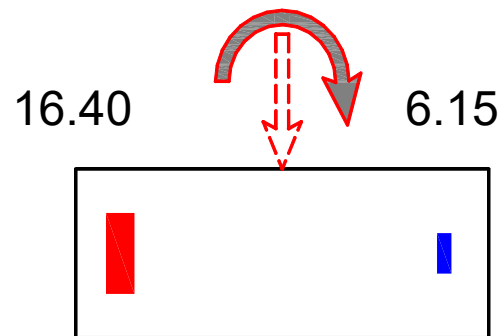
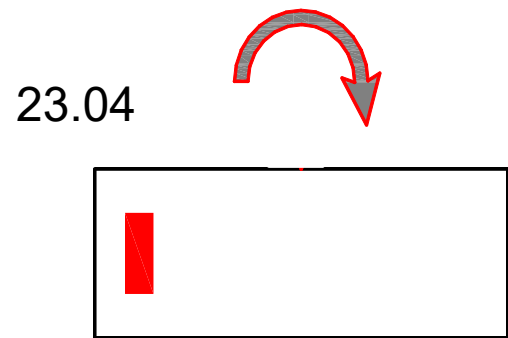
usvojeno $\varepsilon_{s1,lim} = 2.5\text{‰} \Rightarrow k_{lim} = 1.672, \omega_{Rd,lim} = 47.222\%$

$$M_{Rd,lim} = \left(\frac{58}{1.672} \right)^2 \times 25 \times 1.42 \times 10^{-2} = 427.2 \text{ kNm}$$

$$\Delta M = 568.9 - 427.2 = 141.7 \text{ kNm}$$

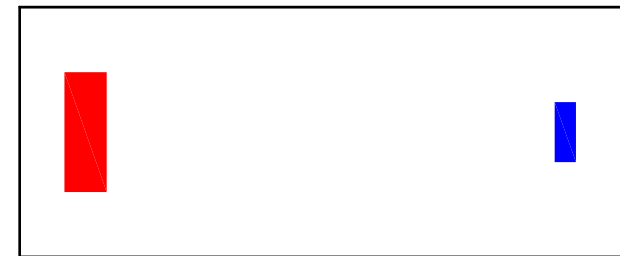
$$\text{pretp. } d_2 = 5 \text{ cm} \Rightarrow A_{s2} = \frac{141.7 \times 10^2}{(58 - 5) \times 43.5} = 6.15 \text{ cm}^2$$

$$A_{s1} = 47.222 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} - \frac{525}{43.5} + 6.15 = 16.4 \text{ cm}^2$$



POTREBNO:

23.04 6.15



$$spolja: A_{s, potr.} = \max. \left\{ \begin{matrix} 23.04 \\ 16.40 \end{matrix} \right\} = 23.04 cm^2$$

$$unutra: A_{s, potr.} = \max. \left\{ \begin{matrix} 0 \\ 6.15 \end{matrix} \right\} = 6.15 cm^2$$

a. zategnuta spoljašnja ivica stuba

a.2 Maksimalni momenat - **MAKSIMALNA** sila pritiska

a.2.2 dominantno promenljivo – **KORISNO**

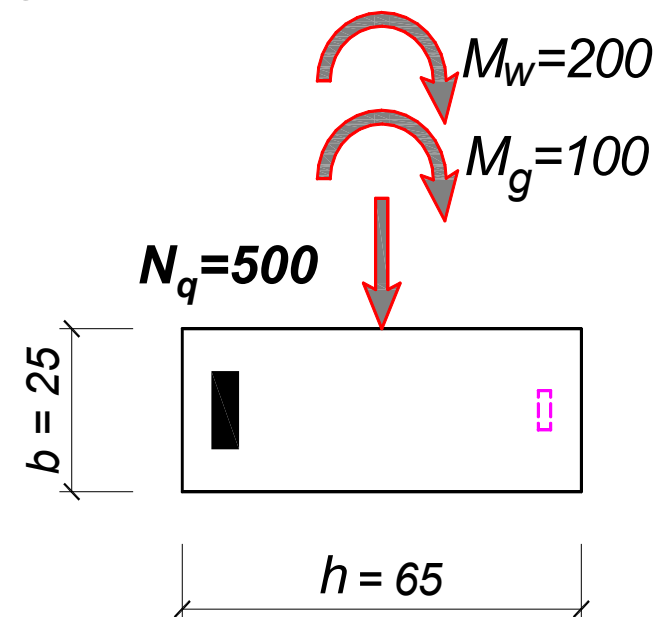
$$1,35 \cdot G_k + 1,5 \cdot (Q_{k,es} + 0,5 \cdot Q_{k,n} + 0,6 \cdot F_{k,w}) = 1,35 \cdot G_k + 1,5 \cdot Q_{k,es} + 0,75 \cdot Q_{k,n} + 0,9 \cdot F_{k,w}$$

$$M_{Ed} = 1,35 \times 100 + 1,5 \times 0,6 \times 200 = 315 \text{ kNm}$$

$$N_{Ed} = 1,5 \times 500 = 750 \text{ kN}$$

$$M_{Eds} = 315 + 750 \times \left(\frac{0,65}{2} - 0,07 \right) = 506,3 \text{ kNm}$$

$$k = \frac{58}{\sqrt{\frac{506,3 \times 10^2}{25 \times 1,42}}} = 1,536 \Rightarrow \varepsilon_{s1} < 2,5\text{‰}$$



Kako je $\varepsilon_{s1} < 2,5\text{‰}$, presek se OBOSTRANO ARMIRA.

usvojeno $\varepsilon_{s1,lim} = 2.5\text{‰} \Rightarrow k_{lim} = 1.672, \omega_{Rd,lim} = 47.222\%$

$$M_{Rd,lim} = \left(\frac{58}{1.672} \right)^2 \times 25 \times 1.42 \times 10^{-2} = 427.2 \text{ kNm}$$

$$\Delta M = 506.3 - 427.2 = 79.1 \text{ kNm}$$

$$\text{pretp. } d_2 = 5 \text{ cm} \Rightarrow A_{s2} = \frac{79.1 \times 10^2}{(58 - 5) \times 43.5} = 3.43 \text{ cm}^2$$

$$A_{s1} = 47.222 \times \frac{25 \times 58}{100} \times \frac{1.42}{43.5} - \frac{750}{43.5} + 3.43 = 8.54 \text{ cm}^2$$

Nije merodavna kombinacija!



b. zategnuta unutrašnja ivica stuba

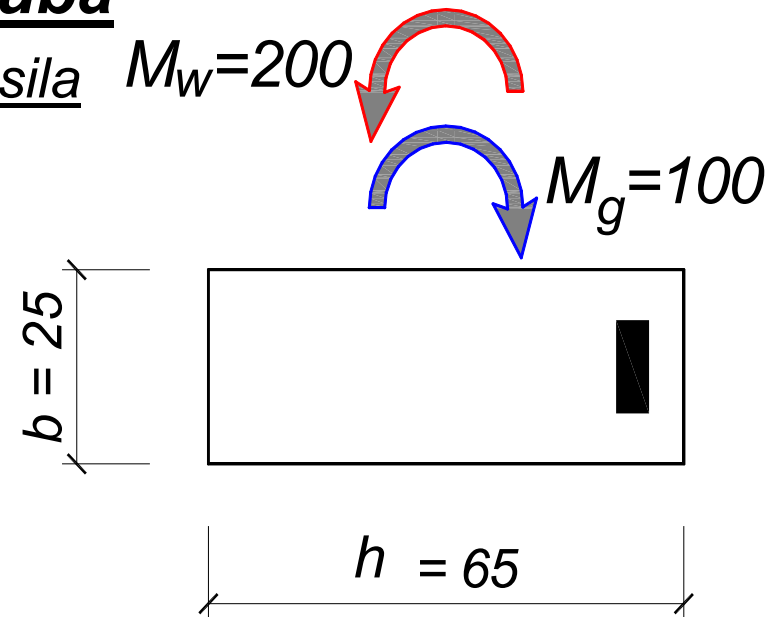
b.1 Maksimalni momenat - MINIMALNA sila $M_w=200$
pritiska $M_g=100$

$$M_{Ed} = 1.0 \times (-100) + 1.5 \times 200 = 200 \text{ kNm}$$

$$N_{Ed} = 0$$

$$\text{pretp. } d_1 = 5 \text{ cm} \Rightarrow d = 65 - 5 = 60 \text{ cm}$$

$$C25/30 \Rightarrow f_{cd} = 14.2 \text{ MPa}$$



$$k = \frac{60}{\sqrt{\frac{200 \times 10^2}{25 \times 1.42}}} = 2.528$$

$$\varepsilon_c / \varepsilon_{s1} = 3.5 / 13.18\text{‰} \Rightarrow \omega_1 = 16.982\%$$

$$A_{s1} = 16.982 \times \frac{25 \times 60}{100} \times \frac{1.42}{43.5} = 8.31 \text{ cm}^2$$

b. zategnuta spoljašnja ivica stuba

b.2 Maksimalni momenat - **MAKSIMALNA** sila pritiska
dominantno promenljivo – VETAR

$$1,35 \cdot G_k + 1,5 \cdot (F_{k,w} + 0,5 \cdot Q_{k,n} + 0,7 \cdot Q_{k,es}) = 1,35 \cdot G_k + 1,5 \cdot F_{k,w} + 0,75 \cdot Q_{k,n} + 1,05 \cdot Q_{k,es}$$

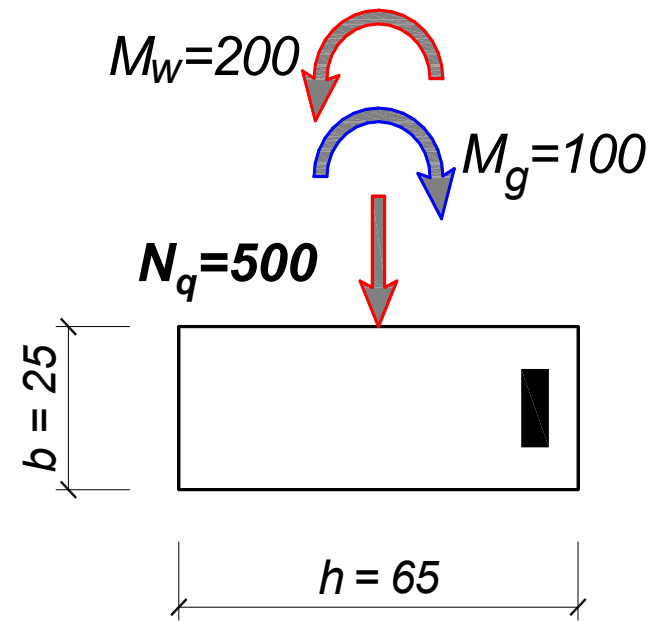
$$M_{Ed} = 1.0 \times (-100) + 1.5 \times 200 = 200 \text{ kNm}$$

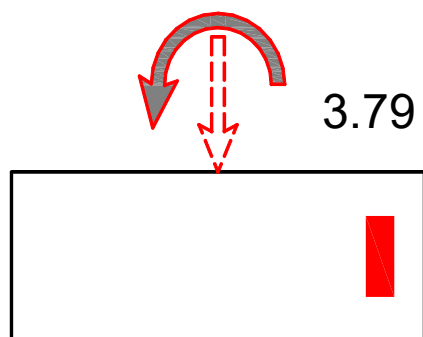
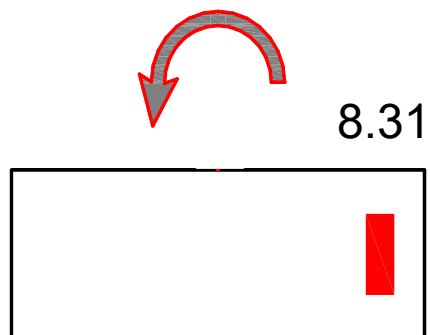
$$N_{Ed} = 1.5 \times 0.7 \times 500 = 525 \text{ kN}$$

$$M_{Eds} = 200 + 525 \times \left(\frac{0.65}{2} - 0.05 \right) = 443.4 \text{ kNm}$$

$$k = \frac{60}{\sqrt{\frac{344.4 \times 10^2}{25 \times 1.42}}} = 1.926 \Rightarrow \begin{aligned} \varepsilon_c / \varepsilon_{s1} &= 3.5 / 5.247\text{‰} \\ \omega &= 32.391\% \end{aligned}$$

$$A_{a1} = 32.391 \times \frac{25 \times 60}{100} \times \frac{1.42}{43.5} - \frac{525}{43.5} = 3.79 \text{ cm}^2$$





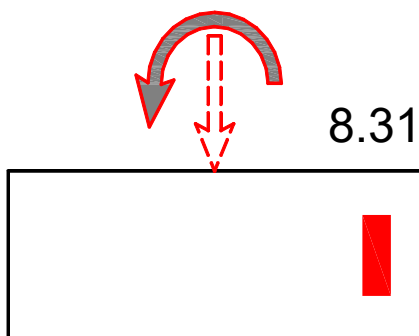
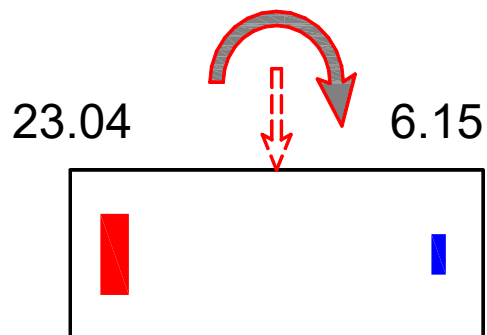
POTREBNO:

8.31



spolja: $A_{s, \text{potr.}} = 0$

unutra: $A_{s, \text{potr.}} = \max. \left\{ \begin{matrix} 8.31 \\ 3.79 \end{matrix} \right\} = 8.31 \text{ cm}^2$



POTREBNO:

23.04 8.31



$$\text{spolja: } A_{s, \text{potr.}} = \max. \begin{Bmatrix} 23.04 \\ 0 \end{Bmatrix} = 23.04 \text{ cm}^2$$

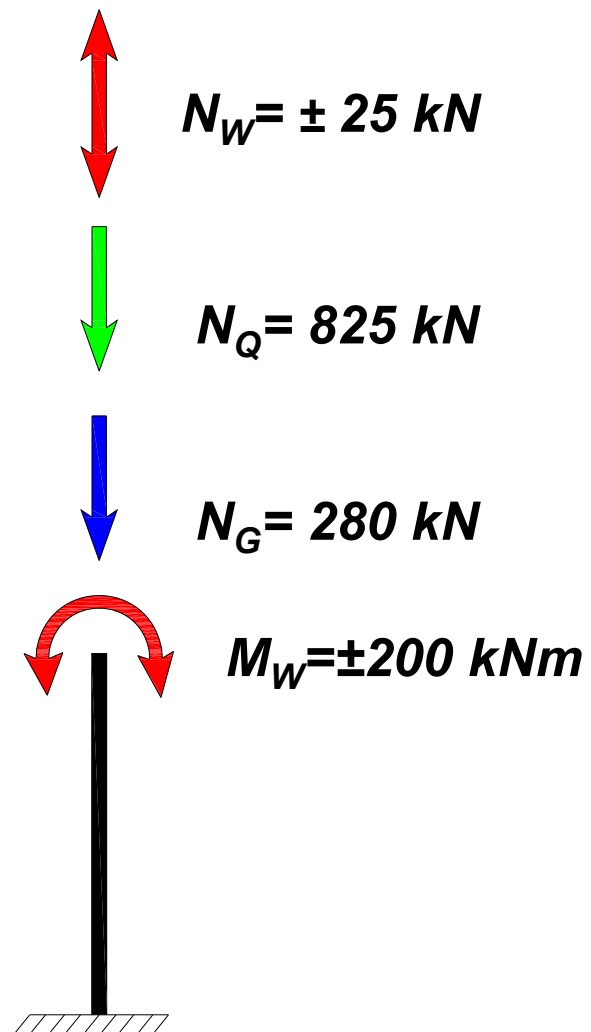
5Ø25 (24.55 cm²)

$$\text{unutra: } A_{s, \text{potr.}} = \max. \begin{Bmatrix} 6.15 \\ 8.31 \end{Bmatrix} = 8.31 \text{ cm}^2$$

2Ø25 (9.82 cm²)

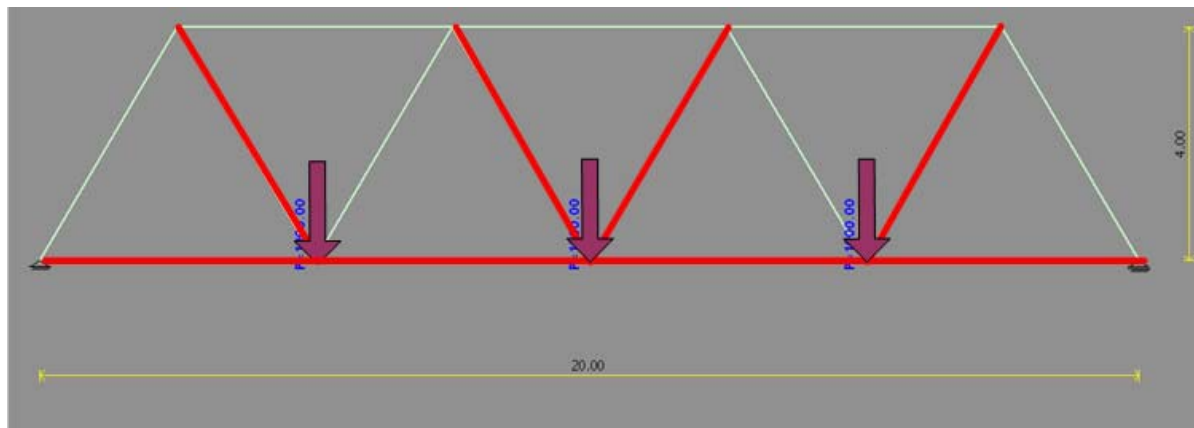
DODATNI ZADATAK 1

Dimenzionisati stub prikazan na skici.



C30/37	B500 B
b/h=30/60 cm	

Centrično zatezanje



Zadatak 17 – CENTRIČNO ZATEZANJE

Odrediti potrebnu površinu armature i oblikovati poprečni presek, pravougaonog oblika, centrično zategnutog elementa. Podaci za proračun:

$$N_{G,k} = -400 \text{ kN}$$

C25/30

XD1

$$N_{Q,k} = -500 \text{ kN}$$

B500 B

Treba li?

C35/45  $f_{cd} = 0.85 \cdot 35 / 1.5 = 19.8 \text{ MPa} = 1.98 \text{ kN/cm}^2$

B500 B  $f_{yd} = 500 / 1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$



Zadatak 17 – CENTRIČNO ZATEZANJE

Granična sila ZATEZANJA:

$$N_{Ed} = 1.35 \cdot (-400) + 1.5 \cdot (-500) = -1290 \text{ kN}$$

Proračun površine armature:

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} = \frac{1290}{43.5} = 29.7 \text{ cm}^2$$

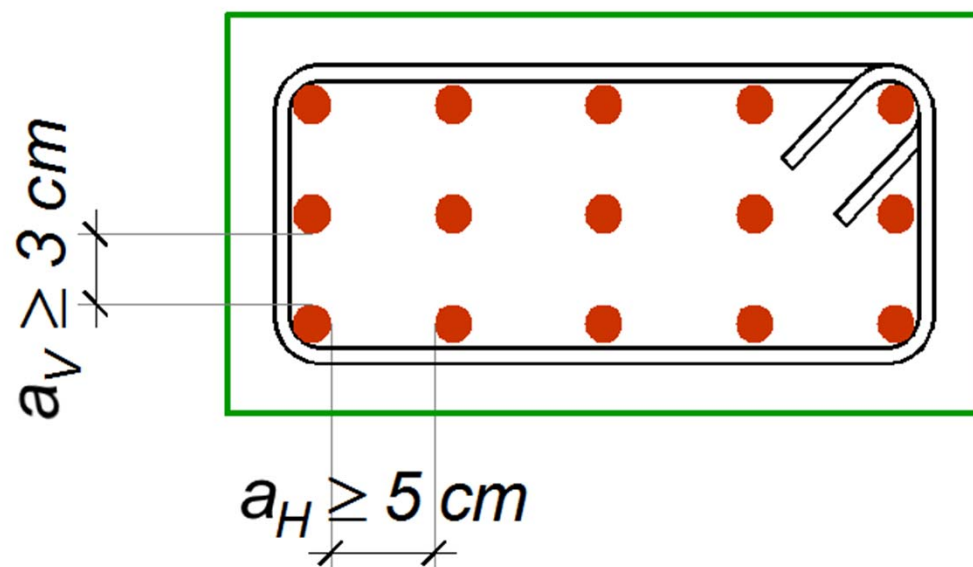
Usvojeno: 15 Ø16 (30.15 cm²)



Zadatak 17 – CENTRIČNO ZATEZANJE

$$b \geq 2c_{\text{nom}} + 2\varnothing_s + m \times \varnothing + (m-1) \times a_H$$

$$h \geq 2c_{\text{nom}} + 2\varnothing_s + n \times \varnothing + (n-1) \times a_V$$



XD1 $\Rightarrow c_{\text{nom}} = 35 + 10 = 45 \text{ mm}$

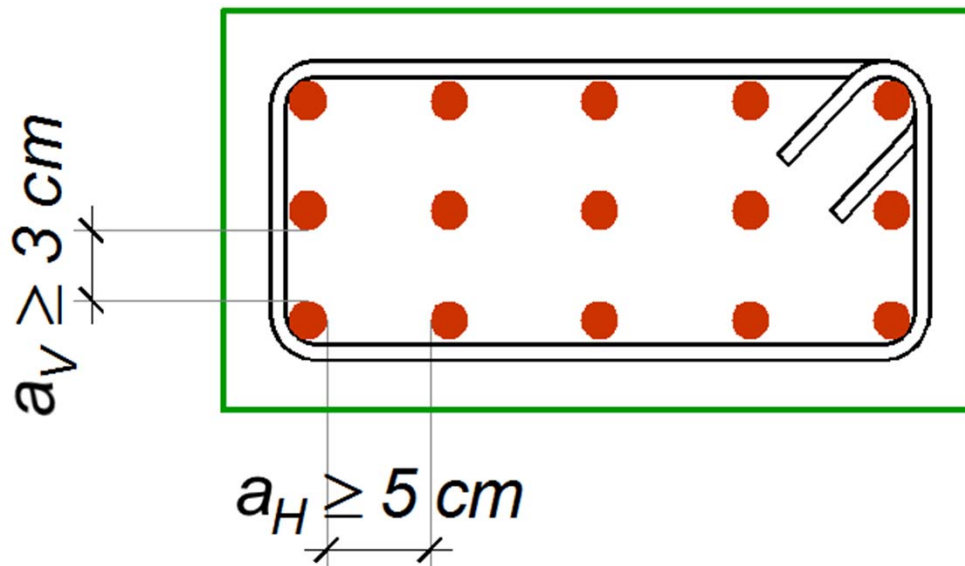
Pretp. $\Rightarrow \varnothing_s = 8 \text{ mm}$

Oznaka klase izloženosti	Minimalni zaštitni slojevi iz uslova trajnosti, <i>c_{min,dur}</i> , za klasu konstrukcija S4									
	10	15	20	25	30	35	40	45	50	55
X0										
XC1										
XC2										
XC3										
XC4										
XD1										
XD2										
XD3										
XS1										
XS2										
XS3										



Zadatak 17 – CENTRIČNO ZATEZANJE

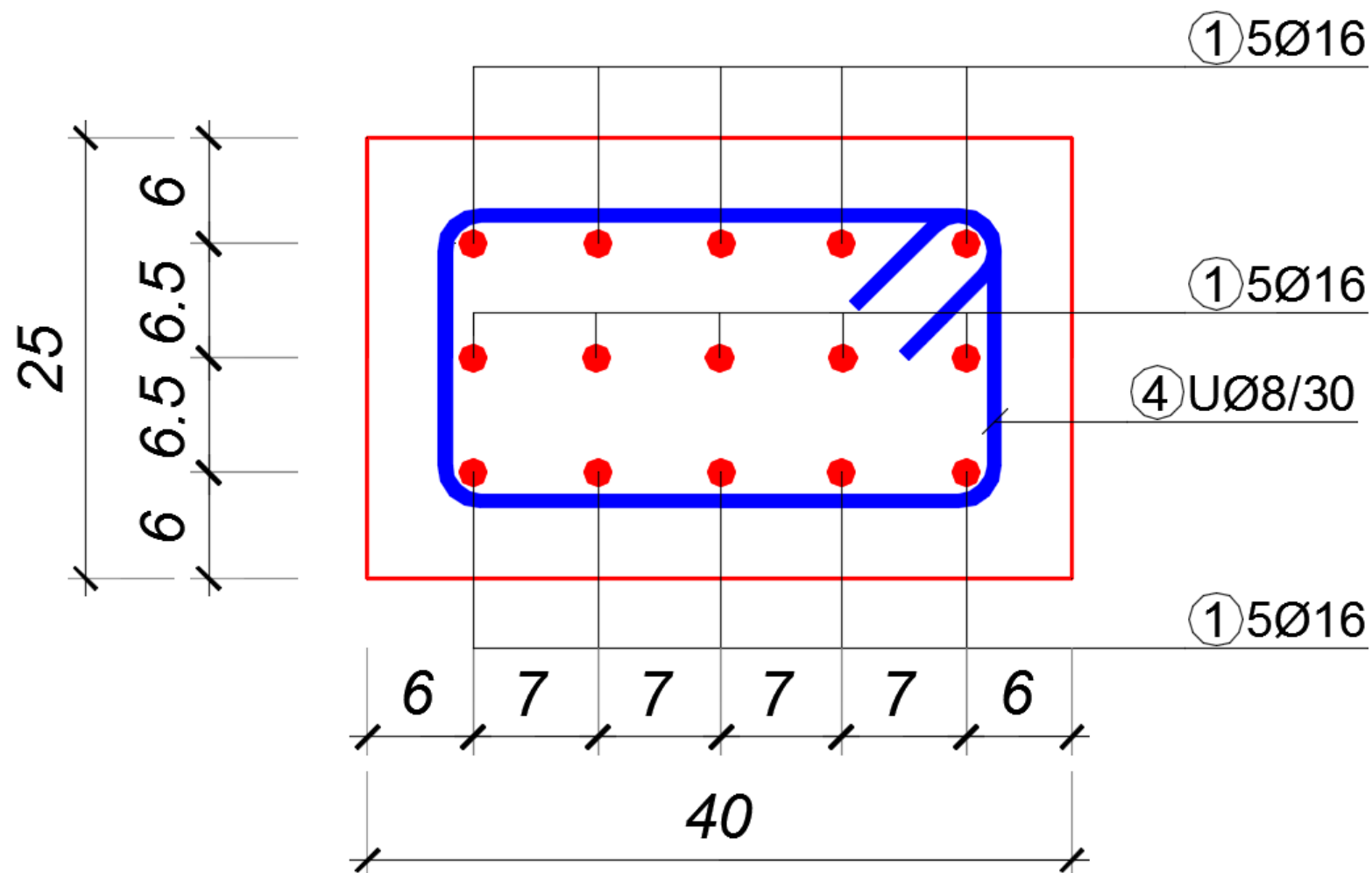
$$\begin{aligned}
 b &\geq 2c_{\text{nom}} + 2\varnothing_s + m \times \varnothing + (m-1) \times a_H \\
 h &\geq 2c_{\text{nom}} + 2\varnothing_s + n \times \varnothing + (n-1) \times a_V
 \end{aligned}$$



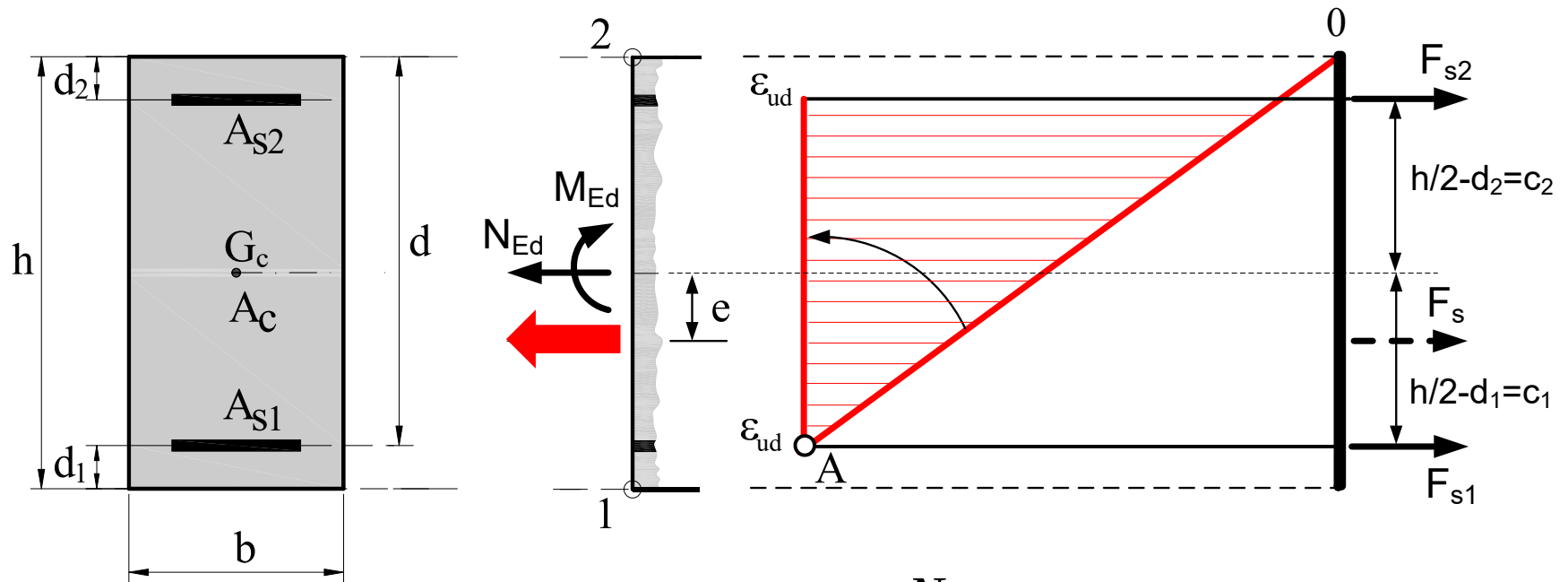
$$b \geq 2 \times 4.5 + 2 \times 0.8 + 5 \times 1.6 + (5-1) \times 5.0 = 38.6 \text{ cm} \Rightarrow b = 40 \text{ cm}$$

$$h \geq 2 \times 4.5 + 2 \times 0.8 + 3 \times 1.6 + (3-1) \times 3.0 = 21.4 \text{ cm} \Rightarrow h = 25 \text{ cm}$$

Zadatak 17 – CENTRIČNO ZATEZANJE



ULS – MALI EKSCENTRICITET, SILA ZATEZANJA

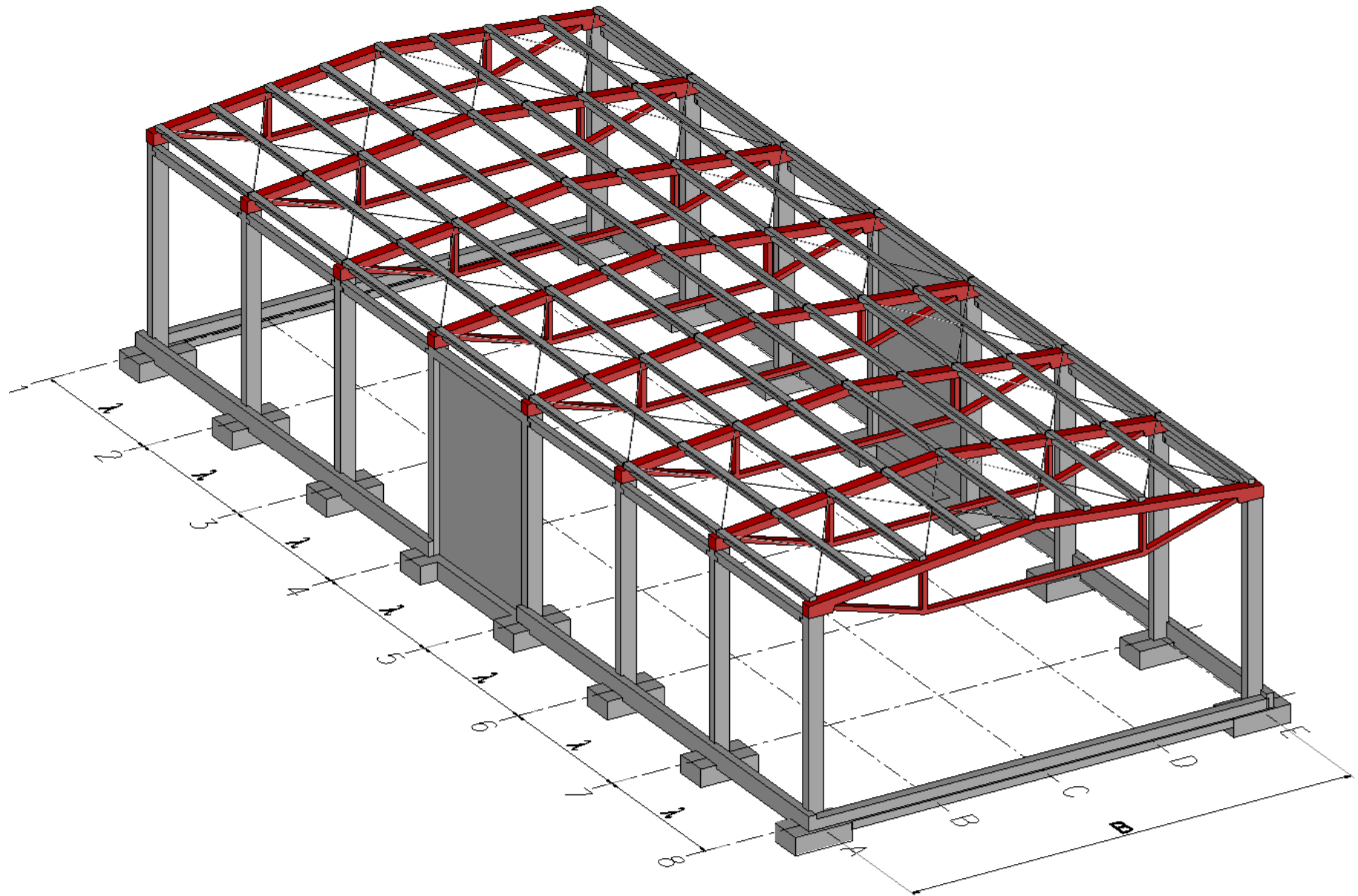


$$A_s = A_{s1} + A_{s2} = \frac{N_{Ed}}{f_{yd}}$$

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2}$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2}$$

ULS – MALI EKSCENTRICITET, SILA ZATEZANJA



Zadatak 18 – **EKSCENTRIČNO ZATEZANJE**

Dimenzionisati pravougaoni poprečni presek zadatih dimenzija, opterećen silom zatezanja i momentom savijanja u fazi malog ekscentriciteta.

Podaci za proračun:

$$N_{G,k} = -400 \text{ kN}$$

$$M_{G,k} = 10 \text{ kNm}$$

$$b = 40 \text{ cm}$$

$$N_{Q,k} = -500 \text{ kN}$$

$$h = 25 \text{ cm}$$

B500 B

$$f_{yd} = 500/1.15 = 435 \text{ MPa} = 43.5 \text{ kN/cm}^2$$



Zadatak 18 – **EKSCENTRIČNO ZATEZANJE**

$$N_{Ed} = 1.35 \cdot (-400) + 1.5 \cdot (-500) = -1290 \text{ kN}$$

$$M_{Ed} = 1.35 \cdot 10 = 13.5 \text{ kNm}$$

$$e = \frac{M_{Ed}}{N_{Ed}} = \frac{13.5 \times 10^2}{1290} = 1.05 \text{ cm}$$

$$\text{pretp. } d_1 = d_2 = 6 \text{ cm}$$

$$c_1 = c_2 = h/2 - d_1 = 25/2 - 6 = 6.5 \text{ cm}$$

$$A_s = A_{s1} + A_{s2} = \frac{N_{Ed}}{f_{yd}} = \frac{1290}{43.5} = 29.7 \text{ cm}^2$$

Zadatak 18 – EKSCENTRIČNO ZATEZANJE

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 + 1.05}{6.5 + 6.5} = 17.2 \text{ cm}^2$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 - 1.05}{6.5 + 6.5} = 12.5 \text{ cm}^2$$

≈ 9Ø16

$$d_1 = 8 \text{ cm}$$

$$c_1 = 25/2 - 8 = 4.5 \text{ cm}$$

≈ 6Ø16

$$d_2 = 6 \text{ cm}$$

$$c_2 = 25/2 - 6 = 6.5 \text{ cm}$$



Zadatak 18 – EKSCENTRIČNO ZATEZANJE

$$A_{s1} = \frac{N_{Ed}}{f_{yd}} \frac{c_2 + e}{c_1 + c_2} = 29.7 \cdot \frac{6.5 + 1.05}{4.5 + 6.5} = 20.4 \text{ cm}^2$$

$$A_{s2} = \frac{N_{Ed}}{f_{yd}} \frac{c_1 - e}{c_1 + c_2} = 29.7 \cdot \frac{4.5 - 1.05}{4.5 + 6.5} = 9.3 \text{ cm}^2$$

≈ 10Ø16

$$d_1 = 8.25 \text{ cm}$$

$$c_1 = 25/2 - 8.25 \\ = 4.25 \text{ cm}$$

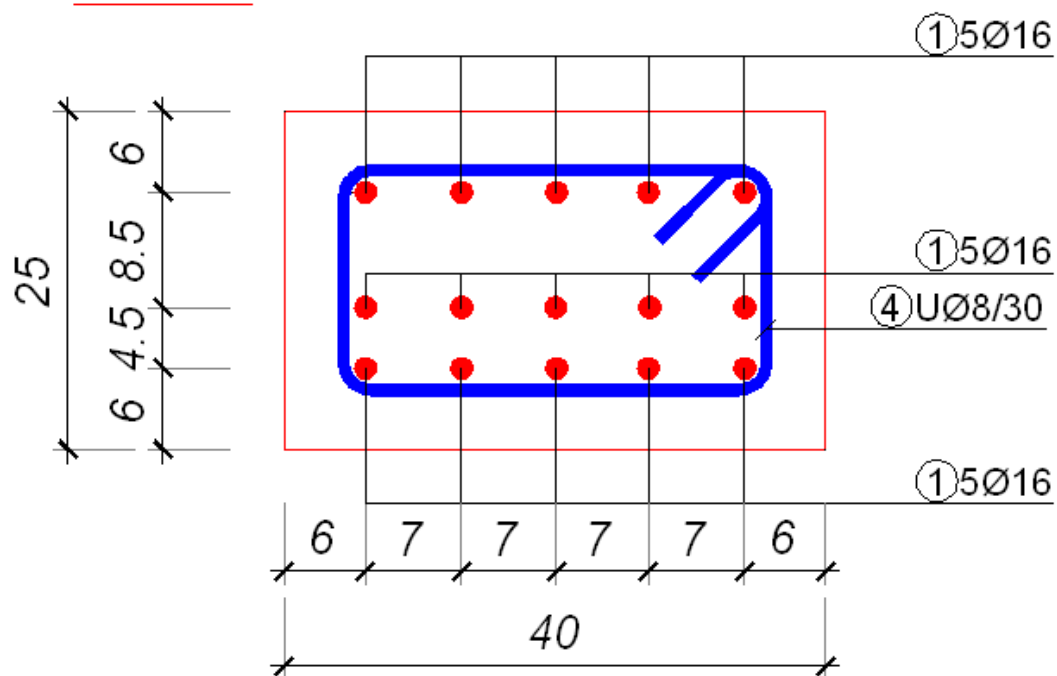
≈ 5Ø16

$$d_2 = 6 \text{ cm}$$

$$c_2 = 25/2 - 6 \\ = 6.5 \text{ cm}$$

Zadatak 18 – EKSCENTRIČNO ZATEZANJE

XD1



$$d^I = c_{\text{nom}} + \varnothing_s + \varnothing/2$$

$$a^I = 4.5 + 0.8 + 1.6/2 = 6.1 \text{ cm}$$

$$\text{usv. } d^I = 6 \text{ cm}$$

$$d^{II} = d^I + a_v + 2 \times \varnothing/2$$

$$d^{II} = 6 + 3.0 + 2 \times 1.6/2 = 10.6 \text{ cm}$$

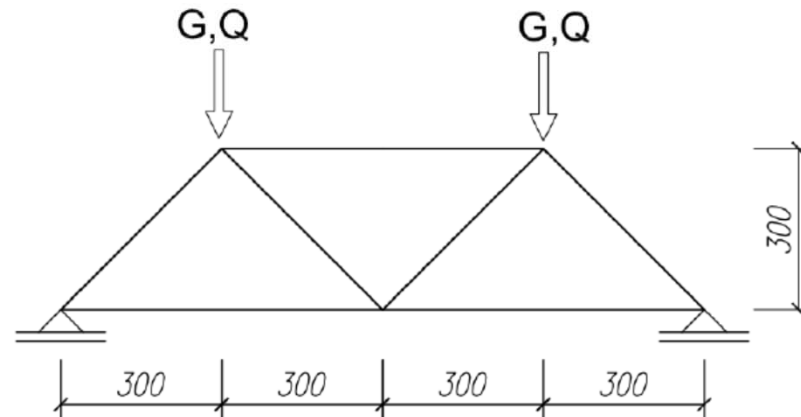
$$\text{usv. } d^{II} = 10.5 \text{ cm}$$

$$a_1 = (5 \times 6 + 5 \times 10.5)/10$$

$$d_1 = 8.25 \text{ cm}$$

3. Dimenzionisati donji pojas rešetke prikazane na skici. Skicirati usvojeni raspored armature u poprečnim presecima uz odgovarajuće oznake i kote.

$G=350\text{kN}$ $Q=200\text{kN}$ C25/30 B500B XC1



3. Zatega preseka prema skici, opterećena je silom $Z_G=600\text{ kN}$ usled stalnog opterećenja. Odrediti koliku silu usled povremenog opterećenja zatega može prihvatiti uz zadovoljenje propisanih koeficijenata sigurnosti.

